# Pressure-gradient-induced Alfvén eigenmodes destabilized by ion temperature gradient

A. Bierwage[1,2,*], L. Chen[1,3], F. Zonca[2]

[1] *University of California, Irvine, USA*

[2] *Associazione EURATOM-ENEA sulla Fusione, Frascati, Rome, Italy*

[3] *IFTS, Zhejiang University, Hangzhou, China*

[*] *Present address: Japan Atomic Energy Agency, Naka, Japan*

In a magnetized plasma, kinetic particle compression or external forcing can excite shear Alfvén waves (SAW) [1]. These waves cover regions of the frequency and wavelength spectrum where oscillations of the ion polarization current (acting as inertia) and the $\boldsymbol{J} \times \boldsymbol{B}$ force (acting as restoring force) are in near balance. In an idealized plasma, assumed to be uniform and of infinite extent, these are plane waves that propagate along the magnetic field with phase velocity $v_{\rm A} = B/\sqrt{\mu_0 m_{\rm i} n_{\rm i}}$. In most realistic situations, the non-uniformity of finite-sized plasmas plays an important role. Parallel non-uniformities (along the magnetic field), such as magnetic field line curvature and variations of the magnetic field strength, create effective potential barriers and wells. These affect the propagation of SAWs with parallel wave numbers $k_{\|}$ that match the typical scales of the potential landscape. In addition, perpendicular non-uniformities (across the magnetic field), such as magnetic shear or gradients in plasma density and temperature, give rise to a continuous SAW spectrum and, through symmetry breaking and further modulation of the potential structure, facilitate the localization of wave energy both along and perpendicular to the magnetic field. The resulting standing waves are known as discrete Alfvén eigenmodes.

In the present work, we are interested in the resonant excitation of Alfvénic eigenmodes with short perpendicular wavelength (comparable to the ion gyroradius) via kinetic compression of thermal ions. We consider a toroidally confined high-beta plasma as envisioned for thermonuclear fusion applications. The kinetic interaction between thermal ions and SAWs becomes important near the first ideal-ballooning-stability boundary [2] and continues to play a significant role throughout the domain of second stability, as is illustrated in Fig. 1. Results of a detailed study of ion-temperature-gradient (ITG)-driven Alfvénic modes were recently reported in two companion

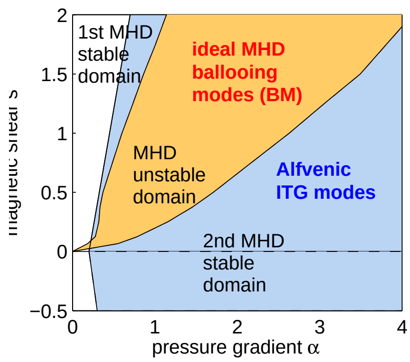


Figure 1: Schematic $s$-$\alpha$ diagram for ideal MHD stability and (blue) regions with Alfvénic ITG activity.

papers [3, 4]. The purpose of this synthetic work is to complement those lengthy articles through a concise summary of the results, with an emphasis on the discussion of their physical meaning.

On a general level, it is interesting to note that dealing with ITG-driven Alfvénic eigenmodes means that we study collective modes in which the same particle species provides the wave's inertia and the resonant drive (and damping). Ion Landau resonances (the mechanism behind ITG drive) are effective only for waves which have perpendicular wavelengths comparable to the ion gyroradius (see Figs. 4–6 in [4]). Recalling the definition of SAWs given in the first paragraph above, this implies that we examine a regime where fluctuations in the diamagnetic current become important and the usual notion of SAW must be generalized: besides the fact that resonant wave-particle interactions affect a part of the ion distribution, non-resonant ion behaviors may vary significantly with the fluctuation wavelength. In the model equations used (see Eqs. (6)–(8) in [4]), this effect is captured by the decay of finite-Larmor-radius (FLR) factors $J_0^2(\lambda)$ (Bessel function) with increasing $\lambda = k_\perp \rho_\text{i} \gtrsim 1$ (perpendicular wave number times thermal ion gyroradius). One such factor also multiplies the ion polarization term. Only modes with frequencies larger than the effective diamagnetic frequency largely conserve their pure SAW character, while experiencing ITG-drive from a relatively small population of ions in the tail of the distribution function (see Fig. 9 in [4]). However, for frequencies close to the diamagnetic frequency, both the above-mentioned FLR effects and kinetic compression begin to affect the eigenfrequency, so that they may alter the physical nature of the mode. The specific target of this study is the second ideal-MHD stable domain, where so-called $\alpha$-induced toroidal Alfvén eigenmodes (in short, $\alpha$TAE) are known exist [5]. $\alpha$TAEs are finite life-time bound states trapped between potential barriers induced by a large pressure gradient (parametrized by $\alpha$). The problem is formulated in "ballooning space", $-\infty < \theta < \infty$, where the SAW equation can formally be written in Schrödinger form,

$$0 = \frac{\mathrm{d}^2\Psi_\text{s}}{\mathrm{d}\theta^2} - V_\text{eff}(\theta|\omega)\delta\Psi_\text{s}, \quad \text{with} \quad \frac{\partial}{\partial t} \rightarrow -i\omega = -i\omega_\text{r} + \gamma; \tag{1}$$

where $V_\text{eff}$ is the effective potential for a given frequency $\omega = \omega_\text{r} + i\gamma$. In ideal MHD, we have $V_\text{eff} = V(\theta|s,\alpha) - \omega^2$, with $\omega^2$ purely real. Due to the limited height and narrow width of the potential barriers, only a small number of these trapped waves are strictly bound states; meaning, they are localized between turning points that lie on the real-$\theta$ axis of the system's Stokes diagram (cf. Figs. 3 and 4 in [3]). However, the transition from weakly bound to strongly bound modes is smooth: when the pressure gradient $\alpha$ is increased, the barriers grow and various bands of $\alpha$TAEs gradually emerge from the continuum as distinct discrete eigenmodes. The formation of two bands of $\alpha$TAE ground states is shown in Fig. 2(a).

For the interpretation of gyrokinetic simulation results for Alfvénic ITG instabilities it is important to understand how FLR effects alter the ideal-MHD nature of modes in the absence of kinetic compression. For $\alpha$TAEs, this was studied in [3] [preceded by numerous works examining FLR ballooning modes (BM)].

One important observation that can be made when comparing the $\alpha$-dependence of the eigenfrequencies $\omega_r$ in the ideal-MHD limit [Fig. 2(a)] and FLR MHD [Fig. 2(b)] is the following. For FLR MHD, the (numerical) threshold $\alpha_{num}$, beyond which a discrete eigenmode can be found inside the continuum, is reduced and the curve $\omega_r(\alpha)$ exhibits a "kink" at some value $\alpha_0 > \alpha_{num}$. Our interpretation of this phenomenon is that $\alpha_0$ marks the transition between weakly and strongly bound $\alpha$TAEs. The attribute "strongly bound" indicates that frequencies and mode structures in the regime $\alpha > \alpha_0$ are determined by the turning points of the $\alpha$-induced potential barriers. For weakly bound $\alpha$TAEs, found in the regime $\alpha < \alpha_0$, this issue is less clear-cut. The mode structure still exhibits components which are localized between potential barriers; however, the frequency has an $\alpha$-dependence which is very similar to that of the diamagnetic frequency and unlike that of strongly bound $\alpha$TAEs. This suggests that the diamagnetic drift is the main factor which sets the eigenfrequency. Thus, the attribute "weakly bound" implies a weak definition of the eigenmode as an "$\alpha$TAE" with respect to two factors: besides the weaker role of the $\alpha$-induced barriers, the importance of FLR effects also implies a weaker pure SAW character; gradually transforming it into a "drift-Alfvén mode." The mode may be loosely thought of as "a localization of wave energy between partially reflecting barriers," which turns into a (strongly bound) $\alpha$TAE when the reflectivity of the barriers becomes sufficiently high (at $\alpha \approx \alpha_0$). The fact that $\omega_r(\alpha)$ scales as the diamagnetic frequency $\omega_{*pi}$ for $\alpha < \alpha_0$ leads us

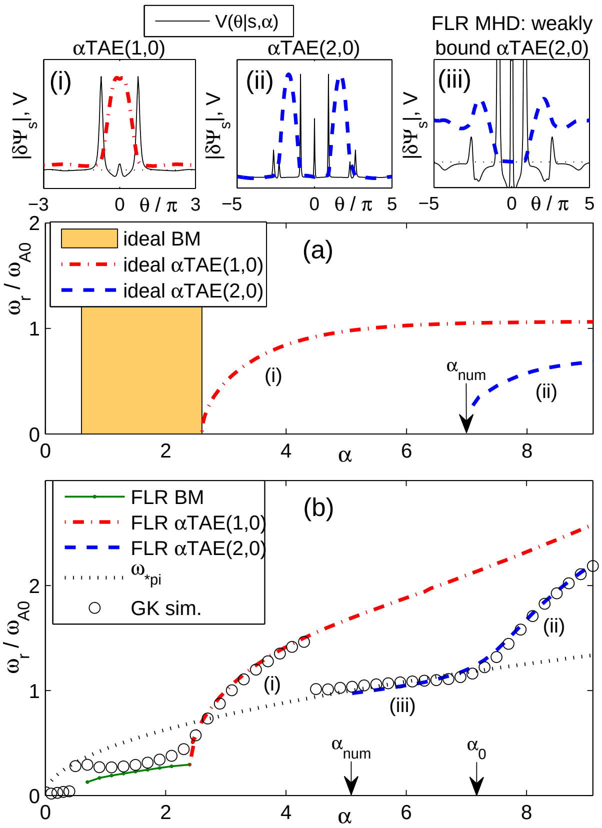


Figure 2: Frequencies of two $\alpha$TAE branches as functions of normalized pressure gradient $\alpha$ for magnetic shear $s = 1.0$. (a) ideal MHD, (b) FLR MHD and gyrokinetic (GK) simulation results. Mode structures $\delta\Psi_s$ and ideal-MHD Schrödinger potential $V$ are shown in (i)–(iii). See also Figs. 1 and 5 in [3], and Fig. 2 in [4].

to conjecture that weakly bound $\alpha$TAEs trace the frequency of the accumulation point of the effective diamagnetic gap, where continuum damping is minimal. Note that the diamagnetic frequency shift is mode-structure-dependent when FLR effects are important (see Fig. 8 in [4]), which is why we speak of an "effective" diamagnetic gap.

When kinetic compression of thermal ions is included, gyrokinetic (GK) simulations show the existence of ITG-driven instabilities in the second ideal-MHD ballooning stable domain, similar to the Alfvén ITG excitation mechanism already known in the first ballooning stable region [2]. Our FLR MHD results allow us to clearly identify these instabilities as resonantly driven $\alpha$TAEs: both the mode structures and frequencies obtained with FLR MHD and GK simulations match well [4]. Furthermore, it is shown that the dominant instability tends to minimize the variation of the ideal MHD potential energy, $\delta W_\mathrm{f}$ (see Figs. 7 and 8 in [4]).

At first glance, it is surprising to find a good match between the frequency traces $\omega_\mathrm{r}(\alpha)$ obtained with FLR MHD and GK simulations, which are reproduced in Fig. 2(b). Scans of the normalized wavenumber $k_\vartheta\rho_\mathrm{i}$ (see Figs. 4, 5 and 8 in [4]) actually suggest that kinetic compression should contribute a significant frequency shift: in the limit $k_\vartheta\rho_\mathrm{i} \to 0^+$ it amounts to about 20–30% of the mode frequency obtained for $k_\vartheta\rho_\mathrm{i} = 0.2$. In one case, we even find evidence of an $\alpha$TAE entering the kinetic thermal ion (KTI) gap: the propagating component of the mode structure vanishes (Fig. 5(f) in [4]). Yet, despite the fact that both the diamagnetic gap and the KTI gap scale as $\sqrt{\alpha}$, the results reproduced in Fig. 2(b) show that the mode frequencies obtained with the GK simulations do not diverge from the frequencies obtained in the FLR MHD study. A possible explanation is provided in the Appendix of [4]: depending on the mode structure, the KTI gap may effectively vanish when $k_\vartheta\rho_\mathrm{i} > \mathscr{O}(0.1)$.

In summary, ITG instabilities in the second ideal-MHD ballooning stable domain were identified as resonantly driven $\alpha$TAEs. Emphasis was put on how the mode frequency is set up, and several peculiar features were discussed which arise when Alfvénic modes feel the effect of finite ion Larmor radii. The Alfvénic ITG modes studied here are expected to be relevant for very high beta plasmas in spherical tori and future reactor-grade thermonuclear fusion devices.

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