Validation Procedure of the Tokamak Equilibrium Reconstruction Code EQUAL with a Scientific Workflow System

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**See the Appendix of F. Romanelli et al., Proceedings 22nd IAEA Fusion Energy Conference 2008, Geneva, Switzerland*

The equilibrium reconstruction code EQUAL (EQUilibrium AnaLysis) is based on the algorithm of EFIT [1] and solves the Grad-Shafranov equation using data from magnetic, MSE and Faraday diagnostics to determine the unknown current profile in the plasma. The code has been developed within the European Task Force on Integrated Tokamak Modelling (ITM-TF), which aims at providing a framework of validated codes for simulation, preparation and analysis of discharges for the ITER device and existing fusion machines. Verification and Validation (V&V) is a key component of the ITM-TF activity, and the simulation infrastructure developed by the ITM-TF has been designed with this in mind. ITM-TF codes are independent of a particular device and interact with each other via predefined data structures [2]. EQUAL is the first of these codes using the simulation infrastructure based on Kepler [http://www.kepler-project.org] for validation, illustrated in figure 1.

The EQUAL algorithm

Neglecting anisotropy, plasma flow and assuming toroidal symmetry of the tokamak and the discharge, the poloidal flux Ψ(R,Z) is given by the Grad-Shafranov equation

$$-\frac{\partial^2\Psi}{\partial R^2}+\frac{1}{R}\frac{\partial\Psi}{\partial R}-\frac{\partial^2\Psi}{\partial Z^2}=\mu_0 R^2 P'(\Psi)+F(\Psi)F'(\Psi)+\mu_0 R J_{ext}(R,Z) ,$$

where P denotes kinetic pressure, $F=RB_{tor}$ with B_{tor} as toroidal magnetic field, and J_{ext} the current distribution of sources outside the plasma. The profiles P' and FF' are parameterised as a linear superposition of suitable test functions. External sources as ferromagnetic materials are modelled with a superposition of sources with known geometry.

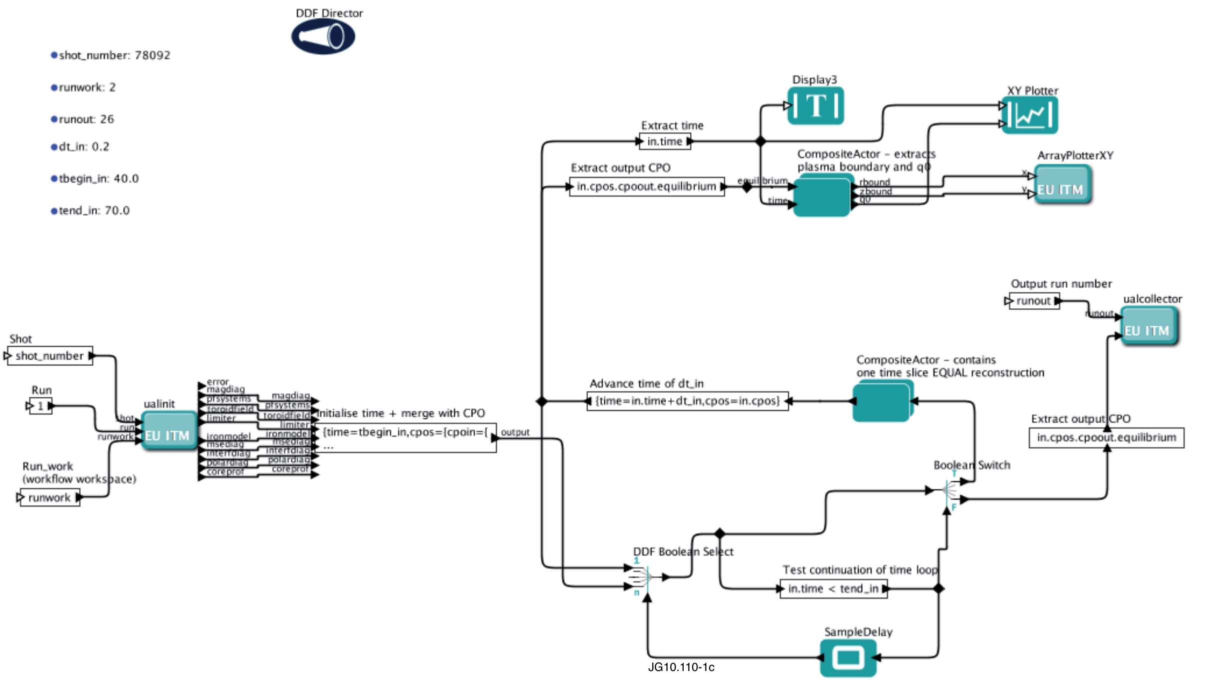

Figure 1. Kepler workflow for time-dependent analysis of a complete discharge with the code EQUAL. The blue boxes represent actors fulfilling complex tasks and can be a complete analysis code, while the rectangles represent parameters or simple actions. The actor "ualinit" retrieves shot data from the ITM database and feeds the data to the time loop in the lower right part of the figure, comprising EQUAL within the composite actor. The top right part shows the plotting facilities of Kepler, displaying the plasma boundary in real time and the central safety factor as a function of time.

The unknown coefficients $\underline{x}$ are determined by minimising the least squares functional

$$\chi^2 = \sum_{m=1}^{N_{meas}} \left(\frac{F_m^{calc}\{\Psi, \underline{x}\} - F_m^{meas}}{\sigma_m}\right)^2 + \lambda^2 \Re(\underline{x})$$

with the Grad-Shafranov type equation (1) as a constraint. F_m^{meas} is the measured value, F_m^{calc} the corresponding synthesised measurement, and σ_m the estimated uncertainty. The Tikhonov regularising term $\lambda^2\Re$ controls unphysical oscillations of the current profile. Further details of the algorithm are given in [4]. The program is written in ANSI Fortran 95, uses public domain libraries, and is highly optimised. Individual modules of EQUAL, in particular field solver and routines for computing physical quantities, were tested against analytical results. The error is shown to scale correctly with the grid size, and is usually below 0.1 % for a grid size of 65x65. For this resolution, the CPU time of a typical JET discharge amounts to 0.2 seconds per time point on a 64 bit workstation (1.4 Linpack GFlops). For a high resolution grid of 512x512 grid points, which might be required for studies of pedestals at the boundary, the CPU time is still less than 20 sec.

Benchmarking against EFIT

The first systematic test of EQUAL is the benchmarking against the code EFIT [5] routinely run for JET discharges. Only data from the magnetic diagnostics is being used. The machine description files of the JET device has been set up for discharges 68613-78157, describing magnetic diagnostics, poloidal and toroidal field coil system, ferromagnetic transformer, and first wall, has been reviewed, mapped into the ITM generic tokamak description and put into the ITM database. JET studies [6] identified a series of JET discharges that contain MHD marker data suitable for comparison with EQUAL. Data required for equilibrium identification of 147 discharges has been transferred with the ITM tool exp2ITM [2] to the ITM database. For benchmarking with EFIT, the same number and type of test functions for P' and FF' are used, polynomials up to second order and a suitable regularisation. First tests confirm the strong dependency of the central safety factor on the amount of regularisation. This is easily explained, since the core structure of the equilibrium is not well constrained by magnetic data only [3]. One reference shot was used to adjust the regularisation parameter, which was then applied to the whole set of 147 discharges. The results of both codes have been compared by producing scatter plots on selected common time points. Results for the central safety factor and the position of the magnetic axis are given in figure 2.

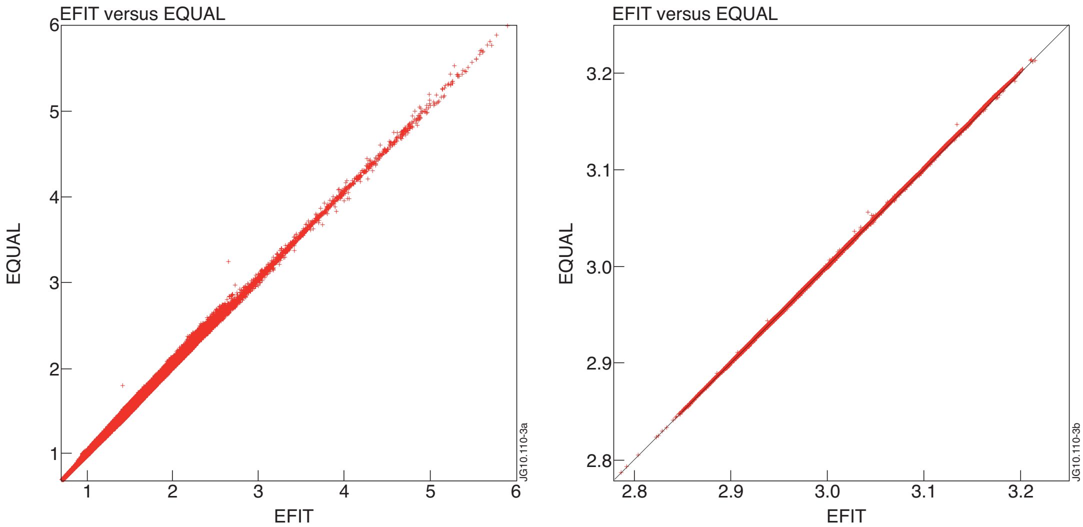

Figure 2. Scatter plot for the comparison of EFIT versus EQUAL, showing the central safety factor (left), and the radial position of the magnetic axis (right).

The result of the comparison of safety factor, position of magnetic axis and X-point, beta poloidal and plasma current are summarised in the table below. There is very good agreement of parameters well constrained by the magnetic diagnostics close to the plasma boundary. The deviation of core related parameters, especially q(0) and β_p, can be explained by differences of the two codes, as different grid size and position, and different algorithms for treating the field contribution of the ferromagnetic transformer.

q(0)	0.05
q(95%)	0.05
Position of magnetic axis	2 mm
Position X-point	1 mm for radial, 3 mm for vertical coordinate
plasma current	1 kA
beta poloidal	0.012

Table 1. Average difference of physical quantities calculated by EQUAL and EFIT

Conclusions and outlook

In the first phase of the validation procedure of the ITM code EQUAL a benchmark comparison was performed against results from the JET code EFIT, giving very good agreement. The geometric and discharge data have been retrieved with the formalised procedure according to ITM standards. Note that since the ITM data description is machine generic, the workflow discussed here can readily be used to process discharges from any other tokamak. The next stage of the validation procedure will use internal plasma data from Faraday and MSE diagnostics to compare with rational surfaces of the safety factor profile obtained from the analysis of MHD instabilities.

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