Reduced kinetic model for relativistic electron transport in dense plasmas

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An accurate description of the transport of high currents of relativistic electrons in dense matter is an important issue for many applications including the fast ignition of thermonuclear fusion targets. A very large difference in energies and densities between the electron beam and the plasma electrons suggests to consider them as two different populations that exchange the energy and particles due to collisions. We developed a new reduced model of two coupled electron sub-systems by using an operator decomposition technique, where the collision operators are interpreted in a systematic manner [1]. The process of energy exchange is described in the Landau-Fokker-Planck (LFP) approximation, where the pitch angle electron-ion and electron-electron collisions dominate. The process of particle exchange between populations, leading to the production of secondary energetic electrons, is described with a Boltzmann term. We demonstrate that electron-electron collisions with small impact parameters make an important contribution in the overall dynamics of the beam electrons.

We consider a relativistic electron beam propagating through a plasma made of electrons and immobile ions of a charge Ze and a density n_i. The electrons of a beam and a plasma are described by a relativistic kinetic equation [2]

$$\partial_t f_e + \mathbf{v}\cdot\nabla f_e - e\partial_{\mathbf{p}}\cdot(\mathbf{E}+\mathbf{v}\times\mathbf{B}) f_e = C_{ee} + C_{ei} \tag{1}$$

where the electron velocity $\mathbf{v} = \mathbf{p}/m_e\gamma$ and momentum $\mathbf{p}$ are related by the relativistic factor $\gamma = \sqrt{1+p^2/m_e^2c^2}$. We consider here the collisional effects in the right hand side of this equation leaving apart the convective terms and fields in the left hand side. The general form of the relativistic collision operator is:

$$C_{ab}(f_a,f_b) = 2c^2\int d\mathbf{q}\int d\tilde{\Omega}\,\left[f_a(\mathbf{p}')f_a(\mathbf{q}') - f_a(\mathbf{p})f_b(\mathbf{q})\right]\,\tilde{p}\,Q^{ab}\tilde{\varepsilon}/\varepsilon_p\varepsilon_q \tag{2}$$

where $Q^{ab}(\tilde{p},\tilde{\mu})$ is the relativistic Rutherford cross section [3], the momenta $\mathbf{p}$, $\mathbf{q}$ and energies ε_p, ε_q correspond to the outgoing particles, after a collision event, whereas the subscripts prime refer to the ingoing particles, before the same collision event. The conservation of the momentum and energy in collisions is assumed. The quantities marked with tilde refer to the center of mass frame for a collision event, in particular, $\tilde{\mu}$ is the cosine of the scattering angle.

The standard approach in the physics of Coulomb collisions consists in developing the collision integrals in series assuming a small momentum transfer in each collision assuming that

Table 1: List of collisional processes considered for the beam and plasma electrons.

	Entering particle	Collisional process (sa)/(la) scattering	Target particle	Exiting particle	Collision model
Self-thermalization: $\mathrm{ST}^{(sa)}$	thermal	small angle scattering (sa)	thermal	thermal	Fokker-Planck-Landau
Self-thermalization: $\mathrm{ST}^{(la)}$	thermal	large angle scattering (la)	thermal	thermal	neglected
Heating: H	thermal	small angle scattering (sa)	beam	thermal	Fokker-Planck
Ionization gain: IO^{+}	thermal	large angle scattering (la)	beam	beam	Boltzmann gain term
Ionization loss: IO^{-}	thermal	large angle scattering (la)	beam	thermal	Boltzmann loss term
Slowing down: $\mathrm{SD}^{(sa)}$	beam	small angle scattering (sa)	thermal	beam	Fokker-Planck
Slowing down: $\mathrm{SD}^{(la)}$	beam	large angle scattering (la)	thermal	beam	Fokker-Planck
Self-thermalization	beam	any scattering angle	beam	beam	neglected

the Coulomb logarithm, $\ln\Lambda \gg 1$. Here, $\Lambda = \Delta p_{\max}/\Delta p_{\min}$ is the ratio between the largest and smallest momentum transfer in a collision. That reduces the general Boltzmann-like collision integral into the Landau-Fokker-Planck (LFP) differential form containing the friction and diffusion terms in the phase space [2]. The hard collisions with a small impact parameter are neglected in this approach. However, there are conditions where the hard collisions could produce qualitatively new effects that are not accounted for in the LFP approximation. Well-known examples are ionization of atoms or molecules by free electrons in partially ionized plasmas and plasma heating due to electron-ion collisions in a strong laser field [4].

We separate the kinetic equation (1) into the fast, f_b, and slow, f_{th}, components and account for the coupling between them. The characteristic beam electron energy, ε_b, is supposed to be much larger than the mean energy, T_e, of the bulk electron population and the beam density is small, $n_b \ll n_e$. The separation energy between two populations, ε_{cut} is supposed to satisfy the following condition: $T_e \ll \varepsilon_{cut} \ll \varepsilon_b$. Its particular choice depends on the problem.

The processes associated with each of two energy scales are listed in Table 1. Concerning the thermal population, only pitch angle scattering between the thermal particles is taken into account (process $\mathrm{ST}^{(sa)}$). The large angle scattering of the thermal particle ($\mathrm{ST}^{(la)}$) is neglected assuming $1/\ln\Lambda$ as a small parameter. The collisions of the thermal particles with the beam particles give raise to three processes. The small angle scattering (H) increases the energy of the thermal particle, while leaving it in its own population. The large angle scattering (IO) promotes the thermal particle into the beam population (IO^{+}) and, at the same time, the thermal population loses this particle (IO^{-}).

Concerning the beam population, the process $\mathrm{SD}^{(sa)}$ is identified as the small angle scattering on the thermal particles. The process $\mathrm{SD}^{(la)}$ can be interpreted as large angle scattering of beam particles on the thermal particles, where the colliding particles are maintained, in the outgoing channel in their original populations. This appears paradoxal, since the large angle scattering event should promote the thermal particle into the beam population. We solve this by choosing a LFP approach for the process $\mathrm{SD}^{(la)}$, that maintains the collision invariants of the bilinear Boltzmann form. Doing so, we forbid the large energy exchanges for thermal particles. This LFP treatment is valid for the beam particles as well, since the large energy exchanges can still be considered small with respect to the variation of the beam distribution function. Collisions between the beam particles are neglected because of a low beam density. Then our model reduces to the following two equations

$$d_t f_b = C_{IO^+}[f_{th}, f_b] + C_{SD}[f_b, f_{th}], \quad d_t f_{th} = C_{IO^-}[f_{th}, f_b] + C_H[f_{th}, f_b] + C_{ST^{(sa)}}[f_{th}, f_{th}] \quad \text{where}$$
$$C_{IO^+} = 2c^2 \int d\mathbf{q} \int d\tilde{\Omega}\, f_{th}(\mathbf{p}') f_b(\mathbf{q}') \frac{\tilde{p}\tilde{\varepsilon}}{\varepsilon_p \varepsilon_q} Q_f^{(la)}, \quad C_{IO^-} = -2c^2 f_{th}(\mathbf{p}) \int d\mathbf{q} \int d\tilde{\Omega}\, f_b(\mathbf{q}) \frac{\tilde{p}\tilde{\varepsilon}}{\varepsilon_p \varepsilon_q} Q_f^{(la)}.$$

The last three terms have a LFP form: $C_\mathrm{A} = \partial_\mathbf{p} \cdot (\mathbf{F}_\mathrm{A} f_b(\mathbf{p}) + \mathbf{D}_\mathrm{A} \cdot \partial_\mathbf{p} f_b(\mathbf{p}))$, where the subscript A stands for SD, H or $\mathrm{ST}^{(sa)}$, respectively, for the friction force, $\mathbf{F}$, and the diffusion coefficient, $\mathbf{D}$. These two equations are respecting the collision invariants – conservation of the number of particles, momentum, and energy – for the complete distribution function $f_{th} + f_b$.

In particular, for the collisional operator of fast electrons, C_SD, one has: $\mathbf{F}_\mathrm{SD} = F_\mathrm{SD}\mathbf{p}/p$, where $F_\mathrm{SD} = 2\nu_0 m_e c \ln\Lambda/\beta^2$, $\beta = v/c$, $\nu_0 = Ze^4 n_i / 8\pi\varepsilon_0^2 m_e^2 c^3$, and $\mathbf{D}_\mathrm{SD} = D_\mathrm{SD}(\mathbf{I} - \mathbf{p}\otimes\mathbf{p}/p^2)$ with $D_\mathrm{SD} = F_\mathrm{SD}(Z+1)/2\gamma$. In addition, the large angle collisions are responsible for production of secondary high energy electrons:

$$C_{IO^+} = (\nu_0 m_e c^2/2\pi\gamma)(\gamma^2 - 1)^{-1/2} \int_{\varepsilon_{cut}}^{\infty} \int \left[\gamma^2/(\gamma-1)^2 + 1\right] f_b' \delta_E d\varepsilon' d\Omega_{p'} \tag{3}$$

where the function accounting for the energy conservation is defined as $\delta_E = \delta(\mathbf{p}\cdot\mathbf{p}'/pp' - \mu_0)$ and $\mu_0 = \sqrt{(\gamma-1)(\gamma'+1)/(\gamma'-1)(\gamma+1)}$. The integral (3) does not contain the Coulomb logarithm, contrary to the friction and diffusion terms. However, it contains a logarithmic dependence on the low energy limit, and that makes it of the same order as the diffusion and friction terms. This is illustrated below for the case of a mono-energetic electron beam, $f_b(\gamma, \mu, t=0) \propto n_b \delta(\gamma - \gamma_b)\, \delta(\mu - 1)$.

The beam energy is defined as $W_b = \int \varepsilon\, f_b d\mathbf{p}$. The beam energy loss due to pitch-angle collisions with plasma electrons is described by the friction term, $d_t W_b^{sa} = -n_b v_b F(\gamma_b)$. It is proportional to the Coulomb logarithm. The energy loss due to hard collisions with plasma electrons is obtained from the ionization integral (3)

$$d_t W_b^{la} = -\nu_0 n_b \gamma_b (\gamma_b^2 - 1)^{-1/2} \int_{\varepsilon_{cut}}^{\varepsilon_b} \left[(\gamma-1)^{-1} + (\gamma-1)/\gamma_b^2\right] d\varepsilon. \tag{4}$$

The integral over the energy has a logarithmic divergence at the lower limit $\varepsilon_{cut} \ll m_e c^2$, corresponding to small angle collisions. Its contribution could be comparable to the LFP term.

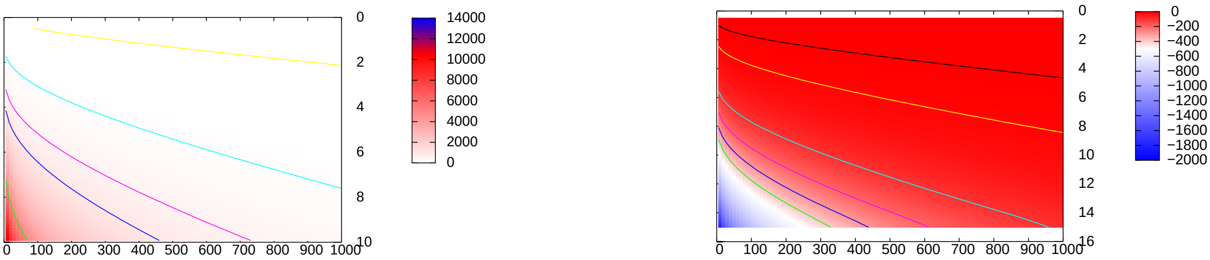

Figure 1: a) Production rate $2(\nu_0 n_b)^{-1} d_t n_b$ of secondary electrons for a beam propagating in a 5 keV plasma as a function of the beam energy (vertical axis in MeV) and the energy cut-off parameter (horizontal axis in keV). Isolines 1, 100, 500, 1000, 5000 mark the corresponding production rates. b) Relative contribution (large over small angle scattering) to the momentum transfer rate. Same beam and plasma parameters. Isolines present the levels of -1, -10, -100, -200, -300, -400 for this ratio.

As an example, we show in Fig. 1-a the ionization rate, that is, the evolution of the number of beam electrons with time for the case of a 5 keV plasma. It decreases if the cut-off energy and it increases with the energy of fast electrons. Both these effects can be easily understood if one accounts for the fact that, even in the pitch angle scattering event, the secondary electron may gain a significant energy. For this reason the choice of the cut-off energy is problem dependent. It must be chosen in such a domain where the dependence of the secondary electron production with respect to the cut-off energy is relatively weak.

We also analyzed the relative contribution of the ionization and slowing down mechanisms to the total momentum gain, $d_t P_b$, where $P_b = \int p_z f_b d\mathbf{p}$. The ratio of the momentum evolution due to the small and large angle collisions, that is, $\int C^{+}_{\rm IO} p_z d\mathbf{p} / \int C_{\rm SD} p_z d\mathbf{p}$, is shown Fig. 1-b. The dependence with respect to the energy cut-off parameter is strong, even if the integral over the energies in P_b is less divergent than the ionization rate. Moreover, this ratio exhibits a negative sign, which implies a positive contribution of secondary electrons into the beam momentum. Large values of this ratio is the signature of the importance of the secondary electron production.

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