Factors influencing the quality of the resonance signal of a hairpin probe in collision-less plasma

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Abstract - The quality of the hairpin resonance signal is investigated as a function of the electron density of the background plasma. Experimental results show that strong attenuation in the resonance signal is observed when the resonance frequency is close to the electron plasma frequency. A qualitative model is presented based on the analysis of the electrical circuit of the resonator which explains the conditions responsible for the attenuation of the resonance signal in the collision-less plasma.

1. Introduction

The hairpin probe is a device for measuring the electron density of a plasma using a U shaped structure which has a characteristic resonance frequency f_r having side band Δf (full width at half maximum) [1-3]. The sharpness of the peak is defined by a parameter called the quality factor Q = $f_r/\Delta f$. Hence the resolution of the electron density measurement is inversely dependent on the width of the resonance peak Δf [3]. When the hairpin probe is in resonance as similar to an L-C-R circuit we observed time varying electric current flowing through the resonator. While the inductance L and capacitance C determines the resonance frequency of the hairpin, the resistance R is a result of the composite resistivity of the probes material and the resistance due to electron-electron and electron-neutral collisions in the plasma. These factors are responsible for the losses in the electro-magnetic energy stored in the resonator resulting in the observed broadening of the resonance peak width Δf. In this paper the experimental results show that the resonance signal is strongly attenuated with increasing electron density as the resonance frequency approach close to the electron plasma frequency.

2. Experimental results

The hairpin probe is a two wire parallel transmission line having length $l = \lambda/4$, which support a standing wave having maximum voltage at the open ends while the time varying current is maximum at the short circuited end of the resonator as shown in fig-1.

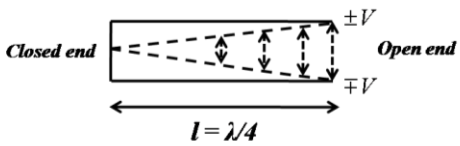

Fig-1: Hairpin showing time-varying electric field distribution between the pins.

The resonance condition in the plasma f_r is related to that obtained in vacuum f_o through the formula,

$$f_r = \frac{c}{4l\sqrt{\kappa_{pr}}} = \frac{f_o}{\sqrt{\kappa_{pr}}} \tag{1}$$

Where, $\kappa_{pr} = 1 - \frac{f_{pe}^2}{f_r^2} = 1 - \eta$ is the plasma permittivity. By substituting the relative plasma permittivity we can obtain a relationship between f_r, f_o and f_{pe} as follows;

$$f_r^2 = f_{pe}^2 + f_o^2 \tag{2}$$

Clearly the resonance frequency f_r is always above the electron plasma frequency f_{pe}.

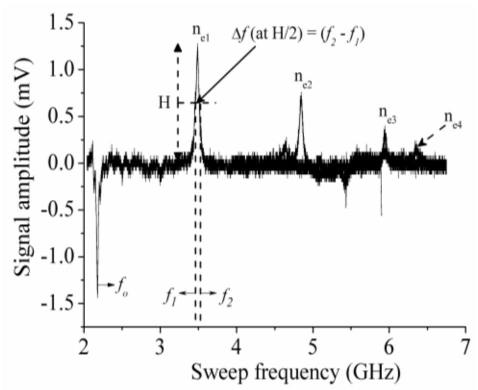

Fig-2: Resonances of hairpin probe obtained at different electron densities in Argon plasma, where (n_{e1}, n_{e2}, n_{e3}, n_{e4}) = (0.92, 2.3, 3.7, 4.3) in $10^{11}cm^{-3}$ and f_o = 2.171 GHz. Δf is the full width half maxima.

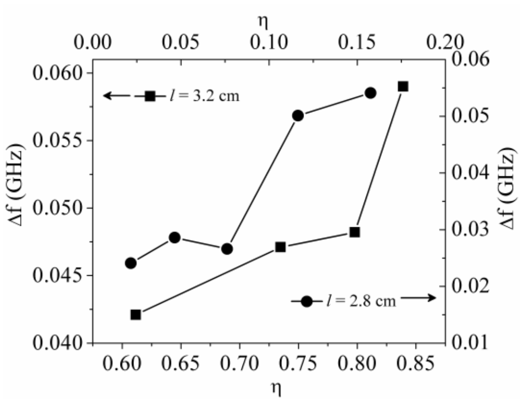

Fig-3: **S**ignal width versus absorption constant η (a) η = 0.61 to 0.84, *l* = 3.2cm (b) η = 0.02 to 0.18, *l* = 2.8cm.

Fig-2 shows a typical plot of resonance signal obtained using the hairpin probe in 13.56 MHz inductive argon plasma for various background electron densities. The inverted peak is the position of the vacuum resonance at f_o = 2.171 GHz whereas the positive peaks are the resonances in the plasma for respective electron densities. Note that the height of the resonance peak diminishes with the electron density.

Fig-3 plots the width of the resonance peak as a function of $\eta = (f_{pe}/f_r)^2$ for two hairpin resonators having lengths l = 3.2 cm (f_o = 2.17 GHz) and 2.8 cm (f_o = 3.22 GHz) respectively. Note that the width of the resonance signal increases as $\eta \approx 1$, which implies that when f_r is close to the electron plasma frequency f_{pe}. The attenuation of the resonance signal is large in the case of shorter hairpin having high frequency resonance.

3. Theoretical analysis of the quality of the resonance signal

Since the attenuation of the hairpin resonance signal is a function of resistive losses in the plasma, we consider the electrical conductivity of cold plasma which is given by;

$$\sigma_p = \frac{\varepsilon_o \omega_{pe}^2}{(\upsilon_{en} + j\omega)} \tag{3}$$

Where, ν_{en} is electron neutral collision frequency, ω are the angular frequency of the oscillating electric field setup between the resonator pins and ω_{pe} is the angular plasma frequency. For the low pressure case, $\omega >> \nu_{en}$, the real part of the conductivity (in S.I. units) represents the ohmic heating in plasma;

$$\sigma_p = \frac{\varepsilon_o \omega_{pe}^2 \upsilon_{en}}{\omega_r^2} = \varepsilon_o \eta \nu_{en} \tag{4}$$

In the case of a loss-less transmission line, Sugai et al [3] obtained the relation;

$$\frac{\Delta f}{f_r} = \frac{2}{\pi}(e^{2\alpha l} - 1) \quad \text{Where, } \alpha \cong \frac{1}{2}\left[R\sqrt{C/L} + G\sqrt{L/C}\right] \tag{5}$$

Where, α is the attenuation constant which determines the losses of the electro-magnetic energy in the transmission line. Using, the transmission line elements R, L, C and G per unit lengths from [3] and substituting in eq. (5) we can further simplify α to obtain ;

$$\alpha \cong \frac{13.18 \times 10^{-7}}{a \ln(2h/a)} \sqrt{\rho f_o (1-\eta)^{0.5}} + 188 \frac{\sigma_p}{\sqrt{1-\eta}} \tag{6}$$

In the above equation 'ρ' is resistivity of the hairpin wire, 'a' is wire radius, '2h' is the separation between pins and $f_o = c/4l$ is the vacuum resonance frequency. Using Eq. (1), & (4-6) we found the relation for signal width as;

$$\Delta f = 0.637 \frac{f_o}{\sqrt{1-\eta}} \left(e^{0.15\alpha / f_o} - 1\right) \tag{7}$$

Clearly, the signal width Δf_r depends on the probe dimensions via f_o (in GHz) and inversely proportional to η and α. Fig-4 shows the theoretical plots of Δf as a function arbitrary values of η. Note that the Δf increases sharply as the f_r approaches to f_{pe} such that $\eta \approx 1$. This is consistent with that observed in the experiment (fig-3).

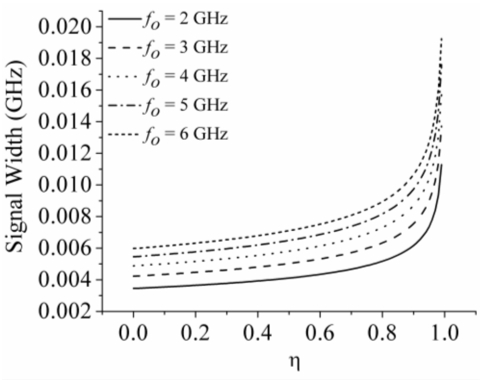

Fig-4: Signal width 'Δf' versus absorption constant 'η' Typical parameters - wire radius 'a' = 0.12mm, resistivity 'ρ' = 5.6x10^{-8} ohm-m, pressure = 10mT, and collisional frequency 'ν_{en}' = 0.024MHz.

4. Discussion

The theoretical estimate shows that the attenuation of the resonance signal is mostly due to the resistive losses in the plasma rather than within the probes resistive material. When the probe is driven in to resonance, the probes time-varying electric field will try to stimulate the local electron population in the vicinity of the pins. Since $f_{pe} < f_r$, the plasma electrons will only respond to a time-averaged field rather than the frequency at which the electric field is changing during the resonance. However at high plasma densities (10^{12} cm^{-3}), the electron plasma frequency f_{pe} is of the order of 9 GHz, therefore as compared to vacuum frequency of f_o = 2-3 GHz, we will find $f_{pe} \sim f_r$ from (2). In this case some of the electron population will absorb energy from the oscillating electric field of the resonator and dissipate the energy by resistive heating of the local plasma electrons. Therefore we observe significant attenuation as the resonance frequency approaches the electron plasma frequency. This condition is observed at plasma densities above 10^{11}cm^{-3}.

In the overall analysis we have ignored the contribution of the external magnetic field, the collision with the background neutrals and radiation losses. This will be a subject of further investigation in our future study.

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