Thomas-Fermi model of an ultra-cold Rydberg plasma

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Abstract

We describe the effects of the electron trapping due to the background ions in a Rydberg plasma. In the early stages, the ions are not thermalized and obey a Gaussian spatial profile, trapping the coldest electrons. In the present work, we provide expressions for both the electrostatic potential and the electron spatial profiles in two different regimes. We show that in the strong confinement regime $\Phi \gg T/e$, a Rydberg plasma can be described by a Thomas-Fermi potential, similar to that obtained for heavy atomic species.

It is a well known fact that plasmas can exist in an extraordinary variety of environments and span a great range of densities and temperatures, but the discipline of plasma physics has mainly focused on the high temperature regimes. Indeed, because the collision energies necessary to ionize atoms and molecules are usually high, plasmas tend to be hot. However, a new plasma regime has emerged in the laboratory in the recent years - the ultracold plasma - where the electrons temperature range from 1-100 K [1, 2, 3]. Such plasmas are obtained through either spontaneous [4] or photoionization mechanism [1]. In the first, high quantum number electronic states (Rydberg states) become populated and because of its huge polarizability, the atoms ionizes giving place to a Rydberg plasma. On the other hand, and more usually, ultracold plasmas are formed through the ionization of atoms or molecules that have been previously cooled well below the 1 K (typically, the temperature is of the order of few hundreds of μK for alkali atoms in magneto-optical traps [5, 6]). Such a cold initial sample is then exposed to a 10-ns laser pulse tuned near the ionization threshold, producing a ultracold plasma with a number density ranging from 10^{10} to 10^{14} cm^{-3}. In the recent years, much effort has been put forward to understand the features of such ultracold plasmas [7, 8, 9, 10].

In the present paper, we derive expressions for the total electrostatic potential in a Rydberg plasma. The main aim here is to show that the effects of the electron trapping lead to a model which is very similar to the well-known Thomas-Fermi model for heavy atomic systems [11]. When the trapped electron population dominates over the ”free” electrons, the Rydberg plasma qualitatively looks like a giant atom, where the ions play the role of a nucleus and the electrons compose the electronic cloud. In the present work, however, we remain in the qualitative picture, but the question of how the electronic energy states are distributed certainly deserves attention in the future.

We start with the Poisson equation describing the potential in a Rydberg plasma

$$\nabla^2\Phi = \frac{e}{n_0}\left(n_e - n_i\right), \tag{1}$$

where $n_{e(i)}$ represents the electron (ion) number density, which is related with the probability distribution $f_\alpha(\mathbf{r},\mathbf{v})$ $(\alpha = e, i)$ as

$$n_\alpha = \int f_\alpha(\mathbf{r},\mathbf{v})d\mathbf{v}. \tag{2}$$

In the early stages of the plasma, right after the photoionization takes place, the ions are approximately described by a Gaussian profile, associated with the neutral atoms confined in the magneto-optical trap (MOT)

$$n_i = n_0 e^{-(r/\sigma)^2}. \tag{3}$$

Those ions will create an electrostatic potential $\Phi > 0$, in such way that the classical energy of the electrons will be given by

$$E_e(\mathbf{r},\mathbf{v}) = \frac{1}{2}m_e v^2 - e\Phi(\mathbf{r}). \tag{4}$$

We immediately observe that the energy can be negative, which is due to the trapping effect of the electrons. Such a trapping occurs for electron velocities satisfying the condition $v < v_t$, where the trapping velocity is given by

$$v_t = \sqrt{\frac{2}{m_e}e|\Phi|}. \tag{5}$$

Putting Eqs. (1-3) together, and defining $\phi = e|\Phi|/T_e$, we can write

$$\nabla^2\phi = \frac{1}{\lambda_D^2}\left(\frac{n_e}{n_0} - e^{-(r/\sigma)^2}\right), \tag{6}$$

where $\lambda_D = v_{th}/\omega_p$, where $\omega_p = (e^2 n_0/\varepsilon_0 m_e)^{1/2}$ is the electron plasma frequency and $v_{th} = \sqrt{T/m_e}$ is the electron thermal velocity. While the "free" (or, more precisely, the untrapped electrons with $v_e > v_t$) electrons follow the Boltzmann distribution associated with the energy Eq. (4), the trapped ones approximately follow an uniform distribution, since we assume that the latter cannot leave the trapping radius $R \sim \sigma$ (at least at the early stages of the plasma, for which the present calculations are valid). Therefore, the electron density can be determined as follows

$$\frac{n_e}{n_0} = \frac{4}{\sqrt{\pi}} \left[\int_0^{u_t} u^2 du + \int_{u_t}^{\infty} e^{-(u^2-\phi)} u^2 du \right], \tag{7}$$

where we have used the dimensionless velocity $u = v/v_{th}$. Using Eqs. (6) and (7), we can obtain a general expression for the electrostatic potential that casts the effects of the electron trapping

$$\nabla^2 \phi = \frac{1}{\lambda_D^2} \left(\frac{4}{3\sqrt{\pi}} \phi^{3/2} - f(\phi) - e^{-(r/\sigma)^2} \right), \tag{8}$$

where the $f(\phi)$ is given by

$$f(\phi) = e^{\phi} \left(1 - \frac{4}{\sqrt{\pi}} \int_0^{\sqrt{\phi}} e^{-u^2} u^2 du \right). \tag{9}$$

The result in Eq. (8) is very hard to solve for general case. Fortunately, approximated expression can be provided in some limiting cases. Therefore, for weak trapping potential $\phi \ll 1$, $f(\phi) \approx 1 - \frac{4}{3\sqrt{\pi}}\phi^{3/2} + \phi - \frac{8}{15\sqrt{\pi}}\phi^{5/2}$ and the potential can be approximately by

$$\nabla^2 \phi = \frac{1}{\lambda_D^2} \left[1 + \phi - \frac{8}{15\sqrt{\pi}} \phi^{5/2} - e^{-(r/\sigma)^2} \right]. \tag{10}$$

On the strong confinement case, $\phi \gg 1$, $f(\phi) \approx 0$ and the potential yields

$$\nabla^2 \phi = \frac{1}{\lambda_D^2} \left[\frac{4}{3\sqrt{\pi}} \phi 3/2 - e^{-(r/\sigma)^2} \right]. \tag{11}$$

This is the main result of the paper. In the strong confinement regime, we obtain an expression which is very similar to the Thomas-Fermi potential obtained for heavy atomic species. The only difference is rooted in the fact that here the ions are not homogeneously distributed, and therefore we have included the ion inhomogeneity, which makes the model more suitable to describe the physical situation in a Rydberg plasma. In order to solve the latter equation numerically, we define the dimensionless variable $\rho = r/\lambda_D$ and the reduced potential $\psi = \rho\phi$. Notice that the ψ represents the potential relative to the Coulomb potential $\phi_{Coul} \sim 1/\rho$, such that $\psi = 1$ for an unscreened plasma. In the present, we restrict the discussion to the case of a spherically symmetric plasma, such that the laplacian contains only the radial derivatives. Due to the trapping effects, the resulting potential significantly differs from the Coulomb case, as it can be observed in Fig. (1). The corresponding electron profile is not a gaussian one. Indeed, the numerical simulations reveal that n_e decays very quickly as a function of r and turns out that the majority of the electrons are trapped inside the radius $R \sim \sigma$ defined by the width of the ion profile n_i. Such results suggest that the ions can efficiently trap a fraction of the electron population inside their cloud.

In conclusion, we have established the electrostatic potential in an electron-ion Rydberg plasma. Because a fraction of the electron population is sufficiently cold to remain inside the ion cloud, the Boltzmann statistics does not hold generally. Casting the effects of the electron trapping, we derived a nonlinear equation to describe the potential which reduces to the case of a Thomas-Fermi potential obtained in a different context to describe heavy atomic systems in the semi-classical regime.

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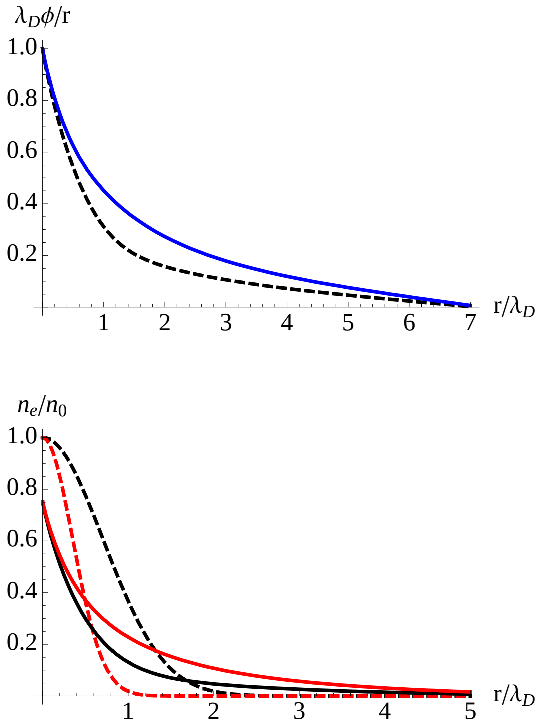

Figure 1: a) Thomas-Fermi potential obtained in the strong trap regime $\phi \gg 1$ for two different ion Gaussian profiles: $\sigma = \lambda_D$, (dashed line) and $\sigma = 0.5\lambda_D$ (full line. b) Ion (dashed lines) and electron (full lines) profiles in the early stages of a Rydberg plasma: $\sigma = \lambda_D$ (black lines) and $\sigma = 0.5\lambda_D$ (red lines).