Plasmon polaritons and slow light in Rydberg plasmas

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Abstract

We discuss the combined effects of both electrons and neutral atoms of a partially ionized Rydberg gas on the dispersion of electromagnetic waves. Within the two-level approximation, we derive the expression for the resulting light-plasmon-atom polariton dispersion relation and discuss the possibility of occurrence of slow-light phenomena in such media.

In recent years, there has been a growing interest in the physics of of ultracold plasmas, where the electron temperature ranges from 1-100 K [1, 2, 3]. Such plasmas are obtained either via spontaneous [4, 5] of photoionization mechanisms [6]. In the first case, the atoms are excited up to high quantum number electronic states (the so-called Rydber states) and spontaneously ionized, giving origin to ultracold neutral plasmas, or Rydberg plasmas [8, 9]. Under the typical experimental conditions, such a plasma consists of a partially ionized gas, where both electrons, ions and neutral atoms coexist. Much effort has been put forward in order to understand the formation and dynamics of these ultracold plasmas [5, 7], but the study of their optical properties still remains an open issue. Motivated by this fact and taking advantage of such a rich environment, we extend the results of our previous work [8] and discuss the quantum effects introduced by the atomic saturation (atomic Rabi oscillation) on the dispersion relation of electromagnetic (EM) waves. We show that a polariton-like dispersion relation arises in Rydberg plasmas due to the combined effect of electrons and atoms.

The starting point is the EM wave equation in a partially ionized Rydberg gas, which reads

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E} = \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2} - \mu_0 \frac{\partial \mathbf{J}}{\partial t}, \tag{1}$$

where $\mathbf{J} = -en_0\mathbf{v}_e$ is the electronic current and $\mathbf{P}$ is the total polarization vector. Because the Rydberg atoms also respond to the EM wave, the total polarization is given by

$$\mathbf{P} = \varepsilon_0 \left(\chi_e + \chi_a\right)\mathbf{E}, \tag{2}$$

with χ_e and χ_a standing for the medium electronic and atomic susceptibilities, respectively. Here, we have considered that the gas is homogeneous and isotropic, which is a good approximation for the bulk of the system, for which this theory is meant to be valid. The electron susceptibility can be split into its real and imaginary parts which, after solving the associated equation of motion, yields

$$\chi_e'(\omega) = -\frac{\omega_p^2}{\omega^2+\nu_e^2}, \qquad \chi_e'' = \frac{\nu_e}{\omega_p}\chi_e', \tag{3}$$

where $\omega_p = \sqrt{e^2 n_0/\varepsilon_0 m}$ is the plasma frequency and ν_e represents the collision frequency between the electrons and the atoms. In order to properly compute the atomic susceptibility, one should go beyond the classical Lorentz-Lorenz model for the atomic susceptibility and take into account the internal electronic structure of the atom. Therefore, we consider the two-level model, where the atom state $|\psi\rangle$ is spanned in the basis of $|g\rangle$ and $|e\rangle$, which respectively represent the ground and the excited states. Here, we consider that the two-level structure of a Rydberg atom can be described by two successive quantum states $|g\rangle$=$|n\rangle$ and $|e\rangle$=$|n+1\rangle$, where n is the principal quantum number. Since $n \gg 1$ in such gases, the energy associated with each quantum number can be approximately given by

$$E_n = -\frac{\mathrm{Ry}}{n^2}, \quad n > 1, \tag{4}$$

where $\mathrm{Ry} = 13.6$ eV is the Rydberg constant. The dominant contribution for the atomic susceptibility will, therefore, come from the radiative transition of energy $\hbar\omega_a = E_{n+1} - E_n$ which, from Eq. (4), immediately reads $\omega_a \approx 2\mathrm{Ry}/\hbar n^3$.

In the semi-classical theory of light-matter interaction [10], the dipole moment of each atom is defined as $\boldsymbol{\mu}_a = -e\langle n|\mathbf{r}|n+1\rangle$, where $\mathbf{r}$ is the electron displacement vector inside the atom. The overall atomic polarization $\mathbf{P}_a = n_a\langle\boldsymbol{\mu}_a\rangle$, where n_a is the atomic number density and the average operation is defined as $\langle\boldsymbol{\mu}_a\rangle = \langle\psi|\boldsymbol{\mu}_a|\psi\rangle$. Solving the Schrödinger equation for the state $|\psi\rangle$, and using the relation between $\mathbf{P}$ and $\mathbf{E}$ expressed in Eq. (2), one obtains

$$\chi_a' = \frac{n_a\mu_a^2}{\hbar\varepsilon_0}\frac{\delta - i\Gamma}{\delta^2+\Gamma^2+\Omega^2}, \qquad \chi_a'' = \frac{\Gamma}{\omega}\chi_a', \tag{5}$$

where $\delta = \omega - \omega_a$ is the EM wave detuning, Γ is the natural linewidth of the atomic transition and $\Omega = \boldsymbol{\mu}_a \cdot \mathbf{E}_0/\hbar$ represents the Rabi frequency, which accounts for the quantum effects of the atomic saturation due to the EM field. Defining the dimensionless oscillator strength $f_a = m_e\omega_a|\langle n|\mathbf{r}|n+1\rangle|^2/\hbar$ and expressing the difference between the two atomic populations as $D = N_a^{(n+1)} - N_a^{(n)}$, we can finally write

$$\chi_a' = -\tau_a\frac{\omega_p^2\delta}{\delta^2+\Gamma^2+\Omega^2}, \tag{6}$$

where $\tau_a = f_a D n_a / n_0 \omega_a$ has the dimensions of a time and can be positive or negative depending on the relative population of the states $|n\rangle$ and $|n+1\rangle$. Therefore, if the excited state is more populated, one obtains $\tau_a > 0$.

After Fourier transform the EM wave equation (1)

$$\frac{k^2c^2}{\omega^2} = \varepsilon(\omega) = 1 + \chi_e(\omega) + \chi_a(\omega) \tag{7}$$

and putting Eqs. (3), (6) and (7) together, we obtain the resulting polariton dispersion relation

$$\omega^2 = k^2c^2 + \omega_p^2 \left[\frac{\omega^2}{\omega^2 + \nu_e^2} + \tau_a \frac{\omega^2(\omega - \omega_a)}{\Gamma^2 + \Omega^2 + (\omega - \omega_a)^2} \right]. \tag{8}$$

Under typical experimental conditions [11], it is estimated an electron-atom collision frequency of $\nu_e/2\pi \sim 100$ MHz. The typical plasma frequency $\omega_p/2\pi \sim 1$ GHz (for an electron density of $n_0 \sim 10^9$ cm^{-3}) and the atomic frequency $\omega/2\pi \sim 10$ GHz (for n ranging from 40-50) both belong to the microwave range, which allow us to neglect ν_e in the dispersion relation. We also consider that Γ is small compared to both ω_a and ω_p, finally writing the lossless light-electron-atom polariton dispersion relation, accounting for the quantum saturation

$$\omega^2 \approx k^2c^2 + \omega_p^2 \left[1 + \tau_a \frac{\omega^2(\omega - \omega_a)}{\Omega^2 + (\omega - \omega_a)^2} \right]. \tag{9}$$

This is the main result of the paper. The main features of the polaritonic dispersion Eq. (9) are summarized in Fig. (1). First, it is shown that in the presence o Rydberg atoms significantly changes the character of the wave, giving origin to new branches in the dispersion relation of the EM waves in the microwave range. Sencond, by allowing the atoms to be sensitive to the field intensity (i.e., by setting $\Omega > 0$), we observe the emergence of a third pice in the dispersion, with frequencies ranging between the values of the two principal branches. The interesting aspect of this (let us say) "new" curve is that of describing waves with very low group velocities compared to the speed of light ($\partial\omega/\partial k \ll c$), as uncommonly expected to be observed in typical laboratory or space plasmas (at least for small wavelengths). This suggests that Rydberg plasmas may be a good candidate to observe slow-light phenomena, which is known to be a feature of major importance in photonic crystals. Moreover, we observe that the increasing of Ω tends to suppress the polaritonic (or hybrid) nature of the wave, in particular the slow-light effect. Indeed, for higher values of Ω, the dispersion relation Eq. (9) approaches to $\omega^2 \approx k^2c^2 + \omega_p^2(1 + \tau_a\omega^2(\omega - \omega_a))$, which explains the suppression of the slow-light band around $\omega \sim \omega_a$.

In conclusion, we have derived the dispersion relation for the electromagnetic waves in a partially ionized ultracold Rydberg plasma. Taking into account the presence of the atoms and using a two-level description to cast the quantum mechanical effects of both the radiative transition and saturation, we show that a coupling between the light and the atoms exists, differing from the usual electromagnetic wave dispersion relation. The resulting polaritonic dispersion exhibits new branches and, in particular, suggests that Rydberg plasmas can provide a stage to observe slow-light phenomena.

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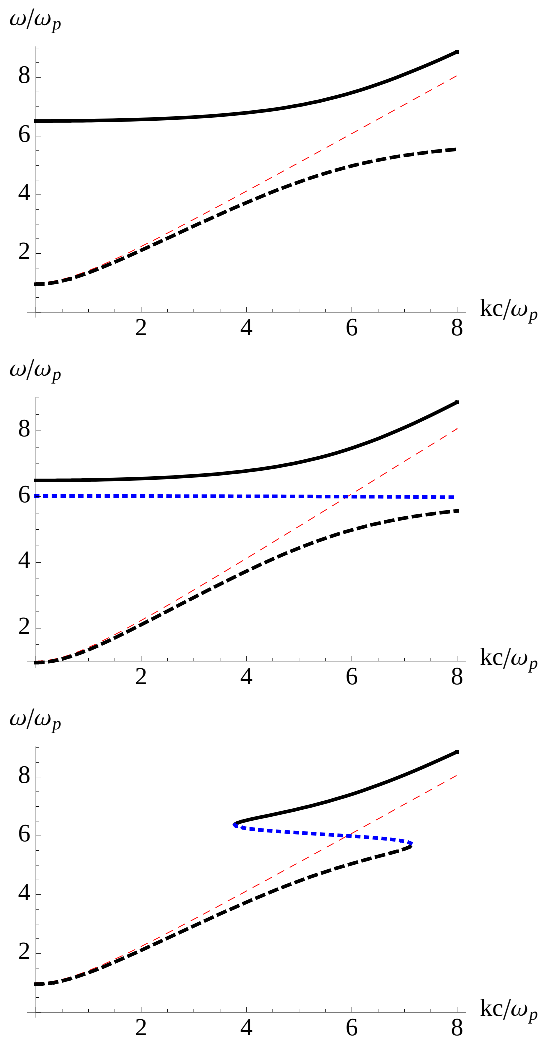

Figure 1: Polariton dispersion relation in a Rydberg plasma, obtained for $\omega_a = 6\omega_p$ and a inverted-population parameter $\tau_a = 0.5\omega_p^{-1}$. **a)** $\Omega = 0$, **b)** $\Omega = 0.1\omega_p$ and **c)** $\Omega = 0.5\omega_p$. In the above plots, the thin dashed line represents the usual EM wave dispersion relation $\omega^2 = \omega_p^2 + k^2c^2$.