Discussion on Slow Light in Magnetized Plasma by Electromagnetically Induced Transparency

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ABSTRACT This paper discusses an analogous state to "dark-state polariton" [M. Fleischhauer and M. D. Lukin, Phys. Rev. Lett. **84**, 5094 (2000)] in magnetized plasma under electromagnetically induced transparency (EIT). Propagation of a coupled state of a probe-wave electric field and longitudinal plasma oscillation can be stopped as its waveform is maintained if a pump-wave is properly controlled in time. The "stop-light" is achieved by adiabatic transition between states in which the probe-wave field is dominant and the longitudinal plasma oscillation is dominant.

Conventional EIT is a phenomenon in which optical properties of a material are drastically changed by quantum interference between eigenstates of matter when irradiated by two lights[1]. The EIT is the basis of several applications, such as slow light, information transfer between matter and light and so on. A theoretical idea by Harris [2] initiated research of a new EIT assuming plasma as a medium, and subsequently possibility of EIT in magnetized plasma was shown by researchers [3] by theoretical/ numerical analyses. Those theories predict that injection of pumping wave alters susceptibility of plasma to right-hand circular polarized (RHCP) electron cyclotron wave. As a result, the RHCP wave no longer resonates with electrons at the electron cyclotron frequency, being transmitted. The EIT in plasmas does not invoke quantum mechanics while the conventional EIT requires quantum mechanics. Instead, the EIT in plasmas is realized by collective excitation of a medium.

Under the EIT condition that two electromagnetic waves of probe and pump are injected into a slowly varying (in space and time) magnetized plasma, the probe-wave frequency component of induced current density $\boldsymbol{J}(\boldsymbol{r},t)$ by the wave field can be expressed as linear response to the probe wave electric field $\boldsymbol{E}_1$ including the pump field $\boldsymbol{E}_0$ as a parameter

$$\boldsymbol{J}(\boldsymbol{r},t) = \left(\vec{\vec{\sigma}}^T_{EIT} + \vec{\vec{\kappa}}^T_{EIT}\right)\cdot \boldsymbol{a}_1(\boldsymbol{r},t)e^{i\psi(\boldsymbol{r},t)} \ . \tag{1}$$

Here, $\boldsymbol{E}_1(\boldsymbol{r},t) = \boldsymbol{a}_1(\boldsymbol{r},t)e^{i\psi(\boldsymbol{r},t)}$, $\boldsymbol{k}(\boldsymbol{r},t) \equiv \nabla\psi(\boldsymbol{r},t)$, $\omega(\boldsymbol{r},t) \equiv -\partial\psi(\boldsymbol{r},t)/\partial t$, where $\psi(\boldsymbol{r},t)$ is rapidly varying over the scale length of a wave and period of the wave. $\boldsymbol{a}_1(\boldsymbol{r},t)$ slowly varies over scale length of the plasma inhomogeneity. See Ref. 6 for definition of other variables. Substitution of eq. (1) to a wave equation $\nabla\times\nabla\times\boldsymbol{E} + \partial^2\boldsymbol{E}/\partial t^2 + \mu_0\partial\boldsymbol{J}/\partial t = 0$ under the assumption that $\boldsymbol{k}\ (\equiv \nabla\psi)$ and $\omega(\equiv -\partial\psi/\partial t)$ are zero order quantities, $\boldsymbol{a}_1, \nabla$ and $\partial/\partial t$ are first order gives a first order equation $\vec{\vec{\varepsilon}}\cdot\boldsymbol{a}_1 = 0$, $\vec{\vec{\varepsilon}} \equiv c^2\left(\boldsymbol{kk} - k^2\vec{\vec{I}}\right)/\omega^2 + \vec{\vec{I}} + \vec{\vec{Q}}_{EIT}/\omega$ which provides the dispersion relation. The second order equation for one-dimension ($\boldsymbol{k} = k_z\boldsymbol{z}$ and $\nabla = (0,0,\partial/\partial z)$) is,

$$\frac{\partial}{\partial t}a_\perp + \frac{c^2k_z}{\omega}\frac{\partial}{\partial z}a_\perp + \frac{a_\perp}{2\omega}\frac{\partial}{\partial t}\omega + \frac{c^2a_\perp}{2\omega}\frac{\partial}{\partial z}k_z = -\frac{1}{2\omega}\frac{\partial}{\partial t}\left\{\left(Q_{xx}+iQ_{xy}\right)a_\perp\right\} - \frac{1}{2\varepsilon_0}\left\{\left(\kappa_{xx}+i\kappa_{xy}\right)a_\perp\right\} \ , \tag{2}$$

if $Q_{xx} = Q_{yy}$, $Q_{xy} = -Q_{yx}$ and $Q_{yz} = Q_{xz} = 0$. Here, $a_\perp \equiv a_{1x} - ia_{1y}$, Q_{ij} and κ_{ij} are i,j component of $\vec{\vec{Q}}_{EIT} \equiv i\vec{\vec{\sigma}}^T_{EIT}/\varepsilon_0$ and $\vec{\vec{\kappa}}^T_{EIT}$, respectively. This equation describes temporal evolution of right hand circular (RHC) component ($a_{1x} - ia_{1y}$) of amplitude of the probe wave. If time scale of frequency change of the probe wave is much slower than that of wave envelope change and spatial variation of wavelength of the probe wave is much smaller than the envelope change, condition $\delta t_{probe} << \delta t_{pump}$ makes $\left(\kappa_{xx} + i\kappa_{xy}\right)a_\perp$ in the right hand side (RHS) of eq. (2) simple.

$$\frac{\partial}{\partial t}a_\perp + \frac{c^2k_z}{\omega}\frac{\partial}{\partial z}a_\perp = -\frac{1}{2\omega}\frac{\partial}{\partial t}\left\{\left(Q_{xx}+iQ_{xy}\right)a_\perp\right\} - \frac{i}{2\varepsilon_0}\left\{\left(\frac{\partial(\sigma_{xx}+i\sigma_{xy})}{\partial\omega}\right)\frac{\partial a_\perp}{\partial t}\right\} \ . \tag{3}$$

Here, δt_{pump} and δt_{probe} are the pulse width of the pump and the probe waves, respectively. The second term of the RHS is RHC component of current which stems from frequency response of susceptibility of the medium. The first term of the RHS is RHC component of current induced by RHCP component of probe-wave electric field. Replacement $-i(\sigma_{xx} + i\sigma_{xy})/(\varepsilon_0\omega) = -(Q_{xx} + iQ_{xy})/\omega = \chi_{EIT}$ can be done in these terms, where the susceptibility for EIT regime χ_{EIT} is given by Hur, et al. [5]

$$\chi_{EIT} = \frac{\omega_p{}^2}{\omega}\frac{\delta\Omega + \delta\Omega_0(k_z)}{\Omega_R{}^2 - \delta\Omega^2} \text{ and } a_{\perp 1} \equiv ea_\perp/(mc\omega)\text{ , where } \delta\Omega \equiv \omega - \Omega_{ce},\ \Omega_R \equiv ck_0|a_0|\sqrt{\frac{\Omega_{ce}}{4\omega_p}}\ . \quad (4)$$

Here, Ω_R, a_0, k_0 and m are Rabi frequency, normalized amplitude and wave number of the pump-wave electric field and electron mass, respectively (see ref. 5 for definition of other variables.). When a_0 is strong enough, $\chi_{imag}\to 0$ indicating disappearance of resonant absorption of the probe-wave at $\omega = \Omega_{ce}$. The EIT window emerges and expands as a_0 increases. Moreover even if a_0 decreases to some extent (but $\neq 0$), χ_{imag} can be small enough and $\partial\chi_{real}/\partial\omega$ is large, indicating slow light propagation without resonant absorption.

When $\chi_{EIT} = \chi_{real} + i\chi_{imag} \sim 0$ such as in case of large a_0, the first term of the RHS of eq. (3) vanishes. The wave equation of envelop of probe-wave can be written in the form:

$$\frac{\partial a_\perp}{\partial t} + v_g\frac{\partial a_\perp}{\partial z} = 0\ , \quad v_g = c^2k_z/\omega_{probe}/\left\{1 + \frac{1}{2}\omega_{probe}\left(\frac{\partial\chi_{EIT}}{\partial\omega_{probe}}\right)\right\}\ . \quad (5)$$

When v_g is a function of only z, achievable lowest group velocity is limited as follows [4 7]: For EIT medium, χ_{EIT} changes sharply within the EIT window, whose width is *w*. Slope of χ_{EIT} is estimated as $\partial\chi_{EIT}/\partial\omega_{probe} \sim \Delta\chi_{EIT}/w \sim \Delta\chi_{EIT}/\Omega_R$, where $\Delta\chi_{EIT}$ is the variation of χ_{EIT} in the window ($\Omega_{ce} \pm \Omega_R$). Note that Rabi frequency Ω_R is proportional to the pump intensity a_0.

$$v_g \sim (c^2k_z/\Omega_{ce})/(\Delta\chi_{EIT}/w) \sim (c^2k_z/\Delta\chi_{EIT}\ \Omega_{ce})2\Omega_R \propto (c^2k_z/\Delta\chi_{EIT}\ \Omega_{ce})a_0\ . \quad (6)$$

Spectrum of the probe-pulse (envelop) must be contained within the EIT window *w*. Namely, the spectral width of the envelope must satisfy the following condition.

$$1/\delta t_{probe} < w \sim 2\Omega_R \propto a_0\ . \quad (7)$$

That is, the lowest achievable group velocity is limited by pulse width of the envelope δt_{probe} when v_g is a function of *z* and constant in *t*. However, if v_g is a function of *t*, the solution has a form of $a_\perp(z,t) = h\left(z - \int_0^t v_g(t')\mathrm{d}t'\right)$, $v_g = c^2k_z/\omega_{probe}/\left\{1 + \omega_{probe}\left(\partial\chi_{EIT}/\partial\omega_{probe}\right)/2\right\}$, indicating envelope of the probe-wave propagates in the z-direction and spatial profile of the wave packet is maintained at any *t*. Deceleration of the wave propagation in this scheme is realized by compression in time in contrast to that by the preceding scheme due to compression in space.

This method prevents the deceleration of light from being restricted by the effective band width of the medium indicated by eqs. (6) and (7) because initial band width of the probe-wave pulse is kept constant even during the wave packet is slowed down.

If $1/\delta t_{probe} << \omega_p, \omega_1$ and $\left| v_{1+}^{osc} \right| \equiv \left| -ea_1^R(t)/m_e\omega_1 \right| << \left| v_{0+}^{osc} \right| \equiv \left| -ea_0^R(t)/m_e\omega_0 \right|$, current induced by the probe-wave $a_1^R(t)$ is written as $j_1 \approx -en_{e0}\tilde{v}_{1+} \approx 2\left(m_e\omega_0/(ek_0)\right)^2\left(\varepsilon_0\omega_{pe}^2/\left\{a_0^R\right\}^2\right)\frac{\partial}{\partial t}a_1^R(t)$. This indicates $\chi_{EIT}=\chi_{real}+i\chi_{imag}$~0 and $2\left(m_e\omega_0/(ek_0)\right)^2\left(\varepsilon_0\omega_{pe}^2/\left\{a_0^R\right\}^2\right) \approx \omega_{probe}\left(\partial\chi_{EIT}/\partial\omega_{probe}\right) \propto v_g^{-1}$. Also the following relation is derived from a simplified equation of motion of electrons,

$$k_0 a_0^R(t)/\omega_0 \approx 2a_1^R(t)/\left(-i\omega_p\tilde{\xi}\right) . \tag{8}$$

This indicates that ratio between the probe field $2a_1^R(t)$ and the longitudinal oscillation $-i\omega_p\tilde{\xi}$ is determined by the pump electric field $a_0^R(t)$. The coupled state of $2a_1^R(t)$ and $-i\omega_p\tilde{\xi}$, which is regarded as a coupled state of light and matter in quantum field, propagates in the direction of z at v_g which is controlled by $a_0^R(t)$ as well. When "stop-light" is accomplished by adiabatic change of $2a_1^R(t)$ and $-i\omega_p\tilde{\xi}$, the plasma oscillation dominates the coupled state. Energy of the probe wave field is stored into the plasma oscillation in the coupled state during the probe-wave is stopped. This is an analogous phenomenon to dark-state polariton in quantum system.[4] Adiabatic control of the ratio between the probe field and the longitudinal oscillation through the pump-wave intensity realizes the manipulation of the group velocity of the probe-wave moreover, freeze of the wave propagation.

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