Kinetic theory of rarefied plasma: the effective action method

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The standard kinetic theory for rarefied plasma is based on the set of integro-differential equations. These are the Boltzmann kinetic equations with collision integral (for particles of each sort) and the Maxwell equations. In the latter, the charge density and current density are expressed through one-particle distribution functions, which, in turn, are determined from the kinetic equations (see e.g. [1]). In this work, the author suggests a kinetic theory of rarefied plasma (a physical equivalent to the standard kinetic theory) that is based on the construction of propagators for distribution functions, these propagators being dependent on these functions.

Consider a volume V that contains N_a particles of sort *a* and N_b particles of sort *b*. In the case of a classical plasma, it is sufficient to take into account only the contribution of the classical trajectory in the path integral [2], which specifies the propagator of a particle. The classical propagator for the density matrix $\rho \;\; \vec{r},\vec{r}',t$ has the form

$$K_a^M(2,1) = \mu \exp \;\; (i/\hbar) \;\; S_a(\vec{r}_2,t_2;\vec{r}_1,t_1) - S_a(\vec{r}_2{}',t_2;\vec{r}_1{}',t_1)$$
$$\mu = \left(\frac{m_a}{2\pi\hbar(t_2 - t_1)} \right)^3, \tag{1}$$

where

$$S_a(\vec{r}_2,t_2;\vec{r}_1,t_1) = \int_{t_1}^{t_2} dt \left\{ \frac{m_a \vec{v}_a^2(t)}{2} - \sum_{i=2}^{N_a} U_{aa}(\vec{R}_i(t) - \vec{r}_a(t)) \right\}$$
$$- \int_{t_1}^{t_2} dt \left\{ \sum_{j=1}^{N_b} U_{ab}(\vec{R}_j(t) - \vec{r}_a(t)) - \vec{F}_a(t)\vec{r}_a(t) \right\} \tag{2}$$

is the classical action of a particle with a mass m_a, $\vec{v}_a(t), \vec{r}_a(t)$ are the velocity and radius vector of the particle, U_{aa}, U_{ab} are the potential energies of particle–particle interaction, $\vec{R}_i$ is the radius vector of a scatterer, and $\vec{F}_a$ is the external force acting on the particle. In a rarefied plasma, the characteristic correlation (interaction) time is much shorter than the characteristic relaxation time [1]. Therefore, one may assume that scatterers in (1) and (2) describe piecewise rectilinear trajectories and pass rectilinear segments for a time that is smaller than the relaxation time but longer than the correlation time. In this case, ensemble averaging of propagator (1) is carried out with a multiparticle distribution function where only small pair correlations are taken into account (polarization approximation [1]). Such averaging results in an effective-action propagator. In the limit $N \to \infty, V \to \infty$ the averaged propagator has the form

$$
\begin{aligned}
(1/\mu)K_a(2,1) = \exp\left\{(i/\hbar)\int_{t_1}^{t_2} dt\left(\frac{m_a\vec{v}_a^2(t)}{2} - \frac{m_a\vec{v}_a'^2(t)}{2}\right) + n_a V_{aa}^{st} + n_b V_{ba}^{st}\right\} \\
\times\exp\ (i/\hbar)\int_{t_1}^{t_2} dt\ \ \vec{F}_a(t)\vec{r}_a(t) - \vec{F}_a(t)\vec{r}_a'(t) \\
+ \sum_{ij=aa,ba,bb} \frac{n_i n_j}{2}\int d\vec{R}_1 d\vec{R}_2 d\vec{p}_1 d\vec{p}_2\, g_{ij}(\vec{R}_1,\vec{p}_1,\vec{R}_2,\vec{p}_2,t_1) \\
\exp\left\{(i/\hbar)\int_{t_1}^{t_2} dt \sum_{k=1,2} -U_{ij}(\vec{R}_k - \vec{v}_k(t_2 - t) - \vec{r}_{a,ij}(t))\right\} \\
\times\exp\left\{(i/\hbar)\int_{t_1}^{t_2} dt \sum_{k=1,2} U_{ij}(\vec{R}_k - \vec{v}_k(t_2 - t) - \vec{r}_{a,ij}{}'(t))\right\} \\
\times\exp\left\{(i/\hbar)\int_{t_1}^{t_2} dt\left(\frac{m_a\vec{v}_{a,ij}^2(t)}{2} - \frac{m_a\vec{v}_{a,ij}'^2(t)}{2}\right) + n_a V_{aa}^{st} + n_b V_{ba}^{st}\right\} \\
\times\exp\left\{(i/\hbar)\int_{t_1}^{t_2} dt\left(\vec{F}_a(t)\vec{r}_{a,ij}(t) - \vec{F}_a(t)\vec{r}_{a,ij}'(t)\right)\right\}.
\end{aligned}
\tag{3}
$$

Here, $g_{ba}(\vec{R}_1,\vec{p}_1,\vec{R}_2,\vec{p}_2,t)$ is the pair correlation function, which is expressed through the single-particle distribution functions $f_{a,b}(\vec{r},\vec{p},t)$ (p is the particle momentum) [1], and

$$
\begin{aligned}
V_{ba}^{st} = \int d\vec{p}d\vec{R}\, f_b(\vec{R},\vec{p},t_1) \\
\left[\exp\left\{-\frac{i}{\hbar}\int_{t_1}^{t_2} dt\ \ U_{ba}(\vec{R} - \vec{v}(t_2 - t) - \vec{r}_a(t)) - U_{ba}(\vec{R} - \vec{v}(t_2 - t) - \vec{r}_a'(t))\right\} - 1\right].
\end{aligned}
\tag{4}
$$

V_{ba}^{st} is the collision volume (see the adiabatic line broadening theory [3]).

In the case of Coulomb interaction, the Weisskopf radius is on the order of the minimal impact parameter r_{min} in the Landau collision integral. The averaged effect of the scatterer fields on the particle trajectory in (3) can be taken into consideration in terms of the perturbation theory where r_{min}/r_D and $nr_{\min}^3$ are small parameters (r_D is the Debye radius, which defines the characteristic correlation length of plasma particles).

The first term in (3) describes the evolution of the distribution function in the self-consistent field approximation, and the term with the collision integral in the exponent defines the deceleration (acceleration) of a plasma particle due to the self-consistent field. The other terms describe the effect of the collision integral on the plasma kinetics.

If an external force is so high that $Fr_{\min} \geq U(r_{\min})$, the effect of such a force on the correlation function should be regarded [1].

IF a relativistic effects in plasma are essential, one should substitute the relativistic expression for the kinetic energy of a particle into (3) and take into consideration both the scalar and the vector potentials of scatterers. The correlation functions of the particles should also be relativistic [1].

For a quantum plasma, when calculating the path integral, which defines the propagator, one must consider the contributions of all trajectories rather than of the classical trajectory alone. Statistical averaging is carried out over the density matrix. In the polarization approximation, the quantum correlation function is expressed through one-particle density matrices in the Wigner representation ([1], part 3 and Refs. therein). The averaged propagator is given by

$$
\begin{aligned}
&K_a(2,1) = \int D\ \vec{r}_a(t)\ \int D\left[\vec{r}_a{}'(t)\right] \exp\left\{ \frac{i}{\hbar} \int_{t_1}^{t_2} dt \left(\frac{m_a \vec{v}_a^{\,2}(t)}{2} - \frac{m_a \vec{v}_a^{\prime 2}(t)}{2} \right) \right\} \\
&\times \exp\ (i/\hbar) \int_{t_1}^{t_2} dt\ \ \int \vec{F}_a(t) d\vec{r}_a(t) - \int \vec{F}_a(t) d\vec{r}_a^{\,\prime}(t)\ + n_a V_{aa}^{st} + n_b V_{ba}^{st} \\
&+ \sum_{ij=aa,ba,bb} \frac{n_i n_j}{2} \int d\vec{R}_1 d\vec{R}_2 d\vec{R}_1' d\vec{R}_2' (g_{ij}(\vec{R}_1, \vec{R}_1, \vec{R}_2, \vec{R}_2, t_1) \rho_i(\vec{R}_1', \vec{R}_1', t_1) \\
&\times \rho_j(\vec{R}_2', \vec{R}_2', t_1) + g_{ij}(\vec{R}_1', \vec{R}_1', \vec{R}_2', \vec{R}_2', t_1) \rho_i(\vec{R}_1, \vec{R}_1, t_1) \rho_j(\vec{R}_2, \vec{R}_2, t_1)) \\
&\int D\left[\vec{r}_a(t)\right] \int D\left[\vec{r}_a{}'(t)\right] \int_{\vec{R}_1(t_1)=\vec{R}_1'}^{\vec{R}_1(t_2)=\vec{R}_1} D\left[\vec{R}_1(t)\right] \int_{\vec{R}_2(t_1)=\vec{R}_2'}^{\vec{R}_2(t_2)=\vec{R}_2} D\left[\vec{R}_2(t)\right] \exp\left\{ (i/\hbar) \int_{t_1}^{t_2} dt \sum_{k=1,2} \left(\frac{m_k \dot{\vec{R}}_k^2(t)}{2} - U_{ij}(\vec{R}_k(t) - \vec{r}_a(t)) \right) \right\} \\
&\times \exp\left\{ (i/\hbar) \int_{t_1}^{t_2} dt \sum_{k=1,2} U_{ij}(\vec{R}_k(t) - \vec{r}_a{}'(t)) \right\} \times \exp\left\{ (i/\hbar) \int_{t_1}^{t_2} dt \left(\frac{m_a \vec{v}_a^{\,2}(t)}{2} - \frac{m_a \vec{v}_a^{\prime 2}(t)}{2} \right) + n_a V_{aa}^{st} + n_b V_{ba}^{st} \right\} \\
&\times \exp\left\{ (i/\hbar) \int_{t_1}^{t_2} dt \left(\int \vec{F}_a(t) d\vec{r}_a(t) - \int \vec{F}_a(t) d\vec{r}_a^{\,\prime}(t) \right) \right\},
\end{aligned} \tag{5}
$$

where

$$V_{ba}^{st} = \int d\vec{R}' d\vec{R} \rho_b(\vec{R},\vec{R},t_1)\rho_a(\vec{R}',\vec{R}',t_1) \int_{\vec{R}(t_1)=\vec{R}'}^{\vec{R}(t_2)=\vec{R}} D\left[\vec{R}(t)\right]$$
$$\left[\exp\left\{-\frac{i}{\hbar}\int_{t_1}^{t_2} dt\left(\frac{m_b\dot{R}^2(t)}{2} + U_{ba}(\vec{R}(t)-\vec{r}_a(t)) - U_{ba}(\vec{R}(t)-\vec{r}_a'(t))\right)\right\} - 1\right]. \quad (6)$$

is the collision volume.

For a rarefied plasma, the path integrals for scattering particles in (5) and (6) can be calculated in terms of the perturbation theory [2] (the zero order is sufficient). The averaged effect of scatterers on a probe particle is also found from the perturbation theory. Exchange interaction is included in the quantum correlation function.

To conclude, expressions (3) and (5) can be considered as a solution to the kinetic problems for short times, i.e., when the distribution function varies insignificantly.

REFERENCES

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