Scaling properties of two-dimensional turbulence in a pure electron plasma

F. Lepreti[1,2], M. Romé[3], R. Pozzoli[3], V. Carbone[1,4], G. Maero[3], S. Servidio[1], F. Valentini[5,1]

[1] *Dipartimento di Fisica, Università della Calabria, Rende (CS), Italy*

[2] *CNISM Unità di Cosenza, Rende (CS), Italy*

[3] *INFN Sezione di Milano and Dipartimento di Fisica, Università degli Studi di Milano, Italy*

[4] *CNR LiCryl Laboratory, Rende (CS), Italy*

[5] *Dipartimento di Fisica and CNISM, Università di Pisa, Italy*

Highly magnetized pure electron plasmas in Malmberg-Penning traps [1] provide the opportunity to experimentally investigate two-dimensional (2D) fluid turbulence. In fact, under suitable experimental conditions, the transverse dynamics of the electron plasma column is well described by the drift-Poisson equations [2], which are isomorphic to the 2D Euler equations for an incompressible, inviscid fluid, whose vorticity corresponds, up to a constant of proportionality, to the electron plasma density. In recent years, the scaling properties and dynamics of 2D turbulence in electron plasmas have been investigated both by Fourier transform [3] and wavelet analysis [4, 5]. In this work the scaling and statistical properties of freely decaying 2D turbulence in a pure electron plasma confined in a Malmberg-Penning trap are studied experimentally. The intermittency properties of turbulence are investigated by analyzing the probability density functions and the structure functions of plasma density (vorticity) increments. The occurrence of the Yaglom law for the third-order mixed moment involving the plasma density and the drift velocity is also studied.

The experimental data have been obtained in the Malmberg-Penning trap ELTRAP [6]. A low density ($n = 10^{12} - 10^{13}$ m^{-3}) and temperature ($T = 1 - 10$ eV) electron plasma is generated by a thermoionic spiral cathode heated with a constant current and negatively biased with respect to a grounded grid. The voltage drop across the filament and the source bias set the initial plasma radius, shape and density. The electrons are axially trapped within a stack of hollow conducting cylinders (with radius $R_W = 4.5$ cm) by two static negative voltages at the trap ends, and radially confined by an axial magnetic field B which keeps the charged column in equilibrium rotation. After being injected into the device, the electrons are trapped for a given time and then dumped onto a phosphor screen. The light emitted by the screen is collected by a 12 bit charge-coupled device (CCD) camera, so that the light intensity measured at a given position on the CCD sensor is proportional to the axially averaged electron density. A 2D image acquired by the CCD thus provides the density distribution and represents also the vorticity $\zeta(x,y,t)$ of the 2D

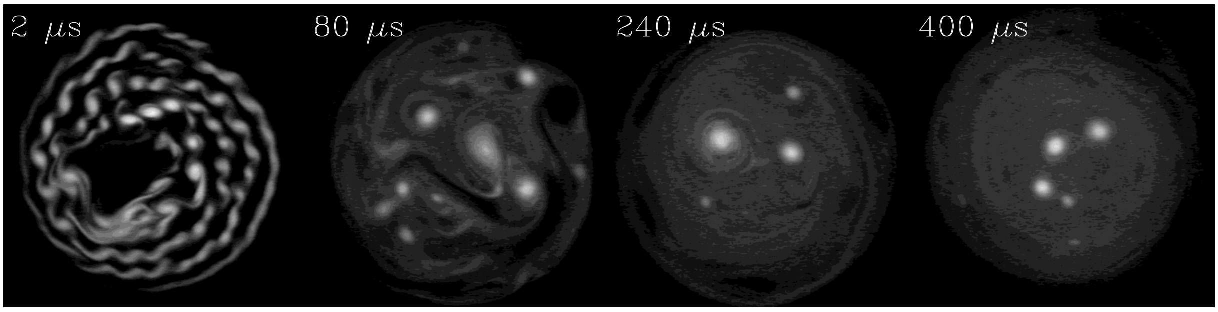

Figure 1: Snapshots of the plasma density for the analysed sequence. The trapping time is indicated at the top left corner of each frame.

fluid. The time evolution is studied by repeating the above described cycle several times with fixed injection parameters and increasing the trapping time τ. The shot-to-shot reproducibility of the initial conditions is very high, as the typical variation of the measured charge at a given position is less than 0.1 %. The sequence under study in this work consists of $N = 250$ frames with a trapping time step of 2 μs. A few snapshots of the plasma density are shown in Fig. 1. The first frame (trapping time $\tau = 2$ μs) reflects the shape of the spiral cathode distorted by the diocotron instability, which rapidly leads to a nonlinear evolution of the flow.

In turbulence studies, the analysis of the scaling properties of the field increment statistics provides information about the presence of coherent structures, such as vortices or shocks, and about their typical spatial scales [7]. For a turbulent field $u(\boldsymbol{r},t)$ the increments across a scale separation $\boldsymbol{l}$ are defined as $\Delta u_{\boldsymbol{l}}(\boldsymbol{r},t) = u(\boldsymbol{r}+\boldsymbol{l},t) - u(\boldsymbol{r},t)$. In the present work the focus is on the properties of the 2D vorticity $\zeta(x,y,t)$ in a pure electron plasma. We thus analyze the scaling behaviour of the increments in both x and y directions

$$\begin{aligned}
\Delta\zeta_l^{(x)}(x,y,t) &= \zeta(x+l,y,t) - \zeta(x,y,t)\,, \\
\Delta\zeta_l^{(y)}(x,y,t) &= \zeta(x,y+l,t) - \zeta(x,y,t)\,.
\end{aligned}$$

Two standard ways to study the statistics of field increments and the intermittency effects in turbulent flows are the structure functions and the Probability Density Functions (PDFs). Structure functions are defined as the moments of field increments, that is, $S_p(l) = \langle \Delta\zeta_l^p \rangle$, where $\langle\cdot\rangle$ denotes spatial averages. A measure of the intermittency of field increments is given by the flatness F, the ratio of the 4th order moment to the square of the 2nd order moment, $F(l) = S_4(l)/[S_2(l)]^2$. The flatness is 3 by definition for a Gaussian PDF. The flatness of the vorticity increments along the x and y directions for different trapping times is shown in Fig. 2. For $\tau = 2$ μs F is very close to 3 at all scales. As τ increases, F becomes larger and larger and, most importantly, a significant increase of F is found in the range 1.5 mm $\leq l \leq$ 8 mm,

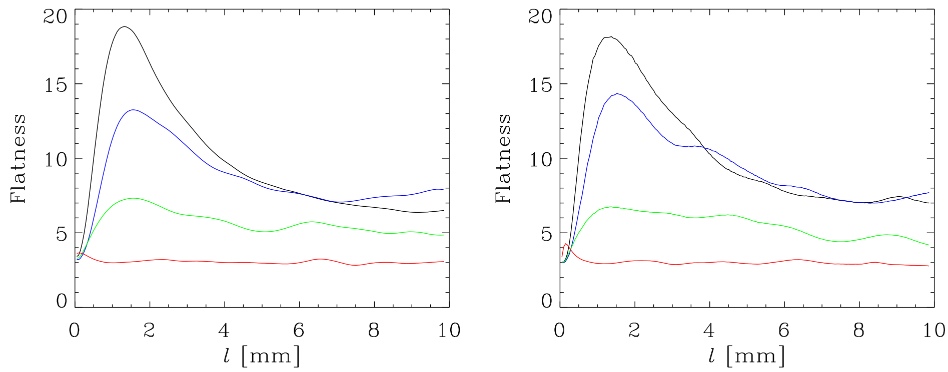

Figure 2: Flatness of the vorticity increments along the x (left panel) and y (right panel) directions for different trapping times: 2 μs (red curve), 80 μs (green curve), 240 μs (blue curve), 400 μs (black curve).

indicating that intermittency gets stronger as turbulence developes. For $l \leq 1.5$ mm, F decreases and tends towards 3, probably due to the effect of instrumental noise. The PDFs of the standardised vorticity increments $\Delta\zeta_{st}$ along the x direction for $\tau = 80$ μs and $\tau = 400$ μs are shown in Fig. 3. The PDFs of increments along the y direction have similar shapes. Fig. 3 shows that the intermittency increase is related to the development of non-Gaussian tails produced by the turbulence evolution.

The mixed third order structure functions $Y_3^{(x)}(l) = \langle \Delta v_l^{(x)} [\Delta\zeta_l^{(x)}]^2 \rangle$ and $Y_3^{(y)}(l) = \langle \Delta v_l^{(y)} [\Delta\zeta_l^{(y)}]^2 \rangle$ were also studied, where $\Delta v_l^{(x)} = v_x(x+l, y, t) - v_x(x, y, t)$ and $\Delta v_l^{(y)} = v_y(x, y+l, t) - v_y(x, y, t)$.

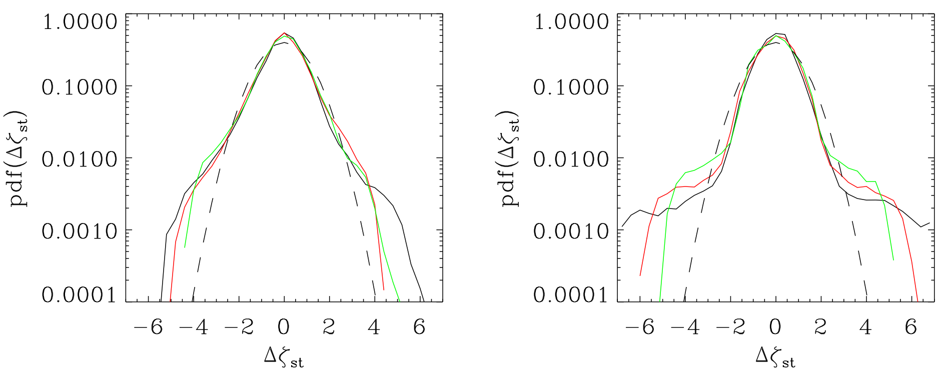

Figure 3: PDFs of the standardised vorticity increments $\Delta\zeta_{st}$ along the x direction for $\tau = 80$ μs (left panel) and $\tau = 400$ μs (right panel) and for different spatial separations l: $l = 0.98$ mm (black solid curve), $l = 4.9$ mm (red curve), $l = 9.8$ mm (green curve). The Gaussian PDF with zero mean and $\sigma = 1$ is also shown for comparison (black dashed curve).

The velocity field $\boldsymbol{v}(x,y)$ is calculated from the electrostatic potential $\phi(x,y)$, obtained by solving numerically the Poisson equation with the measured plasma density field $n(x,y)$. For homogeneous, stationary, isotropic turbulence, forced at large scales, the analog of the Yaglom law $Y_3(l) = -(4/3)\eta l$ is expected to hold in the inertial range of turbulence[8], η being the enstrophy dissipation rate. The typical behaviour of $Y_3(l)$ obtained for the experiment considered in this work is reported in Fig. 4. It can be seen that the range of scales where $Y_3(l)$ follows the behaviour expected from Yaglom law is rather short.

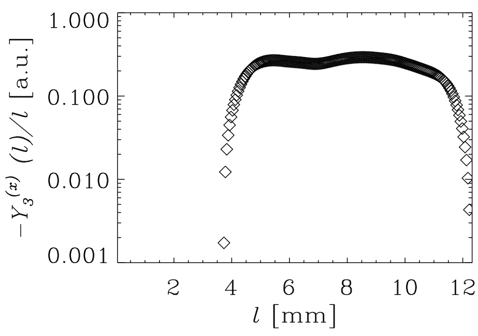

Figure 4: Compensated third order mixed structure functions $-Y_3^{(x)}(l)/l$ for trapping time $\tau = 80\ \mu$s.

In conclusion, in the present work the scaling properties of 2D turbulence in a pure electron plasma were studied. It was shown that, as the turbulence evolution proceeds, intermittency increases, due to the development of strong fluctuations which give rise to non-Gaussian tails in the PDFs of vorticity increments.

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