Reflectometry diagnostics operation limitations caused by strong Bragg back scattering

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The microwave reflectometry is widely used nowadays and considered as key diagnostics in ITER, for plasma positioning, density profile reconstruction, and also able to provide measurements on turbulence. Both ordinary (O) and extraordinary (X) mode reflectometers launching probing wave from high and low magnetic field are developed [1]. On ITER, the fast frequency sweep X-mode reflectometer [1] should be able to diagnose very flat density profiles, which correspond to long probing wave paths (several meters). Under these unfavorable conditions the reflectometers can suffer from destructive influence of plasma turbulence which may lead on one hand to multiple small angle scattering or strong phase modulation and on the other hand to a strong Bragg backscattering (BBS) or to an anomalous reflection. In the BBS case the probing wave reflection occurs far from the cut-off position thus complicating the density profile reconstruction. Recently this effect was analyzed using quasi coherent density fluctuation model. It was shown that in the case of large enough fluctuation the probing wave amplitude decreases exponentially in the vicinity of the Bragg resonance point so that strong reflection occurs far from the cut off point [2]. The threshold level of density perturbations dangerous for reflectometry diagnostic performance determined there appeared to be in the 1% range for fluctuations satisfying the Bragg condition. This threshold provides especially strict limitation for high field side X-mode reflectometry utilizing frequencies in the 40 GHz frequency range and therefore sensitive to backscattering off density fluctuations in centimeter range usually excited in tokamak at 1% level.

In the present paper the probing wave propagation in turbulent plasma is analyzed in the framework of weak turbulence theory approach utilizing a more realistic random density fluctuation model. Equations for averaged incident and reflected wave energy flux densities describing diffusive regime of wave propagation are derived and analyzed. Results of analytical treatment are compared to the outcome of full wave modeling and shown to be in reasonable agreement. The criterion for transition to nonlinear BBS regime is obtained for arbitrary density profile, turbulence spectra and type of probing wave. The turbulence level critical for performance of reflectometry diagnostics at ITER is determined for scenarios discussed in [3].

Basic equations and theoretical approach.

Considering an incident wave propagating perpendicular to the external magnetic field (O- or X-mode), we treat it assuming slab plasma geometry in the framework of equation

$$\left\{\frac{\partial^2}{\partial x^2}+k_\alpha^2\left(x\right)+\delta k_\alpha^2\left(x\right)\right\}E_\alpha\left(x\right)=0\text{, where }\delta k_\alpha^2=h_\alpha\frac{\omega^2}{c^2}\frac{\delta n\left(x\right)}{n_c},\qquad(1)$$

$\alpha=o$ and $\alpha=e$ correspond to O- and X-mode, $E_o\equiv E_z$, $E_e\equiv E_y$, and $k_o^2=\frac{\omega^2}{c^2}\left[1-\upsilon\right], k_e^2=\frac{\omega^2}{c^2}\frac{\left[\left(1-\upsilon\right)^2-u\right]}{1-u-\upsilon}$, $\upsilon=\frac{\omega_{pe}^2}{\omega^2}, u=\frac{\omega_{ce}^2}{\omega^2}, h_o=1, h_e=\frac{\left[1-2\upsilon\right]\left[1-u\right]+\upsilon^2}{\left[1-u-\upsilon\right]^2}$, as shown in [4]. The perturbation of the background density $\overline{n}\left(x\right)$ is given by superposition of m random harmonics possessing random phases φ_j $\delta n\left(x\right)=\sum_{j=1}^{m}\delta n_j\cos\left(\kappa_j x+\varphi_j\right)$. We solve (1) numerically and investigate its solutions analytically using methods developed in the weak turbulence theory [5]. In the later case assuming weak plasma inhomogeneity $\kappa_j x_c>>1$ we seek a solution to (1) in the form of the incident and reflected WKB waves propagating in both directions:

$$E_\alpha\left(x\right)=\frac{a_i\left(x\right)}{\sqrt{v_{g\alpha}\left(x\right)}}\exp\left(i\int^x k_\alpha\left(x'\right)dx'\right)+\frac{a_r\left(x\right)}{\sqrt{v_{g\alpha}\left(x\right)}}\exp\left(-i\int^x k_\alpha\left(x'\right)dx'\right)\qquad(2)$$

where $v_{g\alpha}$ stands for the group velocity. The amplitudes a_i and a_r vary slowly at the wave length scale due to Bragg backscattering which is described by dynamic equations

$$\begin{cases} i\frac{da_i}{dx}=-\left(\frac{\omega}{c}\right)^2\frac{h_\alpha}{2k\left(x\right)}\sum_{j=-m}^{m}\frac{\delta n_j}{n_c}a_r\exp\left(i\Phi_j\right)\\ i\frac{da_r}{dx}=\left(\frac{\omega}{c}\right)^2\frac{h_\alpha}{2k\left(x\right)}\sum_{j=-m}^{m}\frac{\delta n_j}{n_c}a_i\exp\left(-i\Phi_j\right)\end{cases}\qquad(3)$$

where $\Phi_j(x)=\int^x\left[\kappa_j-2k_\alpha\left(x'\right)\right]dx'+\varphi_j$ is the phase mismatch of three wave resonant interaction. Assuming nonlinear wave interaction weak for each fluctuation we represent the incident and reflected wave amplitude in the form of expansion over the parameter proportional to the fluctuation amplitude $a_{i,r}\left(x\right)=a_{i,r}^{(0)}+a_{i,r}^{(1)}\left(x\right)+a_{i,r}^{(2)}\left(x\right)$, where $a_{i,r}^{(0)}$ are amplitudes not affected by Bragg backscattering and $a_{i,r}^{(1)}\left(x\right), a_{i,r}^{(2)}\left(x\right)$ are random corrections describing quick variations of the amplitude due to BBS. These corrections are obtained from (3) using the perturbation method as

$$a_i^{(1)}=-a_r^{(0)}\left(\frac{\omega}{c}\right)^2\int_{x_0}^{x}\frac{h_\alpha}{2ik\left(x'\right)}\sum_{j=-m}^{m}\frac{\delta n_j}{n_c}\exp\left(i\Phi_j\right)dx';\qquad a_r^{(1)}=a_i^{(0)}\left(\frac{\omega}{c}\right)^2\int_{x_0}^{x}\frac{h_\alpha}{2ik\left(x'\right)}\sum_{j=-m}^{m}\frac{\delta n_j}{n_c}\exp\left(-i\Phi_j\right)dx';$$

$$a_i^{(2)}=a_i^{(0)}\left(\frac{\omega}{c}\right)^4\int_{x_0}^{x}\int_{x_0}^{x}\frac{h_\alpha\left(x'\right)}{4k\left(x'\right)}\frac{h_\alpha\left(x''\right)}{k\left(x''\right)}\sum_{l=-m}^{m}\sum_{j=-m}^{m}\frac{\delta n_j}{n_c}\frac{\delta n_l}{n_c}\exp\left(i\Phi_j\left(x'\right)-i\Phi_l\left(x''\right)\right)dx'dx'';$$

$$a_r^{(2)}=a_r^{(0)}\left(\frac{\omega}{c}\right)^4\int_{x_0}^{x}\int_{x_0}^{x}\sum_{l=-m}^{m}\sum_{j=-m}^{m}\frac{\delta n_j}{n_c}\frac{\delta n_l}{n_c}\exp\left(-i\Phi_j\left(x'\right)+i\Phi_l\left(x''\right)\right)dx'dx''\frac{h_\alpha\left(x'\right)}{4k\left(x'\right)}\frac{h_\alpha\left(x''\right)}{k\left(x''\right)}.$$

The obtained random corrections allow estimating the spatial derivative of the averaged probing and reflected wave amplitude squared. Assuming random statistically homogeneous density fluctuations we obtain after averaging the following system for flux density of photons propagating inward ($S_i = \left\langle \left| a_i(x) \right|^2 \right\rangle$) and outward ($S_r = \left\langle \left| a_r(x) \right|^2 \right\rangle$):

$$\begin{cases} \dfrac{dS_i}{dx} = -\nu(x) S_i + \nu(x) S_r \\ \dfrac{dS_r}{dx} = -\nu(x) S_i + \nu(x) S_r \end{cases} \quad (4)$$

Where $\nu(x) = \dfrac{2\pi h_\alpha^2}{\left[2k(x)\right]^2} \left(\dfrac{\omega}{c}\right)^2 \left| \dfrac{\delta n_j}{n_c} \right|^2_{\kappa_j = 2k(x)}$. This system describes the energy exchange between incident and reflected wave due to the BBS. It is easy to show that solution of this system satisfy the conservation of total photon flux $\dfrac{d\Gamma}{dx} = 0$, where $\Gamma = S_i - S_r$ natural in the absence of damping. On the other hand using (4) and introducing the photon density $N = S_i + S_r$ it is easy to show that the total photon flux satisfies the Fick's law $\Gamma = -\dfrac{1}{2\nu}\dfrac{dN}{dx}$.

The general solution of (4) takes a form $S_i = -c_1 \int_0^x \nu(x') dx' + c_2$; $S_r = -c_1 \int_0^x \nu(x') dx' + c_2 - c_1$, where $c_{1,2}$ are constants determined using boundary conditions at plasma boundary $x=0$ and in cut off. In the case the cut off $x=x_c$ is present in plasma, as it should be in the reflectometry experiment, we put in there the condition $S_i = S_r \big|_{x=x_c}$ and obtain the solution of (4) $S_i = S_r = const$ corresponding to vanishing total photon flux. This solution is typical for the stationary problem and provides no indication of transition to strong BBS regime. In the case there is no cut-off in plasma we should put condition $S_r \big|_{x=a} = 0$ at the second boundary at $x = a$ and obtain the following expression for the transmission coefficient:

$$T = \left[1 + \int_0^a \nu(x') dx' \right]^{-1} \quad (5)$$

As it is seen substantial suppression of the probing wave transmission is achieved at condition

$$\int_0^a \nu(x') dx' \geq 1 \quad (6)$$

which determines the transition to the strong BBS regime.

Numerical analysis.

Using equation (1) we performed full-wave modeling of O-mode probing wave propagation in a linear density profile with a density gradient length L_n= 163 λ_o in presence of intensive small scale turbulence possessing spectrum $\left| \delta n_j \right|^2 = const$ for $0 < \kappa_j < 2\omega / c$.

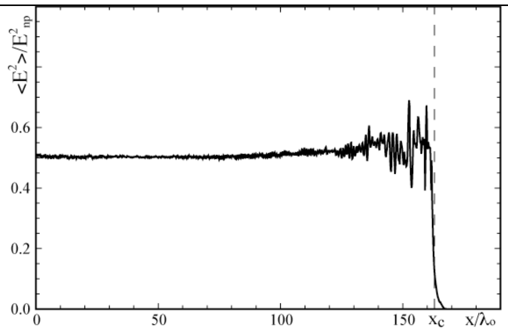

Fig.1 Probing electric field squared averaged over 10^5 samples versus position in the plasma. Density fluctuation RMS value 5%.

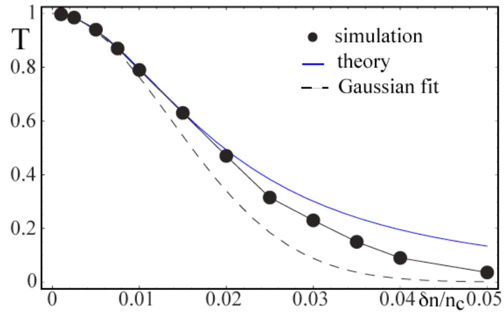

Fig.2. Dependence of the transmission coefficient in the absence of cut off on the density fluctuation RMS.

As it is shown in Fig.1, in the cut-off presence the averaged electric field squared multiplied by group velocity is constant in the plasma volume, far enough from cut-off, in agreement with analytical theory prediction. The computational results for the transmission coefficient are shown in Fig.2 in nice agreement to the analytics. The threshold level of density perturbations obtained for ITER X-mode high field side reflectometry experiment using criterion (6) is shown in Fig.3 against the major radius. The turbulence radial wave number spectrum there was supposed Gaussian with the correlation length equal to the ion Larmor radius calculated for the local ion temperature, whereas the density fluctuation RMS was taken constant along the major radius thus ensuring the growth of relative density perturbation towards the edge. The flat density profile shown in the upper part of the figure was supposed. As it is seen in Fig.3, the strong Bragg backscattering can occur for probing frequency of 40 GHz far from the cut-off already at the density perturbation of 1% and smaller comparable to that found in the present day tokamak experiments.

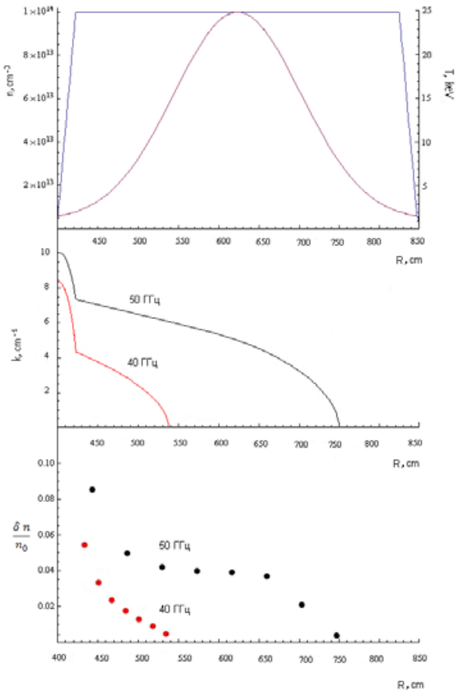

Fig.3: Strong BBS threshold. Density and temperature profiles(top);X-mode wave number (middle); Density perturbation threshold (bottom).

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