Self-consistent non-linear radio-frequency wave propagation and peripheral plasma biasing

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I. Motivation.

The emission of Radio-Frequency (RF) waves by complex antennae and their damping in the core of magnetized plasmas have been described for a long time by sophisticated first principle models in realistic geometry. Comparatively, the simulation of anomalous RF power losses in the plasma edge is still less advanced, although the non-linear wave-plasma interactions in the plasma edge often set the operational limits of RF heating systems. Peripheral Ion Cyclotron (IC) wave damping is attributed to a DC biasing of the edge plasma by RF-sheath rectification [1]. This non-linear process is usually modelled in analogy with a double Langmuir probe driven by an oscillating RF voltage [2]. This RF voltage is estimated as the field line integral of RF fields $E_{//}$ parallel to the confinement magnetic field $\mathbf{B_0}$, as computed from antenna codes in the absence of sheaths. This approach, although not self-consistent, reproduces qualitatively the poloidal variation of RF-induced SOL modifications around powered antennae [3], [4]. Yet its quantitative validity is questionable [5]. Moreover, contradictions appear with recent measurements, e.g. the radial penetration of the plasma bias [6] or the non-linear generation of edge DC currents by RF waves [4], [7], [8]. In view of ITER, improvement of RF-sheath physics towards first principles is needed.

II. Outline of the physical model.

This paper treats self-consistently the slow magnetosonic wave penetration and the edge plasma DC biasing using a two-field fluid approach, inspired by earlier work on RF plasma discharges [9-11] and recently applied to tokamaks [5], [12]. The simulation domain, sketched on figure 1, is a collection of bounded magnetic field lines in the Scrape-Off Layer (SOL) of magnetized plasma. Inside this domain, a wave equation propagates the parallel RF electric field $E_{//}$, i.e. in first approximation the slow wave. Assuming harmonic time variations as $\exp(-i\omega_0 t)$ at the RF pulsation ω_0 the RF wave equation takes the form

$$\varepsilon_{//}\Delta_{//}E_{//} + \varepsilon_{\perp}\Delta_{\perp}E_{//} + \varepsilon_{//}\varepsilon_{\perp}\left(\omega_0/c\right)^2 E_{//} = 0 \qquad (1)$$

Here $\varepsilon_{//}$ and $\varepsilon_{\perp}$ are the diagonal elements of the plasma dielectric tensor at pulsation ω_0. The whole RF+DC system is excited by imposing a map of $E_{//}$ at the outer boundary of the machine from an antenna code. The local DC plasma potential V_{DC} is governed by the continuity equation for DC currents in presence of anisotropic DC conductivity ($\sigma_{//\mathrm{DC}}$,$\sigma_{\perp\mathrm{DC}}$).

$$\sigma_{//DC}\Delta_{//}\phi_{DC} + \sigma_{\perp DC}\Delta_{\perp}\phi_{DC} = 0 \quad ; \quad \phi_{DC} \equiv eV_{DC}/kT_e \qquad (2)$$

In this expression $\sigma_{//\mathrm{DC}}$ is the Spitzer parallel conductivity. Rigorously the transverse DC conductivity takes the form of a diffusion operator $\sigma_{\perp\mathrm{DC}}\Delta_{\perp}\phi_{DC}$ only in the case of ion-neutral collisions [13]. Other processes of transverse effective conductivity were outlined in the literature, due to ion viscosity, inertia or anomalous transport [13]. They generally involve collisions and could be represented phenomenologically as a small effective perpendicular conductivity $\sigma_{\perp\mathrm{DC}}$.

Sheaths at both ends of open flux tubes have a characteristic size δ extremely small compared to that of the simulation domain ($L_{//}$). Instead of being resolved spatially they are described here by RF and DC sheath boundary conditions (SBC). With a non-linear I-V characteristic, sheaths couple RF and DC fields ($E_{//},\phi_{DC}$). For RF waves, sheaths behave as dielectric layers between the main plasma and the conducting walls. The RF sheath capacitance was calculated rigorously for unmagnetized plasma processing reactors [9], [10]. No equivalent presently exists with tilted $\mathbf{B_0}$, like the Chodura's model for DC sheaths [14]. For walls normal to $\mathbf{B_0}$, sheaths are unmagnetized and the RF SBC only involves $E_{//}$ [5].

$$\partial_n E_{//} = -\varepsilon_\perp \Delta_\perp \left(\delta E_{//} / \varepsilon_{sh}\right) \qquad (3)$$

In this expression ε_{sh} is the dielectric constant of the sheath, of order 1. δ represents the width of the dielectric layer. δ is of the order of the sheath thickness and therefore depends on the DC potential *via* the Child-Langmuir law, thus providing a first RF-DC coupling

$$\delta = \lambda_e \phi_{DC}^{3/4} \quad ; \; \lambda_e \equiv \left[\varepsilon_0 k T_e / e^2 n\right]^{1/2} \text{ electron Debye length} \qquad (4)$$

The DC SBCs express the continuity between the DC current in the plasma and the DC conduction current of a sheath subject to a DC potential ϕ_{DC} plus an oscillating RF potential $\phi_{RF}\cos(\omega_0 t)$ whose amplitude ϕ_{RF} depends on the local RF electric field $E_{//}$ and on δ

$$i^+[1-\exp(\phi_b-\phi_{DC})]=kT_e\boldsymbol{\sigma}_{\mathbf{DC}}\nabla_n\phi_{DC} \; ; \; \phi_b\equiv\phi_f+\ln[I_0(|\phi_{RF}|)] \; ; \; \phi_{RF}\equiv -e\delta\varepsilon_{//}E_{//}/\varepsilon_{sh}kT_e \qquad (5)$$

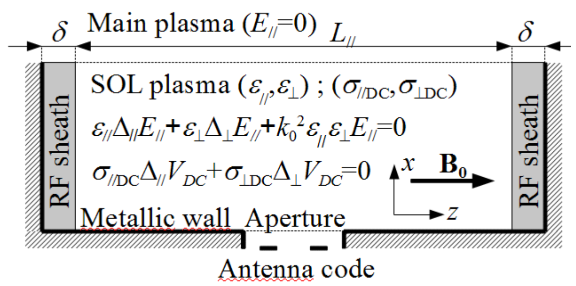


Figure 1: Sketch of the simulation domain. Outline of the model and main notations.

Here $I_0(\phi) \equiv \int_0^\pi \exp[\phi\cos(\varphi)]d\varphi/\pi$ is the modified Bessel function of order 0. i^+ is the ion saturation current and ϕ_f is the plasma potential in absence of RF waves. Equations (2) and (5) define a standard plasma biasing problem: although the vessel walls are electrically grounded, the sheath rectification of ϕ_{RF} acts in practice as an inhomogeneous DC bias at the positive potential ϕ_b-ϕ_f. In the absence of volume source term in eq. (2) this second RF-DC coupling uses RF fields $E_{//}$ to drive the DC biasing from field line extremities. The RF+DC model implicitly assumes negligible role of harmonics of ω_0 in the rectification.

III. Two mechanisms of non-linear DC current generation from RF sheath rectification.

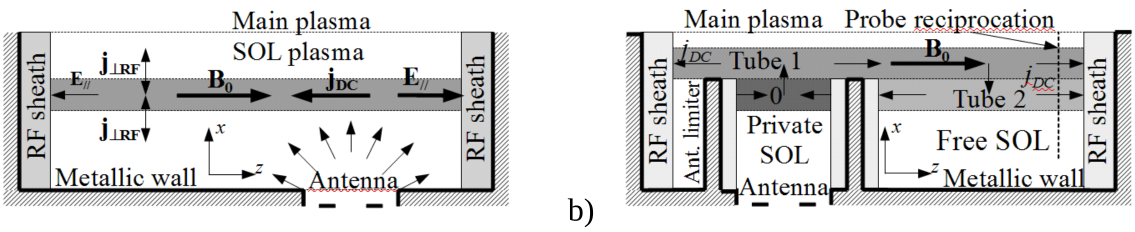
The new approach enforces zero outwards DC current only *on average* over the vacuum vessel and not *in every point* of the boundary. This allows DC currents to flow all over the simulation domain, reducing earlier inconsistency with measurements. Model [5] is recovered when ($\sigma_{//DC},\sigma_{\perp DC}$)=(0,0). Two DC current generation mechanisms, generic of any biasing process and not specific of RF waves, are outlined in figures 2. In steady-state regime, any DC current j_{DC} injected from one extremity of an open field line must exit the flux tube. j_{DC} can flow through the opposite sheath. This first mechanism implies breaking the left/right symmetry of the flux tube, e.g. by unbalanced bias ϕ_b [15,16]. In practice this is produced by an asymmetric map of $E_{//}$ (see figure 2.a). Taking ($\sigma_{//DC},\sigma_{\perp DC}$)=(∞,0) yields [16]

$$j_{DC}/i^+ = \left[I_0(|\phi_{RFl}|) - I_0(|\phi_{RFr}|)\right] / \left[I_0(|\phi_{RFl}|) + I_0(|\phi_{RFr}|)\right] \; ; \; \text{l/r}\equiv\text{left/right} \qquad (6)$$

The magnitude of $|j_{DC}|$ is limited to i^+ by the saturated sheath at low $|V_{RF}|$ extremity. j_{DC} flows from the high-$|V_{RF}|$ side to the low-$|V_{RF}|$ side of the flux tube, consistently with most experimental observations [4] [7] [8]. Detailed study shows that although no DC transverse conductivity is required, transverse RF current exchanges with neighbouring flux tubes *via* $\varepsilon_\perp$ are essential to break the left/right symmetry [15]. When both sheaths of a field line are driven symmetrically, symmetry imposes j_{DC} to exit across the lateral sides of the flux tube.

Within this second mechanism, DC current loops arise from the differential biasing of adjacent flux tubes connected electrically *via* a finite DC transverse conductivity $\sigma_{\perp DC}$ (see figure 2.b). Assuming toroidal homogeneity (e.g. by taking $\sigma_{//DC}=\infty$), eq. (2) yields

$$j_{DC}=0.5L_{//}\sigma_{\perp DC}\Delta_{\perp}V_{DC} \qquad \text{across both sheaths, } L_{//}\equiv\text{flux tube length} \qquad (7)$$



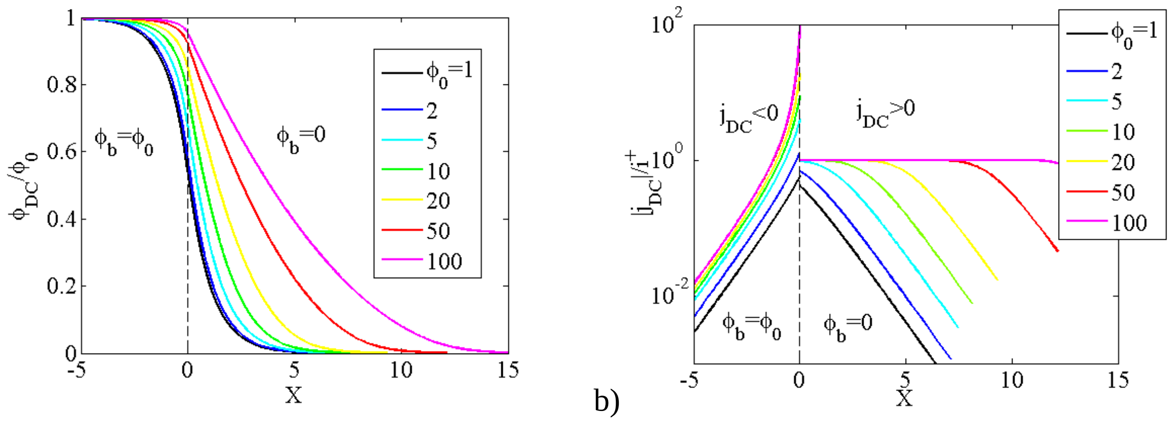
a) b)

Figure 2: Sketch of the 2 DC current generation schemes. Darkness is representative of DC potential magnitude.

To gain insight into the non-linear biasing physics and to benchmark the numerical codes, the 1D (radial) biasing problem (2)+(5) was solved semi-analytically when $\sigma_{//DC}\sim\infty$. A positive bias $\phi_b=\phi_0$ was imposed at both extremities of the outer flux tubes (radial coordinate $x<0$). Figures 3 plot the DC plasma potential ϕ_{DC} and the DC current j_{DC} versus x normalized to a characteristic current diffusion length $\lambda_j\equiv[L_{//}\sigma_{\perp DC}kT_e/2ei^+]^{1/2}$. It can be shown that

$$X\equiv x(\phi_{DC})/\lambda_j=\begin{cases}F(\phi_{DC}(0))-F(\phi_{DC}) \quad ; & 0\le\phi_{DC}\le\phi_{DC}(0)\\ F(\phi_{DC}(0)-\phi_0)-F(\phi_{DC}-\phi_0) \quad ; & \phi_{DC}(0)\le\phi_{DC}\le\phi_0\end{cases} \quad \text{with} \quad (8)$$

$$\phi_{DC}(0)=\ln[\exp(\phi_0)-1]-\ln(\phi_0) \quad ; \quad F(\phi)\equiv\int d\psi/[2(\psi-1+\exp(-\psi))]^{1/2} \qquad (9)$$



a) b)

Figure 3: profiles of ϕ_{DC}/ϕ_0 a) and j_{DC}/i^+ b) versus normalized radial coordinate $X=x/\lambda j$, for several values of ϕ_0.

Figures 3 show that in the linear sheath regime ($\phi_0<1$) the DC current diffusion performs a smoothing of the bias $\phi_b(X)$ over a characteristic width of order $\delta X\sim1$ ($\delta x\sim\lambda_j$), where significant DC currents are found. Similar smoothing was observed for RF potentials in presence of transverse RF conductivity [6]. For large bias ($\phi_0>>1$) the region of large ϕ_{DC} and j_{DC} gets broadened and becomes spatially asymmetric, due to the saturation of the sheaths. Global j_{DC} conservation, illustrated on figure 3, predicts that a Langmuir probe exploring the vicinity of an IC antenna should experience both positive and negative j_{DC}. In order to recover the probe measurements on Tore Supra (always positive j_{DC} [8]) using this mechanism, a "private SOL" between antenna limiters has to be invoked, inaccessible to the probe and biased to high V_{DC} to draw negative j_{DC}. Using the flux tube numbering in figure 2.b it can be shown that ($\phi_{b0}\ge\phi_{b1}\ge\phi_{b2}$) implies ($\phi_{b0}\ge\phi_{DC0}\ge\phi_{DC1}\ge\phi_{DC2}\ge\phi_{b2}$). In these conditions positive j_{DC} can appear at the extremities of both flux tubes 1 and 2 that are accessible to the probe.

Actual DC current flows combine the two basic mechanisms outlined above. Further study is needed to quantify which process is dominant in realistic SOL plasmas and geometry.

IV. Numerical implementation and validation: status and prospects, project SSWICH.

Presently the above coupled RF+DC model is being implemented numerically in two-dimensional (2D: toroidal/radial) rectangular geometry, both with finite difference and finite element methods. Due to the non-linear boundary conditions the DC model is solved iteratively using a damped Newton method. In its stand-alone version the RF code has been successfully compared to analytic expressions of Slow Wave and Sheath Plasma Wave eigenmodes in a homogeneous plasma-filled waveguide with homogeneous sheath width δ [17]. It was checked that the numerical error of the finite difference scheme scales like the square of the spatial grid steps. When ϕ_{DC} was toroidally homogeneous, the 2D numerical DC model was also tested against non-linear 1D solutions (8)-(9). When used alone the DC iterative procedure always converged to a solution. The RF and DC parts were also coupled self-consistently but converge only for low amplitudes of the driving $E_{//}$. Both DC current generation mechanisms could be reproduced numerically, including for the first time the complex geometry with a private SOL and RF sheaths on antenna limiters (see *figure 4*).

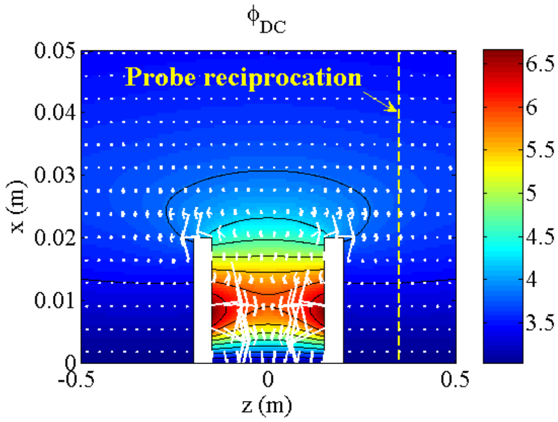


Figure 4: Self-consistent RF+DC test simulation with private SOL, RF sheaths on limiters and symmetric drive. 2D mapping of ϕ_{DC}. Arrows: DC current flow.

More detailed numerical validation is on-going. Simplified forms of the model can already be used for RF-sheath estimates around the ITER IC antenna. The formalism allows improvements towards more realistic edge RF physics description: full RF field polarization, shaped wall, 3D geometry, RF excitation by straps... This scientific program defines project SSWICH (Self-consistent Sheaths and Waves for Ion Cyclotron Heating) by CEA and IJL.

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V. References

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