A Fokker-Planck Code for Fast Self-Consistent Calculations of ICRH

A. Hannan, T. Hellsten and T. Johnson

School of Electrical Engineering, KTH, Association VR-Euratom, Sweden.

Modelling of ion cyclotron heating, ICRH, requires self-consistent calculations of the distribution function of the resonant ion species and of the wave field. A Fokker-Planck code has been developed suitable for fast routine simulations. The distribution function is obtained by solving a pitch angle averaged function with an 1D, time dependent Fokker-Planck code using cubic finite elements. The averaged parallel velocity is calculated from the distribution function with an analytic formula [1, 2], which is necessary in order to be able to calculate the Doppler broadening at the cyclotron resonance and the effect on the wave field near the resonance. The wave-field required in the quasi-linear operator is calculated with the LION code [3, 4]. The global distribution function is obtained by dividing the plasma volume in a number of shells in which the distribution function is calculated. In order to be able to calculate the effect on power partition and wave field of a the RF-heated distribution functions response functions are calculated from the distribution functions compatible with the wave field representation in the LION code. The Fokker-Planck code has been implemented in the self-consistent code SELFO-light enabling self-consistent calculations of wave field and distribution functions.

FOKKER-PLANCK EQUATION

The distribution functions of the resonant ion species are calculated by solving an 1D time dependent Fokker-Planck equation, $F_i(\upsilon,t) = 4\pi\upsilon^2 f_i(\upsilon,t)$, on an arbitrary number of shells defined by magnetic flux surfaces

$$\frac{\partial F_i}{\partial t} = \frac{\partial}{\partial \upsilon}\left[A_i F_i + B_i \frac{\partial F_i}{\partial \upsilon} \right] + S_i$$

where $A_i = \left(\frac{p_i}{\upsilon^2} - \frac{\beta_i + H_i}{\upsilon} \right)$, $B_i = \frac{1}{2}(\beta_i + H_i)$, $p_i = \sum_f C_f \frac{m\upsilon^2}{2\kappa T_f} G(l_f \upsilon)$,

$H_i = \sum_{n_\phi, n_\omega} H_{n_\phi, n_\omega; i} \left| E_{n_\phi, n_\omega +} J_{n-1}\left(k_{\perp n_\phi, n_\omega}\rho_i\right) + e^{-2i\alpha} E_{n_\phi, n_\omega -} J_{n+1}\left(k_{\perp n_\phi, n_\omega}\rho_i\right) \right|^2$ is the quasi-linear operator summed over frequency n_ω and toroidal mode number spectrum n_ϕ and the Larmor radius

$\rho_i = v/\omega_{ci}$. The quasi-linear operator describing the wave absorption is calculated by averaging the wave field and the perpendicular wave number along the flux surfaces with the local absorbed power at each harmonic of the cyclotron frequency. The electric field is calculated to be consistent with a given antenna spectrum, required as an input to the code. $\beta = \sum \beta_f$ is one of Chandrasekhar's collision operators [5] summed over the different background species: ions and electrons, regarding other notations see Ref. [5]. The source term S_i describes NBI or particles arising from thermonuclear reactions.

The $F_i(\upsilon,t)$ is discretised by $F^t = \sum_{i=1}^{n} \varphi_{2i-1}(\upsilon) F_{2i-1}^t + \sum_{i=1}^{n} \varphi_{2i}(\upsilon) F_{2i}^t$, where φ_{2i-1} and φ_{2i} are standard cubic FEM basis functions with $\varphi_{2i-1}(\upsilon_i) = 1$, $\varphi'_{2i}(\upsilon_i) = 1$, $\varphi'_{2i\pm2}(\upsilon_i) = 0$ and $\varphi_{2i\pm2-1}(\upsilon_i) = 0$, where υ_i is the central value of the cell. The time evolution is obtained by solving the following system of linear equation

$$F_i^{t+\Delta t} = \sum_{j=1}^{N} \left[D_{ij} - k\Delta t C_{ij} \right]^{-1} \left[D_{ij} + (1-k)\Delta t C_{ij} \right] F_k^t ,$$

The parameter k changes the scheme continuously from a fully explicit scheme for $k = 0$ to a fully implicit scheme for $k = 1$.

The parallel temperature used in LION code, important for the polarisation of the waves near the fundamental cyclotron resonance, is obtained from the averaged parallel velocity of the ions determined with an heuristic formula [1, 2] from the calculated distribution function $\langle \upsilon_\parallel^2 \rangle = \int \mu_{eff}^2 \upsilon^2 F(\upsilon) d\upsilon$, where $\mu_{eff}^2 = \left(1 + \left(2\upsilon/\upsilon_\gamma\right)^2\right) \Big/ 3\left(1 + \left(2\upsilon/\upsilon_\gamma\right)^2 + \left(2\upsilon/\upsilon_\gamma\right)^4\right)$, υ_γ represents the velocity above which pitch angle scattering ceases to be important [6]. The formula was verified with the 2D BAFIC code [7].

RESULTS

The convergence is studied for in absence of RF-power, by measuring the residual collisional power transfer from the simulated ions to the background electrons. The convergence of this quantities w.r.t the number of grid points in velocity space is shown in Fig. 1. The figure shows how the error is proportional to the inverse of the number of grid points.

The standard scenario of hydrogen minority heating in a deuterium plasma is studied for the following parameters: $n_H = 8\times10^{17}m^{-3}$ and $n_D = 3\times10^{19}m^{-3}$, $T_H = T_D = T_e = 5keV$, $B_0 = 3.4T$, $P_{RF} = 1.6MW$, $f = 47MHz$ and $n_\phi = 22$. The equilibrium parameters are chosen to represent a JET equilibrium calculated with the CHEASE code [8]. Power is absorbed by fundamental heating of

hydrogen and second harmonic heating of deuterium. The cyclotron resonance pass through ρ = 0.35 on the low field side of the magnetic axis, where $\rho = \sqrt{V(\psi)/V(\psi_{\max})}$, $V(\psi)$ is the volume enclosed by the flux surface ψ. Here only the evolution of the hydrogen is calculated. In Fig. 2 the time evolution of the collisional power transfer of hydrogen to the background species is shown. In Fig. 3 the steady state distribution functions are shown versus energy and ρ for 1.6 MW. Fig. 4 shows the collisional power transfer profiles to ions and electrons after 1.5s, at which time they have reached steady state.

CONCLUSIONS

A finite element Fokker-Planck code based on cubic elements has been developed and implemented into the self-consistent code SELFO-light developed for fast routine calculations of the ICRH power deposition profiles. The advantage of the scheme is that it is modular taking any distribution function and its derivative given on a velocity grid and a quasi-linear operator describing the heating. The basic problem with calculating the distribution functions is the attenuation of the distribution function with the velocity, which produces erroneous result at the high velocities. By calculating *F* instead of *f* and by using a cubic FEM scheme we try to improve the solution at high velocities, further optimisation studies is required to make the scheme a robust.

REFERENCES

[1] D. Andersson, W. Core, L.-G. Eriksson, et. al., Physica Scripta **37** (1988) 83.

[2] L.-G. Eriksson and T. Hellsten, Physica Scripta, **T 52** (1995) 70.

[3] L.-G. Eriksson, T. Hellsten, and U. Willén, Nuclear Fusion **33** (1993) 1037.

[4] L. Villard, et al., Nuclear Fusion **35** (1995) 1173.

[5] L. Villard, et al., Comp. Phys. Rep. **4** (1986) 95.

[6] T. H. Stix, Chap. 17 in *Waves in Plasma Physics*, AIP, New York, USA, (1992)

[7] S. Succi, K. Appert, W. Core, et al., Computer Physics Communications **40** (1986) 137.

[8] H. Luetjens, A. Bondeson and O. Sauter, Computer Physics Communications **97** (1996) 219.

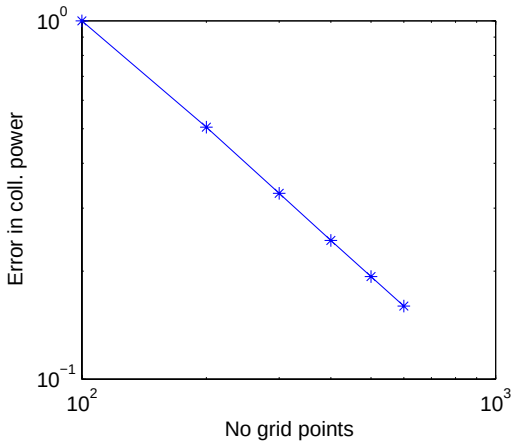

Fig. 1 Convergence of residual collisional power transfer to electrons in absence of ICRF heating.

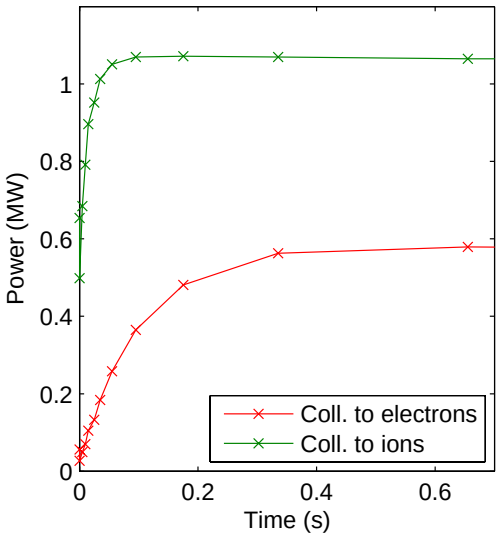

Fig. 2 Time evolution of power transfer to ions and electrons for 1.6 MW.

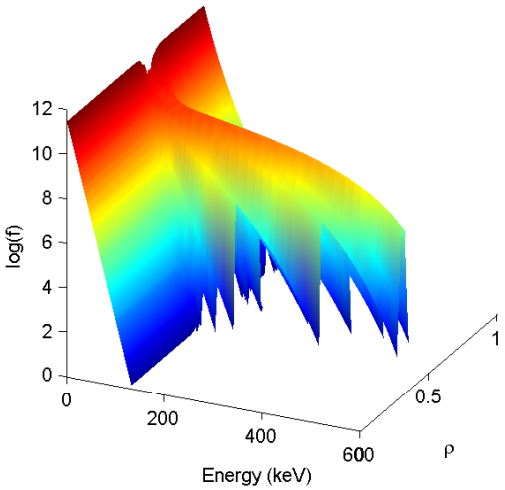

Fig. 3 Distribution functions versus velocity and minor radis, r, in the outer midplane.

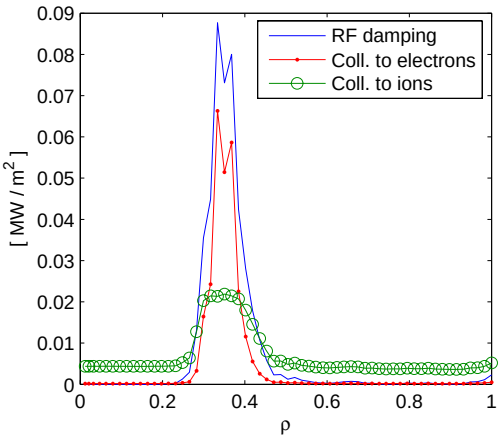

Fig. 4 Absorbed RF power (blue), collisional power transfer to ions and electrons, for a single toroidal mode n_ϕ =22.