Low threshold parametric decay back scattering instabilities in tokamak ECRH experiments

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Introduction. Electron cyclotron resonance heating (ECRH) at power level of up to 1 MW in a single microwave beam is often used in present day tokamak and stellarator experiments and planed for application in ITER for neoclassical tearing mode control. Parametric decay instabilities (PDI) leading to anomalous reflection or absorption of microwave power are believed to be deeply suppressed in tokamak MW power level ECR O-mode and second harmonic X-mode heating experiments utilizing gyrotrons. According to theoretical analysis of PDI thresholds [1], the typical RF power at which these nonlinear effects can be excited at tokamak plasma parameters is around 1 GW, which is only possible with a free electron laser microwave source. The physical reason for such a deep suppression is provided by strong convective losses of daughter waves from the decay region either in the plasma inhomogeniety direction [2] or along the magnetic field. However last year the first observation of the backscattering signal in the 200 – 600 kW level second harmonic ECRH experiment at TEXTOR tokamak was reported [3]. This signal down shifted in frequency by approximately 1 GHz was strongly modulated in amplitude at the *m=2* magnetic island frequency. This observation provides an indication that probably a novel low threshold mechanism of the PDI excitation is associated with magnetic island. In the present paper the experimental conditions leading to substantial reduction of backscattering decay instability threshold in tokamak ECRH experiments are analyzed.

The basic equations. We shall analyze the most simple three wave interaction model in which both the X-mode pump and high frequency X-mode decay wave propagate almost perpendicular to the magnetic field in the density inhomogenuity direction (*x*). For the sake of simplicity we assume the pump frequency exceeding both ω_{ce} and ω_{pe} as in the TEXTOR experiments. We neglect also a weak dependence of the high frequency wave numbers k_{ix} and k_{sx} on coordinate that allows us to introduce the pump wave in the form $E_{iy} = a_i \cdot \exp(ik_{ix}x - i\omega_i t)$ describing a wide microwave beam propagating from the launching antenna inwards plasma with an amplitude $a_i = \sqrt{8P_i/(\rho^2 c)}\exp\left(-\left(y^2+z^2\right)/\left(2\rho^2\right)\right)$, where P_i is the pump wave power and ρ is the beam radius. The equation describing the backscattered wave generation and its convective losses from the decay region in the density inhomogenuity direction is: $\left(\partial^2/\partial x^2 + k_{sx}^2\right)E_{sy} = i\cdot 4\pi\omega_s/c^2\cdot j_{sy}$, where j_{sy} is a product of a electron density perturbation δn_Ω produced by a low-frequency ($\Omega = \omega_i - \omega_s << \omega_i$) small scale decay wave and the quiver electron velocity u_{iy} in the pump wave field $j_{sy} = e\delta n_\Omega u_{iy} \approx -e/(m_e c^2)\cdot\omega_{pe}^2/\omega_{ce}^2\left(\partial^2/\partial x^2\varphi\right)E_{iy}$, where we assume electrostatic low frequency wave $\vec{E} = -\vec{\nabla}\varphi\exp(i\Omega t)$. We seek a solution of the above wave equation in the form $E_{s,y} = a_s(\vec{r})\exp\left(-ik_{s,x}x - i\omega_s t\right)$, where an amplitude $a_s(\vec{r})$ varies slowly due to non-linear interaction in the decay layer. The low frequency daughter wave potential is described in weakly inhomogeneous plasma by the integral Poisson equation

$$(2\pi)^{-3}\int_{-\infty}^{\infty} d\vec{r}'\int_{-\infty}^{\infty} D\left(\vec{q},(\vec{r}+\vec{r}')/2\right)\exp\left[i\vec{q}\left(\vec{r}-\vec{r}'\right)\right]d\vec{q}\varphi(\vec{r}') = 4\pi\rho_\Omega \tag{1}$$

with $D = q^2\left(1+\chi_e+\chi_i\right)$, χ_e and χ_i are electron and ion susceptibility and the nonlinear charge density ρ_Ω responsible for coupling of low and high frequency waves and provided by a ponderomotive force: $\rho_\Omega \sim \partial/\partial x\left[E_{iy}^*\partial E_{sy}/\partial x + E_{sy}\partial E_{iy}^*/\partial x\right]$. Taking into account that the PDI amplification is enhanced when the daughter wave group velocity decreases we shall consider solutions of the Poisson integral equation in the vicinity of the low frequency wave turning point in *x* direction ($x = x_0$), where $D\big|_{q_{x0},\Omega_0,x_0} \equiv D\big|_0 = 0$, $\partial D/\partial q_x\big|_0 = 0$ hold for wave at frequency $\Omega = \Omega_0$, wave number $q_x = q_{x0}$ and its group velocity goes to zero (see Fig. 1). We assume also, as is shown in experiment [6], that a local density maximum is associated with the O-point and therefore conditions $\partial D/\partial x\big|_0 = 0$ and $q_{x0}^2 L_{nx}^{-2} \equiv \partial^2 D/\partial x^2\,|_0 > 0$ hold there. In these circumstances two nearby turning points ("warm" to "hot" mode) exist in plasma for high harmonic IB wave and it can be trapped in *x* direction if additional condition $\partial^2 D/\partial q_x^2\,|_0 > 0$ holds. The corresponding density profile and IB wave dispersion curve $q_x(x)$ calculated for $\Omega_0 = 0.85$ GHz, $T_i = 600$ eV is shown in Fig.2. The described possibility of IB wave localization in a plasma waveguide provides an argument in favor of the PDI threshold lowering in the case of the waveguide eigen mode excitation. The physical reason for it is related to the suppression of convective wave energy losses in the inhomogenuity direction. It is important to note that due to magnetic field dependence on major radius IB wave trapping is possible also in the poloidal direction. In this case the integral equation for low frequency wave in the vicinity of magnetic island O-point, situated in the equatorial plane of the torus

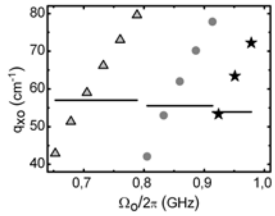

FIG.1. IB wave turning points at $T_i = 600$ eV, $n = 1\times10^{13}\text{cm}^{-3}$ (triangles); $n = 2\times10^{13}\text{cm}^{-3}$ (circles), $n = 3\times10^{13}\text{cm}^{-3}$ (stars); solid lines - $k_{i,x}(x_0)+k_{s,x}(x_0)$ at the corresponding density .

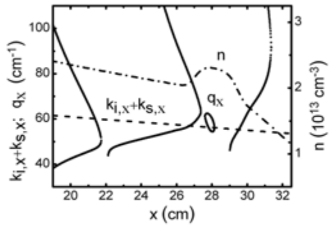

FIG.2. The IB wave dispersion curves (solid line) and plasma density (dash dotted line) in the magnetic island vicinity. Dashed line – $k_{i,x}+k_{s,x}$.

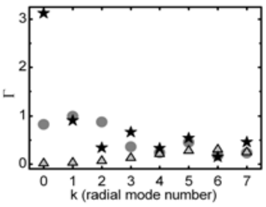

FIG.3. Dependence of the IB wave gain on the radial mode number; $P = 400$ kW, $T_i = 600$ eV; Triangles $n = 1\times10^{13}\text{cm}^{-3}$, $\Omega_0/2\pi = 0.68$ GHz; Circles- $n = 2\times10^{13}\text{cm}^{-3}$, $\Omega_0/2\pi = 0.83$ GHz; Stars- $n = 3\times10^{13}\text{cm}^{-3}$, $\Omega_0/2\pi = 0.92$ GHz.

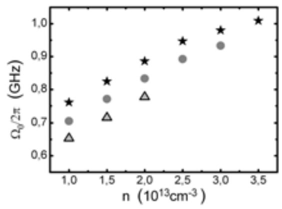

FIG.4. Dependence of frequency corresponding to maximal IB wave gain on plasma density. (Triangles - $T_i = 300$ eV; Circles - $T_i = 600$ eV; Stars - $T_i = 900$ eV)

and coincident with the IB wave turning point, reduces to

$$\left\{ \frac{\partial D}{\partial \Omega}\bigg|_0 \delta\Omega - \frac{\partial^2 D}{2\partial q_x^2}\bigg|_0 \frac{\partial^2}{\partial x^2} + \sin^2\phi \frac{\omega_{pe}^2}{\Omega^2}\bigg|_0 \frac{\partial^2}{\partial y^2} + q_{x0}^2 \frac{(x-x_0)^2}{L_{nx}^2} - q_{x0}^2 \frac{y^2}{L_b^2} - \delta D \right\} b = 4\pi\rho_\Omega \exp\left[iq_{x0}x - iq_z\left(ycot\phi - z\right)\right], (1)$$

where $\varphi = b(x,y)\exp\left[-iq_{x0}x + iq_z\left(y\cdot cot\phi - z\right)\right]$; $L_b^{-2} = \pi\Omega/\omega_{ci}\cdot\omega_{pi}^2/q_x^2\upsilon_{ti}^2\cdot\csc^2\left(\pi\Omega/\omega_{ci}\right)Y/rR\big|_0$; r and R are minor and major radii; $\tan\phi = B_y/B_z$; $\delta D = \delta D\left(q_z^2\right)$ is the perturbation describing convective losses in toroidal direction, $\delta\Omega = \Omega - \Omega_0$. In this consideration we neglected weak poloidal and moreover very weak toroidal density inhomogenuity in the magnetic island and therefore suppose wave number q_z constant.

The PDI analyses and discussion. Assuming the IB PDI pumping and convection in toroidal direction weak, we account for them using the perturbation theory approach. In the zero order approximation we neglect δD and ρ_Ω in (1) and obtain equation which can be solved by separation of variables. The corresponding expression for the IB eigen mode trapped in radial and poloidal direction and possessing $q_z = 0$, which corresponds to suppressed convective losses in toroidal direction is $b(x,y) = \varphi_k\left(x/\delta_x\right)\varphi_l\left(y/\delta_y\right)$, where φ_k is standing for Hermitian polynomial, the size of the IB mode localization region is δ_x, δ_y. At the next step of the perturbation analysis procedure we account for the IB PDI pumping and convection in toroidal direction and find the imaginary part of toroidal wave number $q_z'' = \alpha_{k,l}\left|\sin\phi\right| q_{x0}^{5/2}\delta_x^{3/2}/\sqrt{2}\cdot\omega_{pe}^2\Omega_0/(\omega_i\omega_{ce}\omega_{pi})\cdot a_i/H$, where $\alpha_{k,l}\left(\Delta K\right)$ is the coefficient and $\Delta K = q_{x0} - k_{sx} - k_{ix}$ decay condition mismatch. At $\rho < \sqrt{2l+1}\delta_y cot\phi$ the poloidal gain over the IB mode localization region provided by exponential factor $\exp\left[iq_z y\cdot cot\phi\right]$ dominates and the PDI threshold takes a form $\Gamma = 2\sqrt{2l+1}\delta_y \cot\phi\cdot q_z'' > 1$. Dependence of this gain on the IB radial mode number is shown in Fig.3 for *l=0*, TEXTOR parameters and different plasma densities. As it is seen, because of $\Delta K \neq 0$ the gain is not always maximal for the fundamental mode. As it is shown in Fig.4, the IB frequency corresponding to the maximal gain is growing with the plasma density. This dependence is consistent with corresponding dependence of the back scattered wave frequency shift observed in [5]. The PDI power threshold provided by condition $\Gamma > 1$ is given by formula $P_{th} = cH^2/2\pi\alpha_{k,l}^2\cdot\left(\omega_i^2\omega_{ce}^2\omega_{pi}^2\right)/\left(\omega_{pe}^4\Omega_0^2\right)\cdot\rho^2/\left(2l+1\right)\delta_y^2 q_{x0}^5\delta_x^3$. For TEXTOR parameters ($H = 19$ kGs, $f_i = 140$ GHz, $n = 3\times10^{13}$cm^{-3}, $T_i = 600$ eV, $\rho = 1$ cm),we obtain for the IB mode with *k=l=0*: $\Omega_0 = 0.92$ GHz, $\delta\Omega_{0,0}/\left(2\pi\right) = 10$ MHz, $\delta_y = 0.8$ cm, $\delta_x = 0.6$ cm, $\Delta K\delta_x \ll 1$ and the threshold value $P_{th} \approx 45$ kW which is overcome in the experiment.

Conclusions. The obtained drastic, compared to predictions of the standard theory [1], decrease of the PDI threshold is explained by complete suppression of IB wave radial and poloidal convective losses and their substantial reduction in the third direction. This mechanism is based first of all on magnetic island confinement properties which, we believe, are not specific for the TEXTOR experimental conditions and may lead to easy PDI excitation in ECRH experiments in other toroidal devices where magnetic islands usually exist. Moreover it should be mentioned that not only magnetic island, but also drift wave density perturbations, which are as well elongated along magnetic field, in the case of intensive enough turbulence can result in IB wave trapping. Similar effect leading to reduction of PDI threshold can occur also on the plasma discharge axis. It should be underlined that absorption of parametrically driven IB wave can be responsible for ion acceleration often observed in ECRH experiments (see [5] and references there). Moreover backscattering PDI can lead to reduction of ECRH efficiency and change of its localization.

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