Comparison of EBW heating and current drive simulation models in realistic tokamak conditions

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Introduction

In this paper, we report on our study of models being used in electron Bernstein wave (EBW) simulations. EBW is an electrostatic wave in the electron cyclotron (EC) frequency range that can propagate and be effectively absorbed at any harmonic in overdense plasmas ($\omega_{\rm pe} \gg \omega_{\rm ce}$), where EC O- and X-modes are cut-off. Such conditions are typical in spherical tokamaks (STs) or stellarators. We compare several damping models implemented in our ray-tracing code, showing particularly, in realistic tokamak conditions, the effects of the relativistic damping corrections and electrostatic approximation.

EBW propagation and damping calculation

EBW propagation in plasmas can be simulated with a standard ray-tracing technique. Electrostatic non-relativistic dispersion relation is used in our AMR (Antenna, Mode-conversion, Ray-tracing) [1] for ray propagation, i.e., $\Re(D) = 0$ is solved. Ray power evolution is solved using the radiative transfer equation [2] and necessitates $\Im(D)$, which is more sensitive to relativistic corrections because of its resonant nature [3, 4]. Following models are implemented in AMR and discussed hereafter.

Weakly-relativistic model of Decker and Ram [3]:

$$\Im(D_{\rm WR}) = \sqrt{\frac{\pi}{2}} \frac{1}{\lambda \left|N_\parallel\right| \beta_{\rm T}} \frac{\omega_{\rm pe}^2}{\omega_{\rm ce}^2} \sum_{n=-\infty}^{\infty} \left(I_n(\lambda_\perp) e^{-\lambda_\perp} - \frac{\beta_{\rm T} p_{n0}}{N_\parallel} \left(\lambda_\perp I_n(\lambda_\perp) e^{-\lambda_\perp} \right)' \right) e^{-p_n^2/2}, \quad (1)$$

valid for $|p_{n0}| \ll \left|N_\parallel\right| \beta_{\rm T}$, where

$$\begin{aligned} &\beta_{\rm T} = \tfrac{\sqrt{2}}{2} \tfrac{v_{\rm T}}{c} = \sqrt{\tfrac{kT}{m_{\rm e}c^2}},\ \lambda_\perp = \left(\tfrac{N_\perp \beta_{\rm T} \omega}{\omega_{\rm ce}}\right)^2,\ \lambda_\parallel = \left(\tfrac{N_\parallel \beta_{\rm T} \omega}{\omega_{\rm ce}}\right)^2,\ \lambda = \lambda_\perp + \lambda_\parallel, \\ &p_n = p_{n0}\left(1 + \tfrac{\beta_{\rm T} p_{n0}}{2N_\parallel}\right),\ p_{n0} = \tfrac{1 - n\omega_{\rm ce}/\omega}{\beta_{\rm T} N_\parallel}. \end{aligned} \quad (2)$$

The first term in (1) comes from the relativistic shift of the resonance while the second, typically smaller, is due the relativistic curvature of the resonance. Neglecting the curvature and imposing $p_n = p_{n0}$, (1) simplifies to the usual non-relativistic limit.

Another electrostatic damping term implemented in AMR is due to Saveliev [4]. The usual harmonics expansion is not employed here but rather a cotangent term is expanded into a series. This allows an approximate solution to the double integrals in the Trubnikov relativistic dispersion tensor. The resulting damping term is a series of analytic functions of $\omega_{\rm pe}^2/\omega^2$, $\omega_{\rm ce}/\omega$, $\beta_{\rm T}$, $N_\parallel$, $N_\perp$. This approach has a disadvantage for linear current drive calculations, which require the damping calculated for separate harmonic because of the difference in the current drive efficiency. Attention must be paid in the implementation as the series can converge slowly and hence an accuracy check must be included. From our experience, accuracy of 10^{-6} agrees well with the fully-relativistic calculation, while 10^{-4} is already in a critical disagreement, often worse than the non-relativistic model.

Finally, a routine that integrates numerically the fully-relativistic dispersion tensor in the momentum space along the resonance curve $\gamma - N_\parallel \beta_{\rm T} p_\parallel - n\omega_{ce}/\omega = 0$ has been implemented [5].

We are aware that other approaches exist which are not included in our study, e.g., recently published papers [6, 7].

Comparison of the damping models

In Figure 1 we plot central rays' evolution for three frequencies in MAST-Upgrade model equilibrium. The large magnetic field peak (apparent from the $\omega/\omega_{\rm ce}$ graphs) at the edge causes the second harmonic damping of the larger frequencies. This makes possible to compare both low-field ($\omega > n\omega_{\rm ce}$) and high-field ($\omega < n\omega_{\rm ce}$) side (LFS and HFS) damping. Figure 2 shows the calculation time for 8 rays in MAST-U, averaged over 3 independent runs and normalized to the non-relativistic time. Several points, observed also in many different cases that are not reported here, can be drawn:

- Non-relativistic damping overestimates LFS damping while it underestimates HFS damping. The error can be critical and thus non-relativistic damping should be avoided in EBW simulations.
- The weakly-relativistic model overestimates both LFS and HFS damping. The error is reasonable taking into account the simplicity of the model. Attention must be paid to the validity limits, which are sometimes broken. The calculation time is almost identical to the non-relativistic model (even smaller in our case, which is due to shorter rays' lengths).
- The Saveliev and the fully-relativistic model yield almost identical results (a minor deviation can be seen for 22 GHz). The speed of the fully-relativistic calculation is not

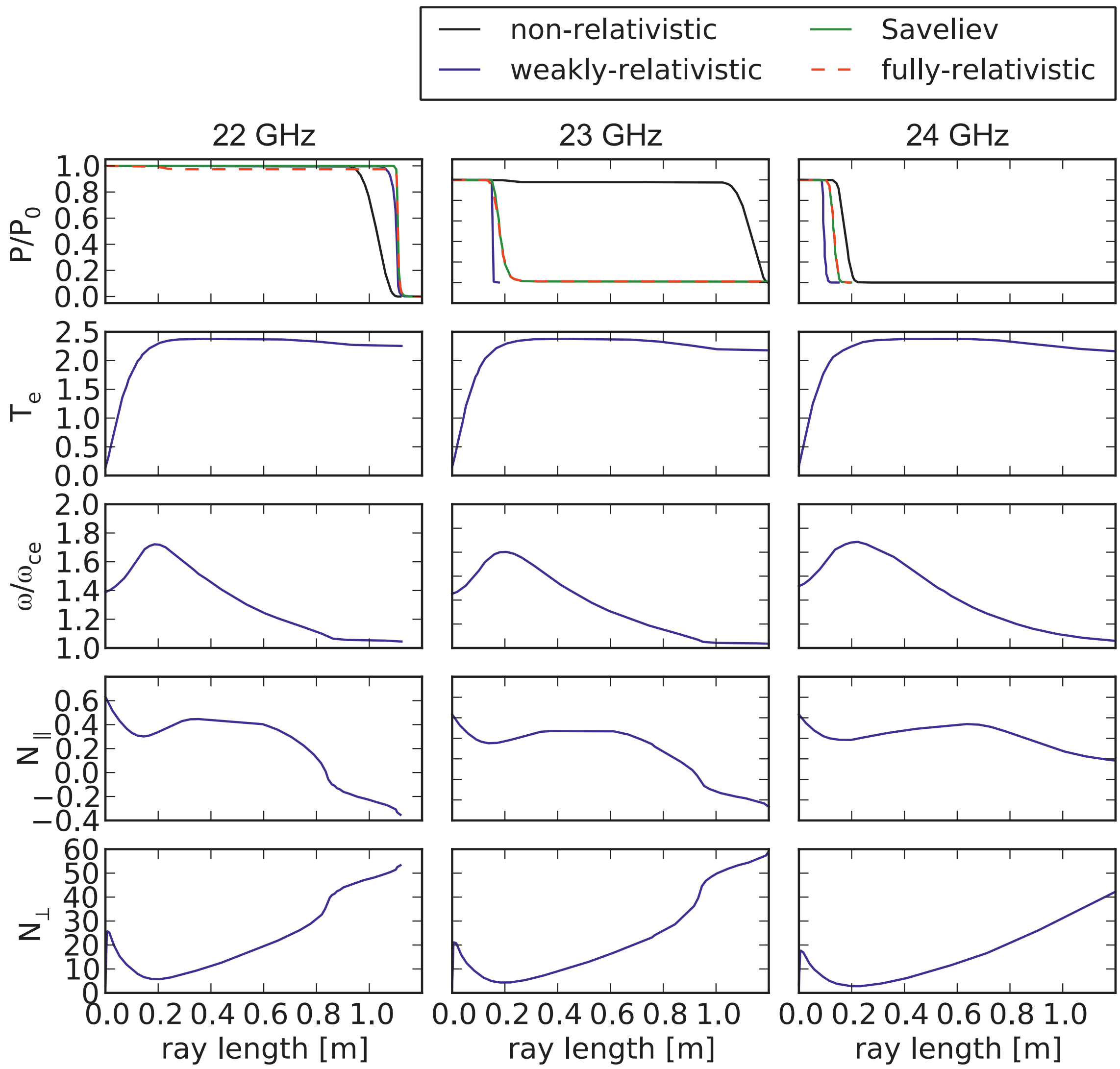

Figure 1: Central EBW rays evolution and damping for three frequencies in MAST-U equilibrium.

much larger compared to non- or weakly-relativistic if p_{n0}=5 or 10, where 10 is a reasonably safe value. Saveliev formula takes much longer time to evaluate, particularly if high accuracy is required. Limitation to $\Im\left(D_{\mathrm{WR}}\right) > D_0$ does not considerably improve the performance.

Conclusions

Relativistic effects in EBW damping are crucial. If speed is of maximum importance, the Decker and Ram weakly-relativistic damping can be used, bearing in mind its limitations. Saveliev and the fully-relativistic calculation are in good agreement. The Saveliev term seems to suffer more from numerical difficulties and also does not provide separate harmonics damping necessary for current drive calculations. Hence, the fully-relativistic calculation, with a proper

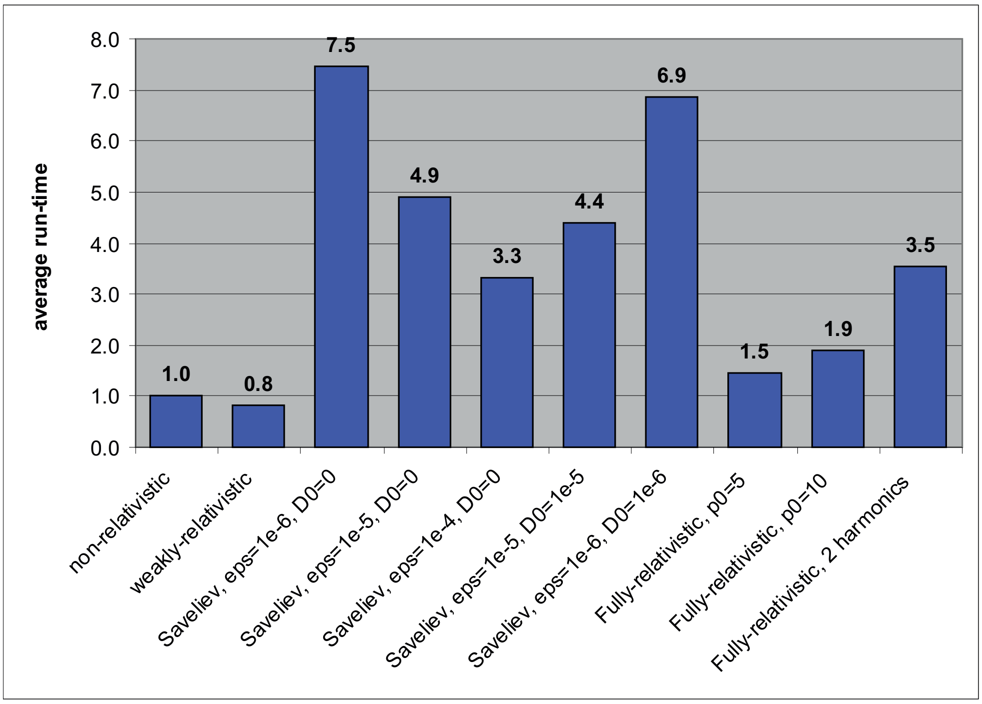

Figure 2: Average run-time for the studied damping models, normalized to the non-relativistic calculation time. eps – relative accuracy, D_0 – evaluated if $\Im(D_{\mathrm{WR}}) > D_0$, p_0 – fully-relativistic damping evaluated if the weakly-relativistic $p_n < p_0$, 2 harmonics – calculated always for n^{th} and $(n+1)^{\mathrm{th}}$ harmonics: $n\omega_{\mathrm{ce}} < \omega < (n+1)\,\omega_{\mathrm{ce}}$.

choice of numerical parameters, seems to be a better choice.

Acknowledgements

The work was partly supported by EFDA, EURATOM, GACR #202/08/0419, AS CR #AV0Z20430508, MSMT #7G09042, and U.S. DoE.

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