

Magnetic field generation with Laguerre-Gauss laser beams

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The quasi-static axial magnetic field in MegaGauss range can be produced due to the interaction of laser beams with a collisionless uniform plasma. For this purpose, we use the orbital and spin angular momentum density associated with the Laguerre-Gaussian photon beams and solve the electron angular momentum conservation equation and Faraday law in a cylindrical coordinate system obtaining the magnetic field. It is found that the excitation of axial magnetic fields is even possible with linearly polarized laser pulses apart from circular polarized pulses at various azimuthal angles. Our results can be useful for the laser wakefield experiments.

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I. INTRODUCTION

Laser beams or photon beams are usually composed of planar wavefronts showing a uniform phase. The wavevectors and linear momentum of these wavefronts are directed along the beam axis. However, the deviation from rotational symmetry may produce helical wavefronts, for which the wavevectors spiral around the beam axis constituting the orbital angular momentum (OAM). It is now well-known that helical wavefronts can be represented in a basis set of orthogonal Laguerre-Gaussian (LG) modes and that each LG mode is associated with a well-defined state of photon OAM. Some efforts [1–3] have been made to explain the physics of photon OAM. Besides, Harwit [4] examined the photon OAM in the context of astrophysical phenomena. Mendonça *et al.* [5] studied the excitation of photon OAM states in a plasma and investigated helical disturbances in a static plasma as well as in a rotating plasma vortex. Recently, Stimulated Raman and Brillouin backscatterings [6] and plasmon (or longitudinal photon) states [7] have been discussed incorporating the photon OAM.

In 1936, Beth [8] and Holbourn [9] measured the mechanical torque due to the transference of angular momentum of circularly polarized laser to a half wave plate. The angular momentum is transferred to matter when it absorbs radiation [10, 11]. Laser absorption is quite complex phenomenon in a plasma specially at high intensities when collisionless processes are dominating. Essentially, the angular momentum of the laser beam is consist of spin and orbital angular momenta associated with the polarization state and the angular beam structure, respectively. Allen *et al.* [12] have recently described photon OAM of a laser beam in terms of LG modes.

Numerous theoretical models [13–17] relating to the generation of a quasi-static axial magnetic field have been presented and verified in different experimental conditions [18–21]. It has now been observed from the studies that the induced magnetic field depends significantly on the laser intensity and the electron number density.

The latter has been modified with relativistic ponderomotive force [17], generating a magnetic field varying as $(n_e I_0)^{1/2}$ in an underdense plasma. Najmudin *et al.* [18] studied the interaction of a circularly polarized laser with underdense helium plasma at relativistic intensities and reported a magnetic field of the order 7 MG. Deschamps *et al.* [19] presented the interaction of circularly polarized microwaves with a plasma at a density $n_e \sim 3 \times 10^{15} \text{ m}^{-3}$ and obtained the field of $2 \times 10^{-2} \text{ G}$ varying as $n_e I_0$. Later, a neodymium-glass laser (wavelength $1.06 \mu\text{m}$) of $n_e \sim 10^{17} \text{ W m}^{-2}$ (10^{18} W m^{-2}) has been used by Horovitz and his coworkers [20, 21] evaluating an axial magnetic field of 10 kG (2 MG). Sheng and Meyer-ter-Vehn [22] estimated the magnetic field as 100 MG incorporating the effects of laser beam inhomogeneity and the electron density in overdense plasmas. Gorbunov and Ramazashvili [23] investigated the magnetic field varying as $n_e^{1/2} I_0^2$ in a uniform plasma for a short circularly polarization. In the beginning of 21st century, Haines [15] explained the axial magnetic field with a dependence of $1/n_e$ due to azimuthal electric field induced by the circularly polarized laser pulses.

In this work, we present the generation of an axial magnetic field in a uniform plasma using Laguerre-Gauss beams containing a finite OAM. It is found that the excitation of axial magnetic fields even possible with linearly polarizations apart from circular polarization at various azimuthal angles.

II. MATHEMATICAL MODEL

The average rate of change of the electron angular momentum per unit volume in a cylindrical coordinate system (r, φ, z) in the presence of photon angular momen-

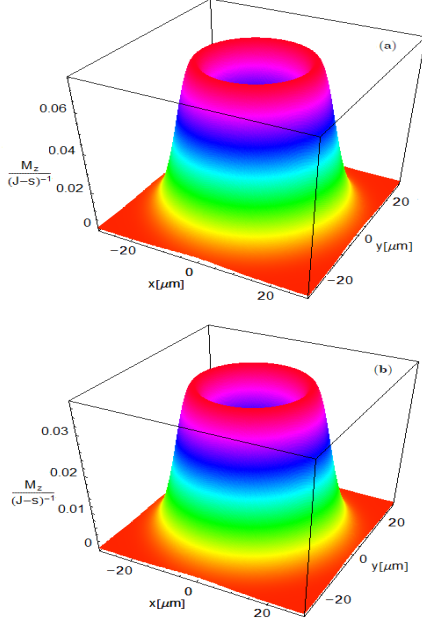


FIG. 1: The orbital angular momentum density (M_z) as a function of x and y for circular polarization $\sigma_z = -1$ and laser intensity $I_0 = 7.3 \times 10^{22} \text{ W m}^{-2}$ at azimuthal angles (a) $\varphi = 0$ and (b) $\varphi = \pi/4$.

tum is governed by the following conservation equation

$$\left(\frac{d}{dt} + \nu_{ei} \right) V_{e\varphi} = -\frac{e}{m_e} (E_\varphi + V_{ez} B_r - V_{er} B_z) - \frac{1}{m_e N_e r} \frac{dM_z}{dt}, \quad (1)$$

where $m_e V_{e\varphi} r$ is the electron angular momentum in the z -direction with m_e the electron mass and $V_{e\varphi}$ the azimuthal electron velocity. $B_r(B_z)$ and $V_{er}(V_{ez})$ are the radial (axial) components of the magnetic field \mathbf{B} and the electron fluid velocity \mathbf{V}_e , respectively. N_e is the electron number density, E_φ is the azimuthal electric field, and ν_{ei} is the electron-ion collision frequency.

The axial component of the photon angular momentum density is given [12] as $M_z = lI/\omega c + (\sigma_z r/2\omega c) (\partial I/\partial r)$, where $l = 0, \pm 1, \dots$ is the quantum number of the orbital angular momentum, corresponding to the azimuthal mode number of a LG mode [7, 12], σ_z is the quantum number of the spin angular momentum, which is -1 ($+1$) for right (left) circularly polarized light and zero for linearly polarized light, ω is the laser angular frequency, I is the laser intensity, c is the speed of light in vacuum and $r = (x^2 + y^2)^{1/2}$.

To derive an expression for the axial magnetic field, we consider the approximation $\tau_{pi} \gg \tau \gg \tau_{pe}$, where $\tau_{pi} (= 2\pi/\omega_{pi})$ is the time period of the ions and $\tau_{pe} (= 2\pi/\omega_{pe})$ is the time period of the electrons. On such a time scale ion motion may be reasonably neglected

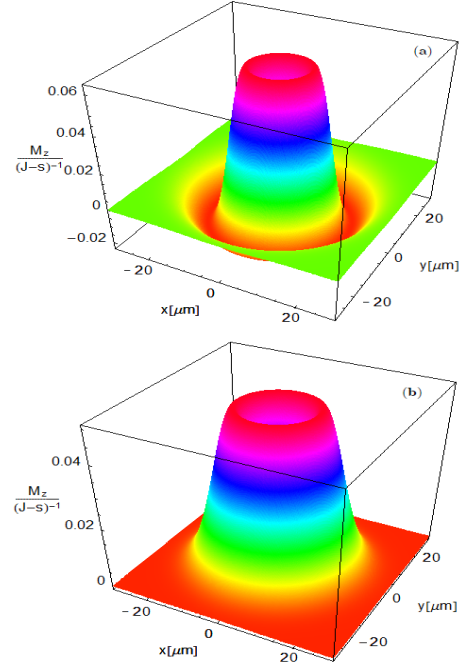


FIG. 2: The orbital angular momentum density (M_z) as a function of x and y for (a) circularly polarized laser pulse $\sigma_z = 1$ and (b) linearly polarized laser pulse $\sigma_z = 0$ at $\varphi = 0$ and $I_0 = 7.3 \times 10^{22} \text{ W m}^{-2}$.

and the electrons will have time to reach a steady state ($dV_{e\varphi}/dt = 0$). Furthermore, neglecting the collision frequency in comparison with the electron plasma frequency, which would lead to a subsequent decay of the magnetic field. Thus, balancing the remaining dominant terms in Eq. (1), we have

$$rE_\varphi \sim -\frac{1}{eN_e} \frac{dM_z}{dt}. \quad (2)$$

The z -component of the Faraday's law can be expressed as

$$\frac{1}{r} \frac{\partial}{\partial r} rE_\varphi = -\frac{\partial B_z}{\partial t}. \quad (3)$$

Substituting Eq. (2) into Eq. (3), we eventually obtain

$$\frac{\partial B_z}{\partial t} = \frac{1}{eN_e r} \frac{\partial}{\partial r} \frac{d}{dt} \left(\frac{lI}{\omega c} + \frac{\sigma_z r}{2\omega c} \frac{\partial I}{\partial r} \right). \quad (4)$$

Performing integration over time from 0 to t and using the relation $I(t) - I(0) = -f_{abs}I$, Eq. (4) gives

$$B_z = -\frac{f_{abs}}{eN_e r \omega c} \left\{ l \frac{\partial I}{\partial r} + \frac{\sigma_z}{2} \frac{\partial}{\partial r} \left(r \frac{\partial I}{\partial r} \right) \right\}, \quad (5)$$

where the absorption coefficient f_{abs} of the laser intensity absorbed over a certain axial distance L can be expressed as $f_{abs} = 1 - \exp(-\kappa_{ib}L)$, here κ_{ib} is the damping rate

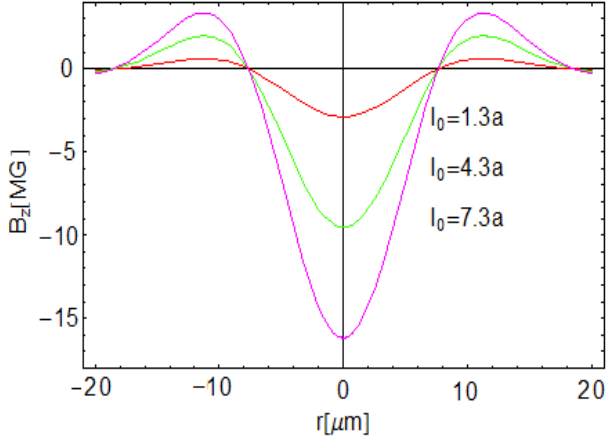


FIG. 3: The variation of axial magnetic field B_z as a function of r for different laser intensities I_0 ($= 1.3a, 4.3a, 7.3a$) with $\sigma_z = 1$ at an angle $\varphi = 0$, where $a = 10^{22} \text{ W m}^{-2}$.

of the laser energy by inverse bremsstrahlung. For weak laser absorption, $\kappa_{ib}L \ll 1$, the absorption coefficient can be approximated as $f_{abs} \simeq \kappa_{ib}L$ and for strong laser absorption, $\kappa_{ib}L \gg 1$, we have $f_{abs} = 1$. As the electron number density $N_e \rightarrow 0$, the absorption coefficient will also tend to zero so in practice Eq. (5) does not have any singularity at $N_e = 0$. It is important to note that even for linearly polarized laser pulses $\sigma_z = 0$, the axial magnetic field can exist due to the orbital angular momentum density involving higher order terms in the laser intensity.

Estimating the order of magnitude of the field in terms of practical units, we assume $r = r_0$ and $\partial/\partial r \sim 1/r_0$ and write Eq. (5) in the following form,

$$B_z \sim f_{abs} \left(\frac{\lambda}{r_0} \right)^2 \left(\frac{N_e}{N_c} \right)^{-1} \left(\frac{\mu\text{m}}{\lambda} \right) \left(l + \frac{\sigma_z}{2} \right) \times \left\{ \frac{I_0 \lambda^2}{7.3 \times 10^{22} \text{ W m}^{-2} (\mu\text{m})^2} \right\} \text{MG}, \quad (6)$$

where λ is the wavelength of the laser beam, N_c is the non-relativistic critical density, which is approximately $1.1 \times 10^{15} / \lambda^2 \text{ m}^{-3}$.

Laser pulses can be described in terms of LG modes [24], which represent a general solution of the paraxial wave equation in cylindrical geometry. In fact, they provide a natural orthonormal basis set for representing a beam in cylindrical geometry. It has been noticed that higher order terms that could be involved in the intensity profiles of the LG modes in case of non-Gaussian beams, will be responsible for the existence of OAM.

The laser intensity profile in the focal plane ($z = 0$)

can be expressed as

$$I(r, \varphi) = I_0 \frac{(-1)^{2p} p!}{(l+p)!} [L_p^l(\rho)]^2 \rho^l \exp(-\rho) \cos^2(l\varphi), \quad (7)$$

where $\rho = (r/r_0)^2$ with r_0 the beam radius and I_0 the maximum axial intensity of the laser beam, p represents the radial mode number, and $L_p^l(\rho)$ is the associated Laguerre polynomials. Equation (7) determines different intensity profiles involving higher order terms of LG modes depending strongly upon the azimuthal and radial mode numbers. For example, firstly, we consider a circularly polarized Gaussian beam assuming that $l = 0 = p$ obtaining $LG_0^0 = 1$, then the intensity profile reduces to $I(r) = I_0 \exp(-\rho)$. The latter may yield the following axial magnetic field

$$B_z(r) = \frac{2I_0 f_{abs} \sigma_z}{e N_e r_0^2 \omega c} \exp(-\rho) (1 - \rho), \quad (8)$$

For a finite orbital angular momentum $l = 1$ and assuming $p = 0$, the laser intensity from (7) reduces to

$$I(r, \varphi) = I_0 \rho \exp(-\rho) \cos^2(\varphi). \quad (9)$$

Accordingly, the axial magnetic field becomes

$$B_z(r, \varphi) = -\frac{2I_0 f_{abs}}{e N_e r_0^2 \omega c} \{1 - \rho + \sigma_z (1 - 3\rho + \rho^2)\} \exp(-\rho) \cos^2(\varphi), \quad (10)$$

It is important to note here that the inverse Faraday effect can also exist for linearly polarized laser pulses due to the presence of an orbital angular momentum density.

The angular momentum for $l = 1$ and $p = 0$ can be expressed by using Eq. (9) as

$$M_z(r, \varphi) = \frac{I_0 \rho}{\omega c} \{1 + \sigma_z (1 - \rho)\} \exp(-\rho) \cos^2(\varphi). \quad (11)$$

A. Results and discussion

For numerical analysis, we choose some typical experimental values [15, 18], namely, $I_0 = 7.3 \times 10^{22} \text{ W m}^{-2}$, $\omega = 1.79 \times 10^{15} \text{ s}^{-1}$, $N_e = 2.1 \times 10^{25} \text{ m}^{-3}$, $r_0 = 10^{-5} \text{ m}$, and the absorption coefficient $f_{abs} = 1$. Assuming $\sigma_z = -1$ in Eq. (8), the axial magnetic field for a parabolic laser profile can be computed as 2.4 MG, which is completely in agreement with Haines's result [15]. Noting that the magnitude of this field is smaller than the experimentally measured value of 4 MG [18], it is also worth mentioning that Eq. (5) can give a larger magnetic field involving higher order terms of the LG modes, which in this case might have been introduced by the quarter wave plate used to produce circular polarization.

We plot the total angular momentum density M_z from Eq. (11) against the transverse distances x and y in

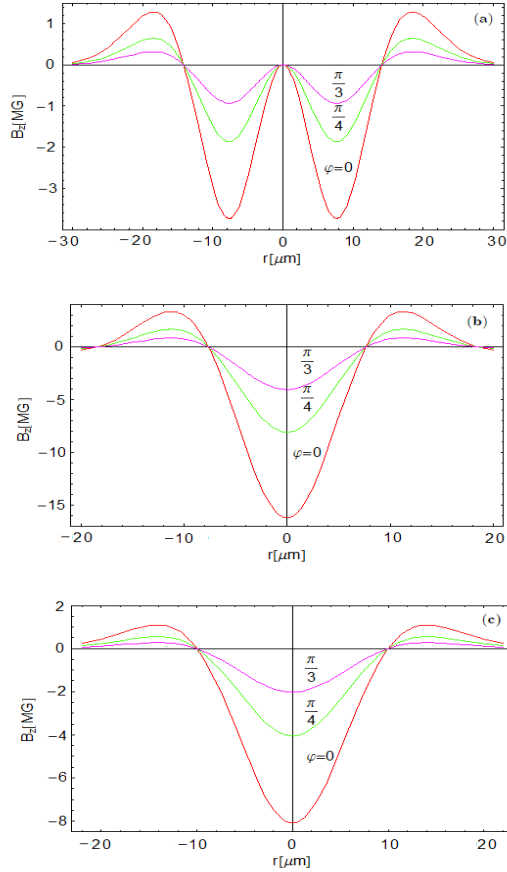


FIG. 4: The axial magnetic field B_z as a function of r with $I_0 = 7.3 \times 10^{22} \text{ W m}^{-2}$ for (a) $\sigma_z = -1$, (b) $\sigma_z = 1$, and (c) $\sigma_z = 0$ with $\varphi (= 0, \pi/4, \pi/3)$, respectively.

Fig. 1 for circular polarization ($\sigma_z = -1$) at azimuthal angles $\varphi = 0$ and $\varphi = \pi/4$ with the Laguerre-Gauss polynomial LG_0^1 . It is found that the strength of the angular momentum density decreases for changing the value of φ from 0 to $\pi/2$. Figure 2 depicts a 3D variation of the angular momentum density for a circularly as well as linearly polarized laser pulses with a fixed $\varphi = 0$. A net modification can be seen in the radial direction and in the strength of M_z for $\sigma_z = 1$ and $\sigma_z = 0$.

We also plot the axial magnetic field as a function of r from Eq. (10) with a polynomial LG_0^1 and a left circular polarization ($\sigma_z = 1$) for various laser intensities $I_0 (= 1.3, 4.3, 7.3) \times 10^{22} \text{ W m}^{-2}$ at $\varphi = 0$. The laser intensities directly enhance the magnitude of the axial magnetic fields and widths of the profiles. Figure 3 displays the variation of the magnetic field B_z as a function of r , produced due to (a) right circular polarization ($\sigma_z = -1$), (b) left circular polarization ($\sigma_z = 1$), and (c) linear polarization ($\sigma_z = 0$) for varying angles $\varphi (= 0, \pi/4, \pi/3)$, respectively. It is observed that the strength of the field decreases with increasing the azimuthal angles and have seen a maximum strength of the magnetic field for left circularly polarization. It is also examined that the inverse Faraday effect can take place even with linearly polarization.

To conclude, we have presented a model for the generation of an axial magnetic field in a collisionless uniform plasma by employing the Laguerre-Gauss laser beams associated with a finite OAM. It is found that the excitation of the axial magnetic fields is even possible with linearly polarized pulses apart from circular polarization at various azimuthal angles. The fields could have implications for the interpretation of electron acceleration and betatron emission in laser wakefield experiments.

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