Langmuir wave dispersion relation in non Maxwellian plasmas: effect of density

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Abstract: We re-derive the Langmuir dispersion relation in partially ionized plasmas, where the electron equilibrium distribution (EDF) is non Maxwellian. A new dispersion relation is obtained that reduces to the Bohm-Gross relation when the non-linear character of the EDF is relaxed. The effect of density on the Langmuir phase and group velocities is investigated. The departure from the Maxwellian case is prominent for dense plasmas.

Introduction

An unmagnetized plasma may support three wave modes, namely, Langmuir, ion acoustic and electromagnetic modes. When the plasma is embedded in a magnetic field, a host of new wave modes appear. The Langmuir wave (LW) that is electrostatic and propagates with a phase velocity greater than the electron and the ion thermal velocities. Langmuir waves may be excited by numerous ways such as electron beam instabilities and laser/plasma interaction at the plasma frequency. The LW phase velocity that is a key parameter in characterizing the wave and which is derived from the wave dispersion relation known as Bohm-Gauss relation appears to be inevasibly critical in many processes. When a non-Maxwellian equilibrium distribution function for free electrons in plasma is adopted a new LW dispersion relation is derived [1]. In this note, we investigate the effect of density on the latter and the phase and group velocities of the Langmuir wave.

Model Equations

In the work by Ouazene and Annou (c.f.Ref.[1]), it has been shown that the appropriate LW dispersion relation for a non Maxwellian distribution function is given by the expression,

$$\omega^2 = \omega_p^2 + \frac{3}{2} k^2 \cdot v_{th}^2 \cdot (1+\varepsilon) , \qquad (1)$$

where, $v_{th}^2 = \dfrac{2T}{m}$, is the squared thermal velocity and $\varepsilon = \sqrt{\pi\, t_0^3} / 4 \propto \sqrt{\dfrac{n}{T^3}}$

The new contribution in the dispersion relation appears to be relevant in dense plasmas as well as low temperature plasmas, since it increases as temperature decreases or as density increases. Equation (1) may rewritten as follows,

$$\frac{\omega}{\omega_p^*} = \left(\frac{n}{n^*}\right)^{\frac{1}{2}} \left[1 + 3k^2 \lambda_D^{*2} \left(\frac{n^*}{n}\right)(1+\varepsilon)\right]^{\frac{1}{2}} \tag{2}$$

where n^* is a reference density and λ_D^* is $\sqrt{2}$ times the Debye length for the considered temperature. Consequently, the phase velocity along with the group velocity may be cast as follows,

$$\frac{V_\varphi}{c} = \frac{V_{th}}{c}\left(\frac{n}{n^*}\right)^{\frac{1}{2}} \frac{\left[1 + 3k^2 \lambda_D^{*2} \left(\frac{n^*}{n}\right)(1+\varepsilon)\right]^{\frac{1}{2}}}{k\lambda_D^*} \tag{3}$$

and,

$$\frac{V_g}{c} = \frac{V_{th}}{c} \frac{3\left(\frac{n^*}{n}\right)^{\frac{1}{2}} k\lambda_D^*(1+\varepsilon)}{\left[1 + 3\left(\frac{n^*}{n}\right) k^2 \lambda_D^{*2}(1+\varepsilon)\right]^{\frac{1}{2}}} \tag{4}$$

where, c is the celerity of light.

Is plotted in Fig.1 the normalized wave frequency versus the normalized wave number for T=400K, $n^*=10^{14}$ cm^{-3} and $n=10^{15}$ cm^{-3}.It is found that for $k\lambda^*_D = 2$ we have an increase of 11% of the frequency.

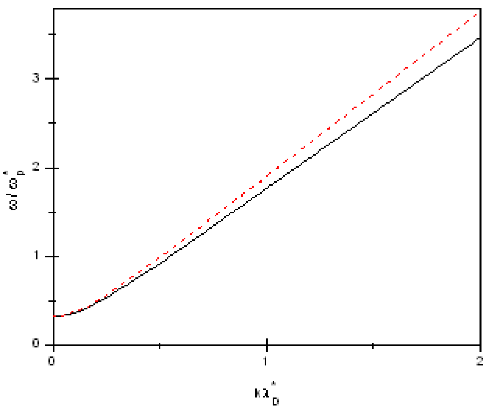

Figure1: The normalized frequency ω/ω_p^* vs $k\lambda_D^*$

for T= 400K and $n=10^{15}$ cm^{-3}.

The normalized LW phase velocity with respect to the normalized wave number is calculated for different densities, i.e., n=10^{15} cm^{-3}, n=10^{16} cm^{-3}, where n^* =10^{14} cm^{-3}. For a given wave number, viz., $k\lambda_D^*$ =2, it is found that LW phase velocity increases with increasing density, i.e., for the above mentioned set of densities, the LW phase velocity calculated with the new dispersion relation is, 1.063 and 1.29 times the phase velocity calculated with Bohm-Gross dispersion relation, respectively.

Moreover, we choose two environments, namely, the cold plasma and the photosphere plasma. The normalized LW phase velocity with respect to the normalized wave number is evaluated for different densities in the case of a cold plasma, i.e., n=10^6 cm^{-3}, n=10^7 cm^{-3} and T= 1 K. For a given wave number, viz., $k\lambda_D^*$ =2, it is found that LW phase velocity increases with increasing density, where for the above mentioned set of densities the LW phase velocity calculated with the new dispersion relation is, 1.036 and 1.042 times the phase velocity calculated with Bohm-Gross dispersion relation, respectively.

Regarding the normalized group velocity it is shown for the same densities that for $k\lambda_D^*$ =2, $v_g(\varepsilon) = 1.036 v_g(0)$ and $v_g(\varepsilon) = 1.137 v_g(0)$ .Furthermore, for the photosphere, the normalized wave frequency increases by an amount of 2.5% for n=10^{15} cm^{-3} and T=1000K. On the other hand, for n=10^{15} cm^{-3}, n=10^{16} cm^{-3} and T=1000 K, we have $v_\phi(\varepsilon)/v_\phi(0) = 1.063$; $v_g(\varepsilon)/v_g(0) = 1.03$ and $v_\phi(\varepsilon)/v_\phi(0) = 1.31$; $v_g(\varepsilon)/v_g(0) = 1.1$.

Conclusion

In conclusion, we recall that for denser plasmas the departure from BG dispersion relation of the proposed dispersion relation is clear. However, the effect of density in most realistic situations (ionosphere, photosphere, etc.) is not essential compared to the effect of temperature.

References

[1] M.Ouazene and R.Annou, Phys.Plasmas, 17, 052105 (2010).