Correct fluid treatment of the collisionless Tonks-Langmuir model via the polytropic-coefficient function

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Abstract

The basic Tonks-Langmuir (TL) model [Tonks & Langmuir, Phys. Rev. **34**, 876 (1929)] represents a quasi neutral collisionlees symmetric discharge with a cold ion source. In [Riemann et al. Plasma Phys. Control. Fusion **47**, 1949 (2005)], a fluid treatment involving the cold-ion approximation was given. This approximation is intrinsically questionable because the ions themselves are not cold but rather have a nonzero effective temperature even if the ion source is cold. Here it is shown that the correct results are obtained if the fluid equations are closed with the appropriate polytropic-coefficient function.

1. Introduction

Tonks and Langmuir [1] gave one of the most basic bounded-plasma models, i.e., a quasi neutral collisionless symmetric discharge with a cold ion source. The kinetic solution of this TL model was found by Harrison and Thompson [2]. Riemann et al. [3] gave an approximate $\left(T^i=0\right)$ fluid treatment of this model, including the matching of the plasma solution with the sheath solution. In [4] this matching was performed using a correct kinetic description. In a related development, Kuhn et al. [5] and Jelić et al. [6] convincingly showed that the ion polytropic coefficient γ^i, which is crucial to the closure of the system of fluid equations via its definition equation

$$\frac{dp^i}{dz}=\gamma^i k_B T^i \frac{dn^i}{dz}=\gamma^i \frac{p^i}{n^i}\frac{dn^i}{dz}, \tag{1}$$

is not a constant and the usual simple assumption $\gamma^i=const.$ can lead to grossly erroneous results especially near the plasma-sheath boundary.

In this work we make use of the kinetic results of [4] to calculate the correct Φ dependence of the ion polytropic coefficient and on this basis formulate the exact ion fluid equations. Solving this system together with the quasi neutrality condition $\left(n^i=n^e\right)$, yields the correct spatial profiles of Φ, n^i, u^i (the ion fluid velocity), p^i, $T^i\equiv p^i/\left(n^i k\right)$, and γ^i, which turn out to visibly differ from the ones obtained with the $T^i=0$ approximation.

2. Model and basic equations

A time-independent, one-dimensional collisionless Tonks-Langmuir (TL) model is considered with the walls at $z = \pm L$ (Fig. 1). The walls are assumed to be non-emissive and perfectly absorbing. There is only one species of ions, which are generated by electron impact ionization of a cold neutral-gas background. The electrons are assumed to be Boltzmann distributed.

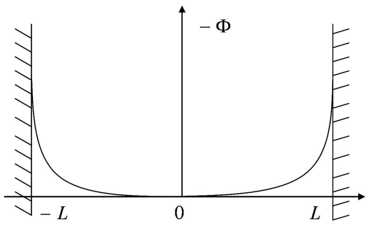


Figure 1: Potential profile of the TL model considered

Using the normalized variables of [4] we write the normalized ion kinetic equation as

$$\frac{\partial f^i(\varphi, y)}{\partial \varphi} + \frac{\partial f^i(\varphi, y)}{\partial y} = \frac{\sigma}{\sqrt{2}} \frac{dx}{d\varphi} \delta(y), \tag{2}$$

where f^i is the normalized ion velocity distribution function (VDF), x, φ and y are the normalized position, potential and velocity variables, respectively, and σ is the normalized ionization frequency. The basic ion fluid equations read

ion continuity equation (CE)

$$u^i \frac{dn^i}{dx} + n^i \frac{du^i}{dx} = e^{-\varphi}, \tag{3}$$

ion momentum equation (ME)

$$u^i \frac{du^i}{dx} + \frac{1}{n^i} \frac{dp^i}{dx} = \frac{d\varphi}{dx} - \frac{u^i}{n^i} e^{-\varphi}, \tag{4}$$

Poisson's equation (PE)

$$\varepsilon^2 \frac{d^2\varphi}{dx^2} = n^i - e^{-\varphi}, \tag{5}$$

and Boltzmann distribution for electrons

$$n^e = e^{-\varphi}, \tag{6}$$

where n^i, u^i, p^i are the ion density, fluid velocity and pressure, respectively and $\varepsilon = \lambda_D / l$ is the smallness parameter, with λ_D the Debye length and l the ionization length.

3. Cold-ion approximation of [3]

In the cold-ion approximation, the ME simplifies to

$$u^i \frac{du^i}{dx} - \frac{d\varphi}{dx} = -\frac{u^i}{n^i} e^{-\varphi}, \tag{7}$$

while the rest of the basic fluid equations remain the same. In the quasi neutral region, the basic fluid equations (with (7) replacing (4)) can be solved analytically, with the results

$$x_s = \frac{\pi}{2} - 1, \quad \varphi_s = ln2, \quad u^i_s = 1, \quad n^i_s = \frac{1}{2}, \tag{8}$$

where the subscript s denotes the values at the sheath edge.

4. Kinetic TL model of [4]

The solution of the ion kinetic equation (2) in the quasi neutral region is given by

$$f^i(\varphi, y) = \frac{2}{\pi}\frac{d}{d\eta}F_D(\sqrt{\eta})H(\eta)H(\varphi-\eta), \tag{9}$$

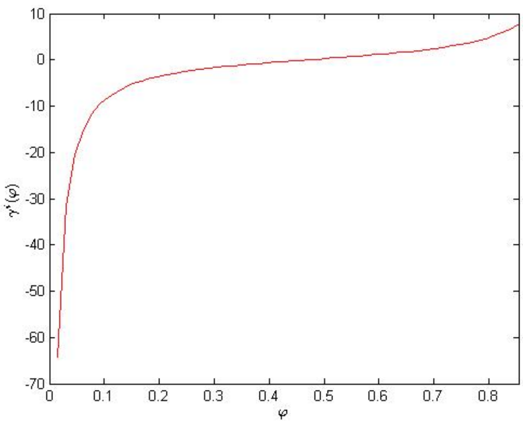
where F_D is the Dawson's function

$$F_D(x) = e^{-x^2}\int_0^x dt e^{-t^2}, \tag{10}$$

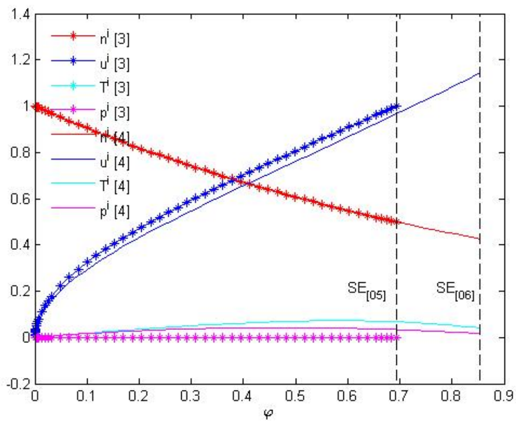
H is the Heaviside function and

$$\eta = \varphi - y. \tag{11}$$

The fluid quantities can be calculated from the quasi neutral ion VDF (9). Figure 2 shows the variation of γ^i with φ and the comparison of the fluid quantities of [3] and [4]. $SE_{[05]}$ represents the sheath potential of [3] and $SE_{[06]}$ represents the sheath potential of [4]. We see that γ^i is not a constant but varies spatially and even becomes negative. Therefore, we conclude that the usual constant values assumed for γ^i are wrong and can give rise to significant errors, especially near the sheath edge. Comparing the fluid quantities, we see that n^i is the same in both cases, (namely the quasi neutral ion density $e^{-\varphi}$), while the ion fluid velocity u^i for [3] is different from that of [4], which discrepancy is due to the cold-ion approximation. Note that to compare the results of cold-ion approximation of [3] with the kinetic results of [4], the basic fluid equations ((3),(5),(6) and (7)) have been solved in terms of φ.



(a) The ion polytropic coefficient



(b) Comparison of the fluid quantities of [3] and [4]

Figure 2: (a) Ion polytropic coefficient as a function of φ and (b) comparison of the fluid quantities of [3] and [4].

5. Correct fluid treatment of the TL model

We now use the ion pressure gradient calculated from the results of [4] and with it solve the basic fluid equations. Figure 3 shows the comparison of the ion fluid velocity calculated from the quasi neutral ion VDF (9) with the one calculated from the basic fluid equations with $T^i \neq 0$. We see that now there is no difference in the ion fluid velocity. Hence, we have shown that solving the fluid equations with the correct dp^i/dz (equivalent to the correct $\gamma^i(z)$) yields the same solution as the kinetic solution, while the cold-ion fluid solution (corresponding to $p^i = 0$) yields wrong results. This is because the ion themselves are not cold, but rather have a non-zero temperature, even if the ion source is cold.

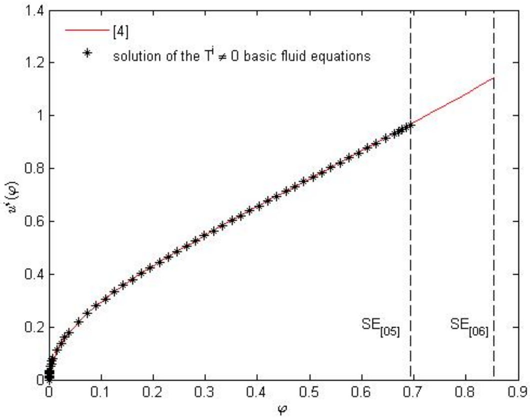


Figure 3: Comparison of the ion fluid velocity calculated from the quasi neutral ion VDF (9) with that calculated from the basic fluid equations with $T^i \neq 0$.

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References

[1] L. Tonks and I. Langmuir, Phys. Rev. **34**, 876 (1929).

[2] E. R. Harrison and W. B. Thompson, Proc. Phys. Soc. London **74**, 145 (1959).

[3] K.-U. Riemann, J. Seebacher, D. D. Tskhakaya sr. and S. Kuhn, Plasma Phys. Control. Fusion **47**, 1949 (2005).

[4] K.-U. Riemann, Phys. Plasmas **13**, 063508 (2006).

[5] S. Kuhn, K.-U. Riemann, N. Jelić, D. D. Tskhakaya sr., D. Tskhakaya jr. and M. Stanojević, Phys. Plasmas **13**, 013503 (2006).

[6] N. Jelić, K.-U. Riemann, T. Gyergyek, S. Kuhn, M. Stanojević and J. Duhovnik, Phys. Plasmas **14**, 103506 (2007).