Transverse kinetic drift wave and particle transport

J. Vranjes

Belgian Institute for Space Aeronomy, Brussels, Belgium

The transverse drift mode has been predicted long ago in [1], and described in detail also in Refs. [2, 3]. Yet, the impression is that it is not enough exploited neither in the laboratory nor in the space plasmas. Features of the transverse drift mode are described by

$$\vec{k} \cdot \vec{B}_0 = 0, \quad \vec{E}_1 \parallel \vec{B}_0, \quad \vec{B}_1 \perp \vec{B}_0, \quad \vec{E}_1 \perp \vec{B}_1. \tag{1}$$

Here, $\vec{E}_1, \vec{B}_1$ are the perturbed components of the electromagnetic field. Hence, the mode is electromagnetic, its geometry implies the perturbed electric field that is in the direction of the background magnetic field vector $\vec{B}_0 = B_0 \vec{e}_z$, the magnetic field perturbation is in the x-direction, $\vec{B}_1 = (k_y E_{z1}/\omega)\vec{e}_x$, while the mode propagates in the y-direction, $\vec{k} = k_y \vec{e}_y$, i.e., perpendicular to both, the magnetic field vector, and the gradients of the background plasma parameters (i.e., the density and the magnetic field) that are assumed to be in the x-direction.

In the limit $k_y \rho_i < 1$, and using the usual local approximation $k_y L_n, k_y L_b > 1$, where L_n, L_b denote the characteristic inhomogeneity lengths of the density and the magnetic field, $L_n = [(dn_0/dx)/n_0]^{-1} \equiv 1/\varepsilon_n$, $L_b = [(dB_0/dx)/B_0]^{-1} \equiv 1/\varepsilon_b$, the frequency and the growth rate of the mode are given by [1]:

$$\omega_r = -\frac{k_y \kappa T_e}{e n_0 B_0} \frac{dn_0}{dx} \frac{1}{1 + k_y^2 c^2/\omega_{pe}^2}, \tag{2}$$

$$\frac{\gamma}{\omega_r} = \pi \frac{m_e}{m_i} \frac{\varepsilon_n}{\varepsilon_b} \left(1 + \frac{k_y^2 c^2}{\omega_{pe}^2}\right)^{-2} \left(1 + \frac{k_y^2 c^2}{\omega_{pe}^2} + \frac{T_e}{T_i}\right) \exp\left\{-\frac{\varepsilon_n}{\varepsilon_b} \frac{T_e}{T_i} \frac{1}{1 + k_y^2 c^2/\omega_{pe}^2}\right\}. \tag{3}$$

In the opposite limit, $k_y \rho_i > 1$ the mode is described by:

$$\omega_r = -\frac{2\kappa T_e k_y \varepsilon_b (k_y \rho_e)^{-2}}{m_e \Omega_e (1 + T_i/T_e)}, \quad \frac{\gamma}{\omega_r} = \frac{\pi}{k_y^2 \rho_e^2} \frac{T_i}{T_e} \left(\frac{m_e}{m_i}\right)^{3/2} \left(\frac{2 + 2T_i}{T_e}\right)^{-1/2} \exp\left[-\frac{2T_e/T_i}{k_y^2 \rho_e^2 (1 + T_i/T_e)}\right]. \tag{4}$$

This perpendicularly propagating mode may cause a new stochastic motion mechanism, which is of a completely different nature as compared to the 'standard' obliquely propagating drift waves. To show this we may follow two adjacent particles a and b in the wave field, that are initially at positions $\vec{r}_a, \vec{r}_b$ such that $\vec{r}_b = \vec{r}_a + \vec{\delta}r$, and in general have different velocities $\vec{v}_a, \vec{v}_b$ determined by

$$\frac{d\vec{v}_a}{dt} = \frac{q}{m}\left[\vec{E}(\vec{r}_a, t) + \vec{v}_a \times \vec{B}(\vec{r}_a, t)\right], \quad \frac{d\vec{v}_b}{dt} = \frac{q}{m}\left[\vec{E}(\vec{r}_b, t) + \vec{v}_b \times \vec{B}(\vec{r}_b, t)\right]. \tag{5}$$

Here $\vec{E}(\vec{r}_{a,b},t)$ is the wave electric field in the z-direction and the following notation will be used: $\vec{B}(\vec{r}_a,t) = \vec{B}_0(\vec{r}_a) + \vec{B}_1(\vec{r}_a,t)$, $\vec{B}(\vec{r}_b,t) = \vec{B}(\vec{r}_a,t) + (\delta\vec{r}\cdot\nabla)\vec{B}(\vec{r}_a,t)$, $\vec{E}(\vec{r}_b,t) = \vec{E}_1(\vec{r}_a,t) + (\delta\vec{r}\cdot\nabla)\vec{E}_1(\vec{r}_a,t)$. Subtracting these two equations yields the distance between particles in the x-direction

$$\frac{d^2\delta x}{dt^2} = \Omega\frac{d\delta y}{dt} + v_{b,y}\Omega\delta x\frac{d\ln B_0}{dx} \quad \Rightarrow \quad \frac{d\delta x}{dt} \simeq \Omega\delta y. \tag{6}$$

Here, and further in the text the background magnetic field inhomogeneity is dropped out as it introduces higher order terms only. Similarly

$$\frac{d^2\delta y}{dt^2} + \Omega^2\left[1 - \frac{v_{b,z}}{\Omega}\frac{\partial}{\partial y}\left(\frac{B_{x1}}{B_0}\right)\right]\delta y = \Omega\frac{d\delta z}{dt}\frac{B_{x1}}{B_0}. \tag{7}$$

Here $\vec{B}_1 = B_{x1}\vec{e}_x$. It is seen that for a positive term $1 - \delta$, $\delta = [v_{b,z}/(\Omega B_0)]\partial B_{x1}/\partial y$, Eq. (7) describes forced harmonic oscillations due to the term on the right-hand side. In the other limit $1 - \delta < 0$, the distance between the two starting particles grows and particle dynamics becomes stochastic. For a sinusoidal magnetic field perturbation $B_{x1} = \widehat{B}_{x1}\sin(ky - \omega t)$, and using

$$\frac{dv_{b,z}}{dt} \simeq \frac{q\widehat{E}_{z1}}{m}\sin(ky - \omega t)$$

together with the Faraday law, the stochastic behavior will take place provided that

$$\frac{\widehat{E}_{z1}^2}{B_0^2} > \frac{\omega^2}{k_y^2} \equiv v_{ph}^2. \tag{8}$$

The condition (8) is a completely new result, obtained for strictly perpendicular waves $k \equiv k_y$, which differs essentially from the well condition dealing with the oblique drift waves.

Note that the classic condition dealing with the oblique drift waves can be obtained following exactly the same procedure. This may be demonstrated by starting from Eqs. (5) but taking $\vec{B}$ as the unperturbed, equilibrium magnetic field [this is because the classic stochastic motion is a linear effect due to essentially electrostatic oblique drift wave]. Repeating the procedure for such a case one obtains

$$\frac{d^3\delta y}{dt^3} + \Omega^2\left(1 - \frac{1}{\Omega^2}\frac{\partial E_{y1}}{\partial y}\right)\frac{d\delta y}{dt} = \delta y\frac{\partial}{\partial y}\frac{\partial E_{y1}}{\partial t}. \tag{9}$$

For harmonic waves the left-hand side directly yields the classic stochastic condition

$$k_y^2\rho_i^2\frac{e\phi_1(t)}{\kappa T_i} \geq 1, \tag{10}$$

In the condition (8) no mass dependence appears, so in principle it does not exclude a possibility for both ions and electrons to simultaneously behave stochastically, contrary to the case of the oblique drift wave which does not act on both species in the same time. The condition
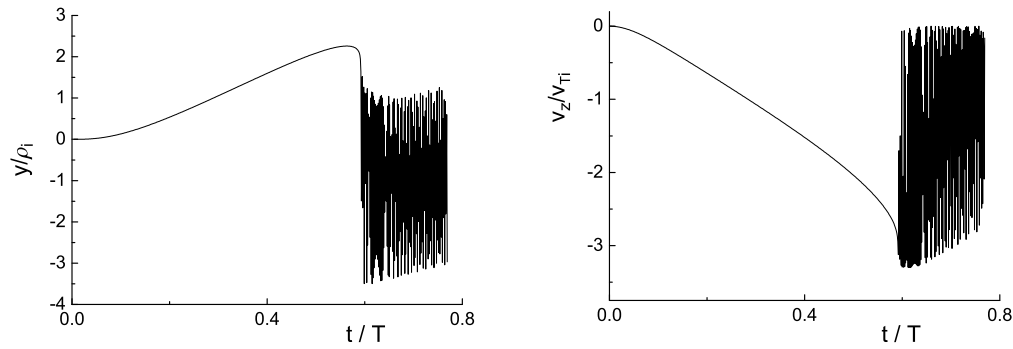
Figure 1: Left: displacement $y(t)[= v_x(t)/\Omega]$ normalized to the ion gyro-radius ρ_i, showing stochastic behavior within one wave-period T, for a tokamak plasma. Right: particle velocity $v_z(t)$ along the magnetic field for a tokamak plasma, normalized to the ion thermal speed, corresponding to the displacement from the figure left.
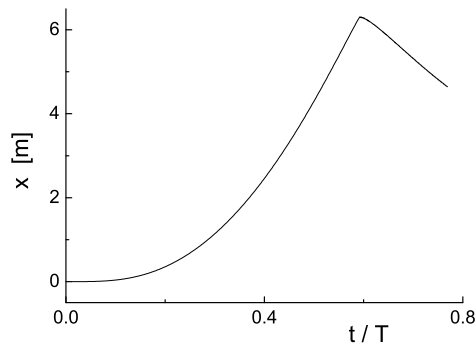

Figure 2: Particle transport in the x-direction, i.e. along the density gradient, in a tokamak plasma, simultaneous with the dynamics from Fig. 1.

(8) implies electromagnetic perturbations that are transverse, contrary to the case of the oblique drift wave which implies electrostatic and longitudinal perturbations. Further, the condition (8) follows from the nonlinear Lorentz force term in Eqs. (5), while in the case of the oblique drift wave the corresponding condition follows from a linear description of an electrostatic mode because the same Lorentz force term contains the unperturbed magnetic field. However, the condition (8) includes the parallel velocity of the particle, which means that only those particles that are pre-accelerated by the same wave to large enough velocities, will in addition also be subject to stochastic motion. Because the electric field is in the direction of the background magnetic field, this pre-acceleration is expected to be very effective. Hence, although it appears as a nonlinear phenomenon, it may be substantial. While the classic condition for the oblique drift wave yields stochastic motion (and heating) in the direction perpendicular to the magnetic field

vector, the stochastic motion caused by the transverse drift wave takes place in both directions, perpendicular and parallel to the magnetic field vector.

In application to the laboratory plasma, as one example we may use some typical tokamak parameters at fusion temperatures $T_i = T_e = 10^8$ K, $n_0 = 10^{20}$ m^{-3}, $B_0 = 2.8$ T, and also assume L_n to be equal to the minor radius $\simeq 1$ m. The initial conditions are as follows: $x(0) = y(0) = z(0) = 0$, $v_x(0) = v_z(0) = 0$, $v_y(0) = 1$ m/s. The previously introduced x, y, z directions in this case correspond to the radial, azimuthal and axial directions, respectively. Taking the wavelength $\lambda_y = L_n/30$ yields the frequency $\omega_r \simeq 5.7 \cdot 10^5$ Hz, and the critical electric field becomes $E_c = 8.5$ kV/m. For comparison, in the present case the classic stochastic threshold yields the electric field $E_{y1} \simeq 4$ MV/m. Hence, it is much higher, as predicted by the theory presented above. The particle dynamics is presented in Fig. 1, for the electric field $E_c = 9$ kV/m.

In addition to this, in Fig. 2 the proton drift is presented in the direction of the background density gradient. It is seen that the proton is subject to a very effective transport in the radial (or x-)direction. Within around $1/3$ of the wave period, the particle is already transported to the distance of 1 m (the assumed minor radius). Therefore in reality its further transport (up to 6 meters in the figure) will not happen, and the particle will be lost at the wall before the stochastic effects take place.

However, the onset of stochastic motion can be controlled by several parameters. As an example, taking tritium instead of protons, and with the temperature $T_i = 10^7$ K and a shorter wavelength $\lambda_y = L/80$, the stochastic effects take place earlier, at the moment $t \simeq T/1.5$, where now $T = 4.3 \cdot 10^{-6}$ s. Instead of 6 m obtained in Fig. 2, the maximum value for x is now shifted to around $x \simeq 0.33$ m only, so both the stochastic motion and the transport will take place. One can conclude that the stochastic phenomena associated with the transverse drift wave imply an enormously efficient radial transport of particles which clearly represents a risk for the plasma confinement and deserves a more detailed investigation.

References

[1] N. Krall and M. N. Rosenbluth, Phys. Fluids **6**, 254 (1963).

[2] N. Krall, in Advances in Plasma Phys., ed. A. Simon and W. B. Thompson, Interscience, New York 1968, p. 153.

[3] A. B. Mikhailovskii, Electromagnetic Instabilities in an Inhomogeneous Plasma. IOP Pub., London 1992, p. 40.