Collision Frequency Dependence of Neoclassical Tearing Mode Threshold

K. Imada and H.R. Wilson

York Plasma Institute, Department of Physics, University of York, Heslington, York YO10 5DD, U.K.

Introduction

Neoclassical tearing modes (NTMs) are resistive MHD instabilities characterised by magnetic islands. They limit tokamak performance by reducing the core pressure. Controlling NTMs in future devices such as ITER is crucial. In toroidal geometry, the finite banana width of trapped particle orbits gives rise to the neoclassical polarisation current, induced when the magnetic island is moving relative to the plasma. It has been suggested that the neoclassical polarisation current may provide the threshold mechanism [1]: sufficiently small magnetic islands (typically below 1cm in width) shrink away, while larger ones grow due to the bootstrap drive [2]. Understanding this threshold physics is essential for developing an effective NTM control system.

This paper employs drift kinetic theory developed in Refs.[1, 3] to study the collision frequency dependence of the neoclassical polarisation current contribution to the modified Rutherford equation, characterised by the coefficient $g(\nu_{ii}, \varepsilon, \omega)$. Here, ν_{ii} is the ion-ion collision frequency, ε is the inverse aspect ratio and ω is the island propagation frequency. The neoclassical polarisation current depends strongly on the collision frequency regime: $O(\varepsilon^{3/2})$ smaller in the collisionless limit ($\nu_{ii} \ll \varepsilon\omega$) than in the collisional limit ($\nu_{ii} \gg \varepsilon\omega$) [1, 3]. The leading order dependence on ν_{ii} in the low collision frequency limit is found to scale as $\sqrt{\nu_{ii}/\varepsilon\omega}$. This originates from the collisional layer in the vicinity of the trapped/passing boundary in the pitch angle space, $\Delta\lambda \sim \sqrt{\nu_{ii}/\varepsilon\omega}$. We extend these analytic results to determine the full collision frequency dependence of the neoclassical polarisation current. As in Ref.[1], the electrostatic potential is determined by quasineutrality. In order to focus on the role of collision frequency, additional physics such as finite Larmor orbit effects are not taken into account here. In addition, we consider the contribution to the current perturbation away from the island separatrix only, avoiding the complicated physics associated with a layer that exists there (such as finite radial diffusion). Our new results in the low to intermediate collision frequency regime ($\nu_{ii} \lesssim \varepsilon\omega$) is of particular interest, as the plasma parameters for the ITER steady state scenario [4] lie in this regime.

Ion Response

The ion response to the perturbed magnetic geometry is described by the drift kinetic equation. The non-adiabatic response, g_i, is expanded in terms of two small parameters: $\Delta = w/r$ and $\delta_i = \varepsilon^{1/2}\rho_{\theta i}/w$. The full derivation of ion and electron responses is given in Ref.[1]. At leading order ($O(\delta_i^1\Delta^0)$), g_i involves a free function $\bar{h}_i$, which arises from integration along unperturbed field lines; this function carries the leading order collision frequency dependence. $\bar{h}_i$ is determined from a constraint equation obtained by averaging the higher order equations over these unperturbed field lines:

$$-Rqk_\parallel \left\langle \frac{Rq}{v_\parallel} \frac{\omega}{m\tilde{\psi}} \frac{\mathrm{d}h}{\mathrm{d}\Omega} \left. \frac{\partial g_i^{(1,0)}}{\partial \xi} \right|_\Omega \right\rangle_\theta + \left\langle \frac{Rq}{v_\parallel} C_i \left(g_i^{(1,0)} \right) \right\rangle_\theta = 0 \tag{1}$$

for the passing particles, where $\langle ... \rangle_\theta$ denotes averaging over a period in poloidal angle θ and C_i is a model collision operator. The function h represents the electrostatic potential profile in the vicinity of the island (related to the electron density profile) and Ω is the normalised perturbed flux, with $\Omega = 1$ labelling the island separatrix (See Ref.[1] for other parameters, which are standard). For the trapped particles, the constraint equation is:

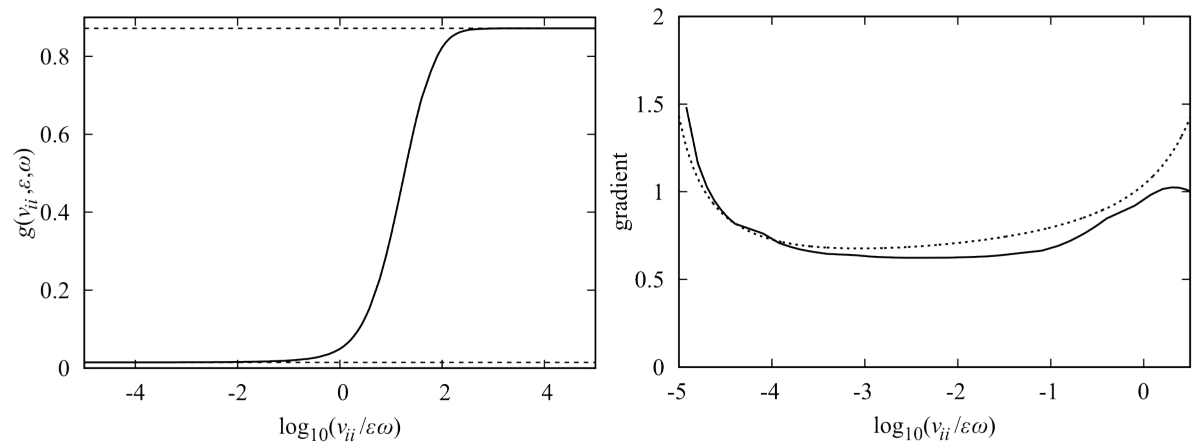
$$-Rqk_\parallel \frac{\omega}{m\tilde{\psi}} \frac{\mathrm{d}h}{\mathrm{d}\Omega} \left\langle \frac{Rq}{|v_\parallel|} \right\rangle_\theta \left. \frac{\partial \bar{h}_i}{\partial \xi} \right|_\Omega + \left\langle \frac{Rq}{|v_\parallel|} C_i \left(\bar{h}_i \right) \right\rangle_\theta = 0, \tag{2}$$

where $\langle ... \rangle_\theta$ is averaging between the bounce points. Eqs.(1) and (2) have been solved analytically in the collisionless and collisional limits by neglecting appropriate terms. Here, the full constraint equation is solved numerically for an arbitrary value of $\nu_{ii}/\varepsilon\omega$.

Results

The numerical result for $g(\nu_{ii}, \varepsilon, \omega)$ as a function of $\nu_{ii}/\varepsilon\omega$ is presented in Figure 1(a). A positive value of g corresponds to a stabilising contribution of the neoclassical polarisation current to the modified Rutherford equation. The numerical result is in excellent agreement with the analytic solutions in the collisionless and collisional limits. The transition from the collisionless to the collisional limit occurs between $\nu_{ii}/\varepsilon\omega \sim 0.1$ and $\nu_{ii}/\varepsilon\omega \sim 100$, for $\varepsilon = 0.1$. In Figure 1(b) the gradient of $g(\nu_{ii}, \varepsilon, \omega)$ in the low collision frequency limit is plotted, illustrating that the leading order collisional correction to g is indeed $O(\sqrt{\nu_{ii}/\varepsilon\omega})$ in the low collision frequency limit, aside from the weak logarithmic dependence which offsets the gradient from the expected value of 1/2.

Figure 1(a) shows a case where $g(\nu_{ii}, \varepsilon, \omega)$ is positive across all of the collision frequency range. However, the sign of g depends on the relative size of ω with respect to the ion diamagnetic frequency, ω_{*i}. The variation of g with ω/ω_{*i} is plotted in Figure 2(a), for $\varepsilon = 0.1$ and $\eta_i = 0.1$, where η_i is the ratio of the density and ion temperature gradient length scales. Considering the explicit dependence of g on ω in the analytic limits given in Refs.[1, 3], we note



(a) (b)

Figure 1: (a): Plot of $g(\nu_{ii},\varepsilon,\omega)$ as a function of $\nu_{ii}/\varepsilon\omega$ (solid curve), for $\varepsilon = 0.1$, $\omega/\omega_{*i} = 2.5$ and $\eta_i = 1.0$. The dashed lines represent the values of g in the analytic limits. (b): Plot of the gradient: $\mathrm{d}\log\left(g(\nu_{ii}) - g(0)\right)/\mathrm{d}\nu_{ii}$ against $\nu_{ii}/\varepsilon\omega$ for $\varepsilon = 0.1$, $\omega/\omega_{*i} = 2.5$ and $\eta_i = 0.5$. The dashed curve is the gradient taken from the analytic result, while the solid curve is the gradient taken from the new numerical result.

that g is negative if $0 < \omega/\omega_{*i} < (1+\eta_i)$ in the collisionless limit, or if $0 < \omega/\omega_{*i} < 1 + (1+k)\eta_i$ in the collisional limit. As is shown in Figure 2(a), g is negative everywhere in the collision frequency domain for sufficiently small ω ($< \omega_{*i}$), and is positive everywhere for $\omega > \omega_{*pi}$, as expected. Here, $\omega_{*pi} = \omega_{*i}(1+\eta_i)$ and $k(\varepsilon) = -1.17 f_c(\varepsilon)$ [5] is the familiar coefficient from the neoclassical theory with $f_c(\varepsilon)$ the passing particle fraction. However, when $0 < \omega \lesssim \omega_{*pi}$, the sign of g changes as the collision frequency is increased. Furthermore, there is a range of ω where g has a maximum in the intermediate collision frequency regime (see Figures 2(a) and 2(b)). This new result implies that whether the neoclassical polarisation current can stabilise the magnetic island depends not only on the plasma parameters (e.g. collision frequency), but on ω as well.

Lastly, we examine the relationship between the coefficient $k(\varepsilon)$ and the critical value of ω in the collisional limit, ω_{crit}, for which the sign of g reverses. From the analytic expression for $g(\omega)$ in the collisional limit [3], $\omega_{crit} = \omega_{*pi} + k\eta_i\omega_{*i}$. Explicitly evaluating $f_c(\varepsilon)$ gives $k = -0.67$ and this expression correctly predicts our observed $\omega_{crit} = 1.33\omega_{*i}$, for $\varepsilon = 0.1$ and $\eta_i = 1.0$ (Figure 2(a)). Similarly, for $\varepsilon = -0.15$ and $\eta_i = 1.0$ (Figure 2(b)), $k = -0.57$ correctly gives our observed $\omega_{crit} = 1.43\omega_{*i}$. It should be noted that in both cases the value of k is substantially different from its asymptotic value, $k(\varepsilon = 0) = -1.17$, even though ε is small.

Conclusion

We have determined the full collision frequency dependence of the neoclassical polarisation current using drift kinetic theory. Our new results show that the collisional correction to the neoclassical polarisation current becomes important even for very low collision frequency,

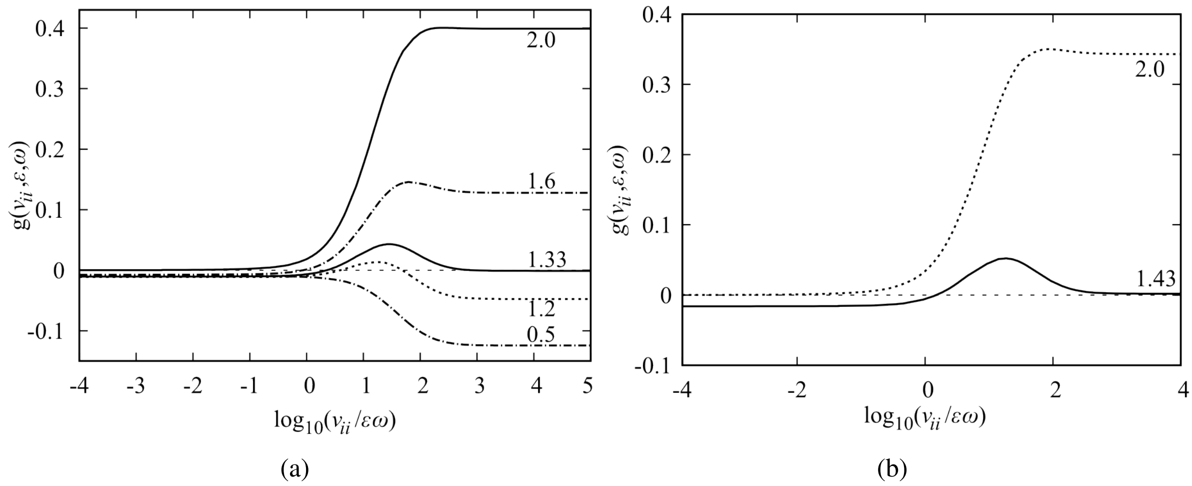


Figure 2: (a): Plot of $g(\nu_{ii},\varepsilon,\omega)$ vs. $\nu_{ii}/\varepsilon\omega$ for $\varepsilon = 0.1$ and $\eta_i = 1.0$, for different values of ω. The numbers on the right hand side of the graph are the values of ω/ω_{*i} for each curve. (b): $g(\nu_{ii},\varepsilon,\omega)$ vs. $\nu_{ii}/\varepsilon\omega$ for $\varepsilon = 0.15$ and $\eta_i = 1.0$. The numbers on the right hand side of each graph shows the values of ω/ω_{*i}.

$\nu_{ii}/\varepsilon\omega \gtrsim 0.1$ ($\varepsilon = 0.1$). We have also revealed the complicated dependence of the neoclassical polarisation current on both the collision frequency regime and the island propagation frequency. Whether the neoclassical polarisation current can stabilise the NTMs depends crucially on the relative rotation between the plasma and the island, as well as the collision frequency regime.

Using the parameters given in Ref.[4], we find that the collision frequency regime of the ITER steady state scenario lies in the range $0.06 \lesssim \nu_{ii}/\varepsilon\omega \lesssim 0.34$, with $\varepsilon \simeq 0.32$. This is the range where the neoclassical polarisation current is particularly sensitive to the collision frequency, which may have implications for the NTM control strategy on ITER.

Acknowledgement: This work is funded by the UK Engineering and Physical Sciences Research Council, under grant EP/D065399/1.

References

[1] H.R. Wilson, J.W. Connor, R.J. Hastie and C.C. Hegna, Physics of Plasmas **3**, 248 (1996)

[2] R. Carrera, R.D. Hazeltine and M. Kotschenreuther, Physics of Fluids **29**, 899 (1986)

[3] K. Imada and H.R. Wilson, Plasma Physics and Controlled Fusion **51**, 105010 (2009)

[4] C. Gormezano *et al*, Nuclear Fusion **47**, S285 (2007)

[5] P. Helander and D.J. Sigmar, Collisional Transport in Magnetized Plasmas (Cambridge: Cambridge University Press, 2002) p.195