Propagation of ultrastrong femtosecond laser pulses in PLASMON-X

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Plasmon-X is a project based in the Frascati INFN laboratories using the Ti:Sa laser FLAME and electrons' linac SPARC. FLAME (Frascati Laser for Acceleration and Multidisciplinary Experiments) has a pulse with maximum energy $E_M = 7$ J, maximum duration $\tau_M = 25$ fs, maximum power $W_M = 250$ TW, wavelength $\lambda = 800\,\mathrm{nm}$, and repetition rate $\nu_{rep} = 10$ Hz (see, f.i., [1]). The pulse duration $\tau = 25\,\mathrm{fs}$ corresponds to the pulse length $L_z = 7.5\ \mu$m (~ 10 wavelengths within the pulse). An upgrade that includes the polarization control (S, P, circular) is planned for the near future [1]. The plasma density in different Plasmon-X experiments [2, 3] ranges as $n_e = 0.6 - 1\times 10^{19}\,\mathrm{cm}^{-3}$, up to $4\times 10^{19}\,\mathrm{cm}^{-3}$. Note that $n_e = 10^{19}\,\mathrm{cm}^{-3}$ corresponds to the plasma frequency $\nu_{p,e} = \omega_{p,e}/2\pi \simeq 28\,\mathrm{THz}$. Thus, the pulse duration is $\tau = 0.7\,T_p$, where T_p is the plasma period $T_p \equiv 2\pi/\omega_{p,e} = 35\,\mathrm{fs}$, while the collisionless skin depth, $d_e \equiv 2\pi c/\omega_{p,e} \simeq 10.6\ \mu$m, is an order of magnitude longer than the laser wavelength ($\lambda = 0.8\ \mu$m) and close to the pulse length ($L_z = 7.5\ \mu$m). The laser wake field (LWF) accelerator scheme [2] envisages that an electron bunch with the energy of 150 MeV and the transverse normalized emittance of 1 mm·mrad, whose transverse and longitudinal rms sizes are 5 μm and 2.5 μm, respectively, is injected in the second bucket of the Langmuir wave excited by a Ti:Sa pulse delivering 7 J of energy in 30 fs. The laser pulse with initial waist size of $L_\perp = 130\ \mu$m and minimum size of $L_\perp = 32.5\ \mu$m is guided by a matched channel profile. Plasma is 9.88 cm long and its density profile has a positive and varying slope with starting and ending densities of 1.5×10^{17} cm^{-3} and 2.5×10^{17} cm^{-3}.

We derive the nonlinear equations that describe the propagation of ultrashort laser pulses in a plasma, in the Plasmon-X device. We consider the interaction of the high frequency electromagnetic and Langmuir waves, while the acoustic phenomena are disregarded. The laser electric field is so strong that the electrons achieve relativistic jitter velocities. The nonlinear effects come mostly from the interaction between the electromagnetic pump wave with a Langmuir wave, whose frequency is considerably lower than that of the electromagnetic (laser) pump. The electrons are regarded as cold, i.e. the phase velocity of the nonlinear modes involved are much higher than the electron thermal velocity, see. e.g. Refs. [4, 5, 6]. The characteristic frequency of the laser light is sufficiently high and the ions are essentially immobile, i.e. $n_i = n_0$

and $\vec{v}_i = 0$. The component of the Ampere's law perpendicular to the direction of propagation of the laser beam and the Poisson's equation have the form

$$\frac{\partial^2 \vec{A}_\perp}{\partial t^2} - c^2 \left(\nabla_\perp^2 + \frac{\partial^2}{\partial z^2} \right) \vec{A}_\perp + \nabla_\perp \frac{\partial \phi}{\partial t} = \frac{\vec{j}_\perp}{\varepsilon_0}, \quad \left(\nabla_\perp^2 + \frac{\partial^2}{\partial z^2} \right) \phi = -\frac{\rho}{\varepsilon_0}, \tag{1}$$

where ρ and $\vec{j}_\perp$ are the charge density and the perpendicular component of the current density $\vec{j}$, respectively, that are calculated as $\rho = \sum_\alpha q_\alpha n_\alpha$ and $\vec{j} = \sum_\alpha q_\alpha n_\alpha \vec{v}_\alpha$, where q_α is the charge of the particle species α, and the hydrodynamic densities n_α and velocities $\vec{v}_\alpha$ are calculated from the appropriate hydrodynamic equation. The electron continuity and momentum equations take the form

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{v}) = 0, \quad \left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla \right) \vec{p} = q \left[-\nabla \phi - \frac{\partial \vec{A}}{\partial t} + \vec{v} \times \left(\nabla \times \vec{A} \right) \right], \tag{2}$$

where, for simplicity, the subscript for electrons has been omitted and $q = -e$. Here $\vec{p}$ is the electron momentum, related with the electron velocity $\vec{v}$ through $\vec{v} = \vec{p}/m_0\gamma$, m_0 is the electron rest mass, $\gamma = (1 + p^2/m_0^2c^2)^{\frac{1}{2}}$, and c is the speed of light. Under the Plasmon-X conditions, the solution is slowly varying in the reference frame moving with the velocity $\vec{e}_z u$ and we can use the approximation from [4, 5]. Using the dimensionless quantities $\vec{p} \to \vec{p}/m_0c$, $\vec{v} \to \vec{v}/c$, $\phi \to q\phi/m_0c^2$, $\vec{A} \to q\vec{A}/m_0c$, $n \to n/n_0$, $u \to u/c$, $t \to \omega_{p,e}t$, $\vec{r} \to (\omega_{p,e}/c)(\vec{r} - \vec{e}_z ut)$, where $\omega_{p,e}$ is the plasma frequency of stationary electrons, $\omega_{p,e} = (n_0q^2/m_0\varepsilon_0)^{\frac{1}{2}}$. The solution of the hydrodynamic equations (2) is sought in the almost 1-D, i.e. $\nabla_\perp \ll \partial/\partial z$, quasistatic regime, i.e. $\partial/\partial t \ll u\,\partial/\partial z$, and u is adopted close to the speed of light, i.e. $1 - u \ll 1$. Then, with the accuracy to the leading order, they are integrated as

$$(v_z - 1)\,n + 1 = 0, \quad -p_z + \gamma - 1 + \phi = 0, \quad \vec{p}_\perp + \vec{A}_\perp = 0, \quad A_z = 0. \tag{3}$$

Using the definition for γ, after some straightforward algebra, we obtain the dimensionless charge- and current densities as $n = [(\phi-1)^2 + \vec{A}_\perp^{\,2} + 1]/2\,(\phi-1)^2$ and $\vec{v}_\perp n = \vec{A}_\perp/(\phi - 1)$, which permits us to rewrite our basic equations (1) as

$$\left[\frac{\partial^2}{\partial t^2} - 2u \frac{\partial^2}{\partial t\, \partial z} - \left(1 - u^2\right) \frac{\partial^2}{\partial z^2} - \nabla_\perp^2 + \frac{1}{1-\phi} \right] \vec{A}_\perp = -\left(\frac{\partial}{\partial t} - u \frac{\partial}{\partial z} \right) \nabla_\perp \phi, \tag{4}$$

$$\frac{\partial^2 \phi}{\partial z^2} = \frac{(\phi-1)^2 - 1 - \vec{A}_\perp^{\,2}}{2\,(\phi-1)^2}. \tag{5}$$

The above equations constitute a Zakharov-like description of a modulated electromagnetic wave, coupled with a Langmuire wave via the nonlinearities that arise from the relativistic effects. Thus, besides the standard nonrelativistic three-wave coupling phenomena (the Raman scattering), in the relativistic case there arises also a possibility for the four-wave processes, that may lead to the modulational instability, soliton formation, etc. The actual nonlinear dynamics of the pulse strongly depends on the physical conditions in each particular device and can not be generalized. We apply our Eqs. (4) and (5) to the Plasmon-X conditions, for *moderately focussed* laser beams. With the present power of the laser in the Plasmon-X device, in most experimental setups we have $\vec{A}_\perp^{\,2} < 1$, i.e we can take the pulse to have a moderate intensity and expand into series the nonlinear terms in the above equations. We seek the solution in the form of a modulated electromagnetic wave, viz. $\vec{A}_\perp = \vec{A}_{\perp 0} \exp\{-i[\omega' t - k'(z+ut)]\} + c.c.$, where the dimensionless frequency ω' and the dimensionless wavenumber k' are defined as $\omega' = \omega/\omega_{p,e}$, $k' = ck/\omega_{p,e} = d_e/\lambda$, where ω, k, and λ are the frequency, the wavenumber, and the wavelength of the electromagnetic laser wave, respectively, while $\omega_{p,e}$ and d_e are the electron plasma frequency and the collisionless skin depth. They satisfy the linear dispersion relation of electromagnetic waves, $\omega = \sqrt{c^2k^2 + \omega_{p,e}^2}$, whose dimensionless version has the form $\omega = \sqrt{k^2+1}$. Here and in the following, for simplicity, we drop the primes. We adopt u to be equal to the group velocity of the electromagnetic wave $u = d\omega/dk = k/\omega$. Substituting these into the wave equation and the Poisson's equation, dropping the nonresonant zero- and double pump frequency terms in Eq. (4) (the latter is absent for a circularly polarized laser wave), these are further simplified to

$$\left[2i\omega\frac{\partial}{\partial t} + \frac{1}{\omega^2}\frac{\partial^2}{\partial z^2} + \nabla_\perp^2 - \phi\right] A_{\perp 0} = 0, \quad \left(\frac{\partial^2}{\partial z^2} + 1\right)\phi = -\frac{|A_{\perp 0}|^2}{2}. \tag{6}$$

We studied the 2-D evolution of a moderately focussed pulse, typical for the Plasmon-X accelerator scheme, by the numerical solution of Eqs. (6) in the 2-D regime $\nabla_\perp^2 = \partial^2/\partial x^2$. The initial condition was adopted in the form of an unchirped NLS soliton, modulated in the perpendicular direction by a Gausssian, $A_{\perp 0}(x,z,0) = 2\sqrt{C_1}\,\exp(-x^2/2L_x^2)\,\exp(i\,\delta k z)\,\mathrm{sech}(\sqrt{C_1}\,z)$, with $L_x = 25$, $C_1 = 0.07$, and $\delta k = 0.5$, with $\phi\,(x,z,0) = 0$. We followed its evolution until $t = 15$. During this time, the pulse travels approximately 4.5 cm, which is about half the length of the interaction chamber. The results are displayed in Fig. 1. The folding of the pancake-like pulse and the creation of a V shape was observed after $t \sim 5$ and the simultaneous longitudinal stretching due to the negative chirp. The related diminishment affected mostly the laser envelope, while the electrostatic potential still featured a sizable amplitude; the depth of its first minimum was more than 50% of its largest value, achieved immediately after the launch of the pulse. The potential minimum obtained a V-shape in the *x*-*z* plane and its perpendicular contraction due to collapse was rather small.

In conclusion, we have shown that the experimental scheme for the LWF electron acceleration, with the laser power of $W \leq 250$ TW, the pulse duration $\tau \geq$ 25fs, and the spot size of 130 μm, can be described by a nonlinear Schrödinger equation with a reactive nonlocal nonlinear term, that produces an oscillating electrostatic wake to the laser pulse. Such moderately focussed pulses develop a chirp, that is roughly proportional to the intensity. The pulse spreads, relatively slowly, in the direction of propagation

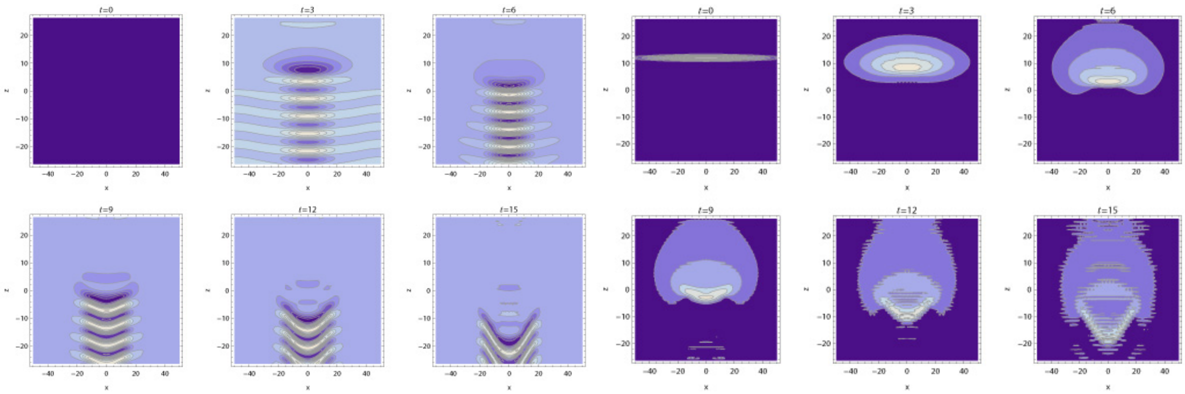

Figure 1: The electrostatic potential (left) and the amplitude (right) of a moderately focussed laser pulse, typical for the Plasmon-X accelerator scheme.

and its amplitude diminishes, and simultaneously a non-selfsimilar transverse collapse takes place. We do not expect that the latter constitutes a critical limitation for the Plasmon-X accelerator scheme. For very large amplitudes, ~ 30 times bigger than the potential in Fig. 1, the collapse is quenched by the saturation of the nonlinear term in the wave equation. The presence of the chirp offers a new venue for the stabilization of the collapse even at smaller amplitudes, known in nonlinear optics and in Bose-Einstein condensate, and it permits one to introduce some sort of external control, such as the appropriate profile of the plasma density, to stabilize the structure.

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