Plasma-sheath resonance in a plasma reactor: a model with finite geometry

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Abstract: A three dimensional model of plasma-sheath resonance in a plasma reactor is described and a complete solution is given for the case of a spherical plasma.

Plasma-sheath resonance in a finite geometry.

Previous theoretical studies of the phenomena of plasma-sheath resonance in an R.F. plasma have been restricted to plane geometry [1-3]. Uniform and non-uniform plasmas were considered and in the latter case, for certain values of the parameters, the plasma-sheath resonance coincides with the local plasma frequency, leading to an absorption of energy [3]. The present work addresses the problem of finite geometry and, as an illustration, a complete solution is presented for case of a spherical plasma. In the previous work the plasma was represented by an equivalent circuit as shown in Figure 1.

In the present work the plasma is described in terms of its dielectric properties. It is found that the plasma-sheath frequency depends on both the geometry and the plasma frequency as in the one-dimensional (plane) case.

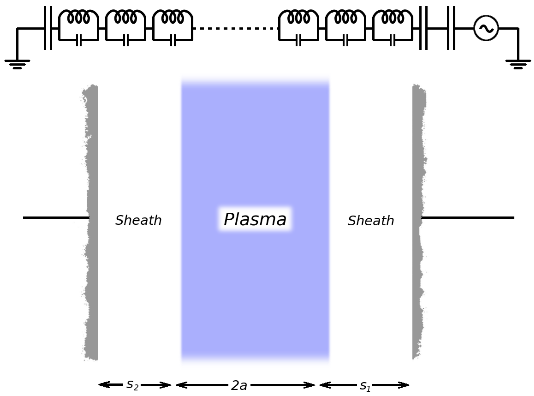

Figure 1: A circuit model to illustrate plasma-sheath resonance. Previous work included both uniform and non-uniform plasmas [1,2,3]

The plasma is considered to be uniform and to consist of cold collision-free electrons and immobile ions, the relative permittivity being given by $\varepsilon_r = \left(1 - \frac{\omega_{pe}^2}{\omega^2}\right)$. The sheath is represented by a vacuum region since there are relatively few electrons, and the positive ions are little perturbed by R.F. fields. Assuming that the plasma is uniform $\nabla^2 V = 0$ in both regions, using the quasi-static approximation $\nabla \times \mathbf{E} = 0$. The latter is an excellent approximation for a range of radio frequencies and reactor dimensions. The boundary conditions are $[D_\perp] = 0$ and $[E_\parallel] = 0$ at the plasma boundary, and on the electrodes $E_\parallel = 0$ and $E_\perp = \sigma/\varepsilon_0$, where σ is the surface charge density. In the absence of an applied magnetic field magnetic forces are negligible; it is interesting to note, however, that the magnetic field still plays a rôle in that the Poynting vector, $\mathbf{E} \times \mathbf{H}$, describes the flow of electromagnetic energy in the system. Let us consider first the case of a dielectric sphere in a uniform field $\mathbf{E}_0$, Laplace's equation can readily be solved [4] to yield the following result for the electric field inside the sphere where ε_1 and ε_2 are the permittivities of the sphere and the outer region.

$$\mathbf{E}_1 = \frac{3\varepsilon_2}{\varepsilon_1 + 2\varepsilon_2}\mathbf{E}_0 \tag{1}$$

In the case of a plasma sphere this becomes

$$\mathbf{E}_1 = \frac{1}{\left(1 - \frac{\omega_{pe}^2}{3\omega^2}\right)}\mathbf{E}_0 \tag{2}$$

It is seen that the plasma sphere has a resonant frequency $\omega_{pe}/\sqrt{3}$, where ω_{pe} is the electron plasma frequency, and that the electric field changes sign at this frequency.

Referring to the complete solution of Laplace's equation in both regions [4] we find that, inside the sphere

$$V_1 = -\frac{1}{\left(1 - \frac{\omega_{pe}^2}{3\omega^2}\right)}E_0 r\cos\theta \tag{3}$$

and outside the sphere

$$V_2 = -\left(1 + \frac{r_p^3}{r^3}\cdot\frac{\omega_{pe}^2/\omega^2}{(3 - \omega_{pe}^2/\omega^2)}\right)E_0 r\cos\theta \tag{4}$$

Let $\gamma = \omega_{pe}/\sqrt{3}\omega$, then

$$V_1 = -\frac{1}{(1-\gamma^2)}E_0 r\cos\theta \tag{5}$$

and

$$V_2 = -\left(1 + \frac{r_p^3}{r^3}\cdot\frac{\gamma^2}{1-\gamma^2}\right)E_0 r\cos\theta \tag{6}$$

where the potential is measured with respect to the midplane ($\theta = \pi/2$). The electric field is given by $\mathbf{E} = -\nabla V$ so that inside the sphere

$$\mathbf{E}_1 = \mathbf{u}_r\frac{1}{(1-\gamma^2)}E_0\cos\theta - \mathbf{u}_\theta\frac{1}{(1-\gamma^2)}E_0\sin\theta \tag{7}$$

which is a uniform field, and outside the sphere

$$\mathbf{E}_2 = \mathbf{u}_r\left[1 - \frac{2r_p^3}{r^3}\cdot\frac{\gamma^2}{(1-\gamma^2)}\right]E_0\cos\theta - \mathbf{u}_\theta\left[1 + \frac{r_p^3}{r^3}\cdot\frac{\gamma^2}{(1-\gamma^2)}\right]E_0\sin\theta \tag{8}$$

which can be rewritten as

$$\mathbf{E}_2 = \left[\mathbf{u}_r E_0 \cos\theta - \mathbf{u}_\theta E_0 \sin\theta\right] - \frac{r_p^3\gamma^2}{(1-\gamma^2)}\left[\mathbf{u}_r \frac{2E_0\cos\theta}{r^3} + \mathbf{u}_\theta \frac{E_0 \sin\theta}{r^3}\right] \tag{9}$$

It is seen that the field is a combination of a uniform field $\mathbf{E}_0$ and a dipole field due to the polarisation of the plasma sphere.

Application to a plasma reactor

We now seek a solution for a system exhibiting plasma-sheath resonance in which a spherical plasma contains a uniform electric field. It is seen from the above analysis, eqn.(6), that this is the case for two hemispherical electrodes of radius R, given by

$$R^3 = r_p^3 \frac{\gamma^2}{(\gamma^2-1)} \tag{10}$$

At this radius $V_2 = 0$, $E_{2\theta} = 0$ and $E_{2r} = 3E_0\cos\theta$. At the surface of the plasma sphere

$$V_1 = \frac{E_0}{(\gamma^2-1)} r_p \cos\theta \tag{11}$$

Figure 2 illustrates the field configuration for $\gamma^2 = 10/9$ and $\gamma^2 = 5/4$ respectively.

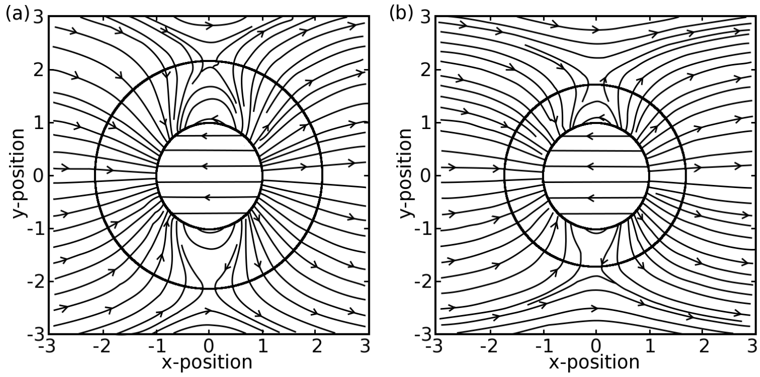

Figure 2: The electric field configuration applicable for plasma-sheath resonance for (a) $\gamma = (10/9)^{1/2}$ and (b) $\gamma = (5/4)^{1/2}$. The unit of distance is the radius of the plasma sphere. The density of the lines of force is not proportional to the strength of the electric field.

The plasma sphere becomes polarized and most, but not all, of the associated electric field lines leaving the surface charge extend from the plasma to the surrounding R.F. electrodes. The remaining field lines begin and end on the plasma sphere. Figure 3 illustrates the application to a radio frequency reactor which is fed by a current source.

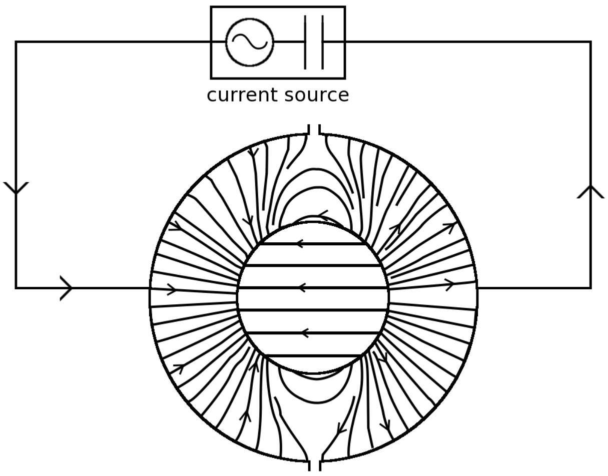

Figure 3: A schematic diagram of a spherical system employing plasma-sheath resonance. An external current source is connected to two closely spaced hemispherical electrodes.

The resonant frequency of the plasma sphere itself is $\omega_{pe}/\sqrt{3}$, where ω_{pe} is the electron plasma frequency, and corresponds to $\gamma^2 = 1/3$. In the present study the plasma-sheath resonance is found for values of γ exceeding unity. The value of the plasma-sheath resonant frequency depends on the radii of the plasma and the hemispherical electrodes. This behaviour is analogous to that of the one-dimensional case in which the resonance frequency depends on both the electron plasma frequency and the geometry. At the plasma-sheath resonance the impedance seen by the external circuit is zero, such that high currents could be injected. In practice some collisions will take place with a corresponding injection of energy into the plasma. Extension to other geometries is straightforward, simply involving solutions of Laplace's equation, but the case of a non-uniform three-dimensional plasma remains to be considered. The latter situation would involve the absorption of energy associated with the coincidence of the plasma-sheath resonance with a local plasma resonance referred to above [3].

References

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