

## On the proton to electron mass ratio in particle-in-cell simulations

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Choosing the electron to proton mass ratio in Particle in cell simulations has become an art where the modeler needs to ponder the processing time required by heavy ions, with the need to render reality. Lighter protons speed up calculations but may render the simulation unrealistic. Since a  $e^-p^+$  plasma with protons weighting the same that electrons is eventually a pair plasma  $e^-e^+$ , a critical mass ratio must be crossed between 1836 and 1 where a given simulation no longer describes reality.

A dimensional analysis of the problem shows that regardless of the number of dimensionless parameters introduced, the mass ratio  $R = m_p/m_e$  necessarily remains a part of the problem [1]. In the current absence of any rigorous criterion allowing to choose a proper mass ratio, the concept of modes hierarchy introduced above allows to define the critical mass ratio as *the smallest mass ratio leaving the mode hierarchy unchanged*.

Consider the simple and heavily studied system formed by a cold relativistic electron beam entering a cold electron/proton plasma. Two-stream unstable modes can be found with wave vector aligned with the flow. Filamentation modes are found propagating perpendicularly to the flow. Finally, modes arbitrarily oriented can equally be unstable so that the full unstable spectrum eventually looks like the one pictured in Figure 1. The whole problem relies on only 3 independent parameters, namely the beam to plasma density ratio  $\alpha$ , the beam Lorentz factor  $\gamma_b$  and the mass ratio.

Hierarchy maps [2] such as the one pictured on Fig. 2 are  $R$ -dependant because finite mass protons introduce new unstable modes, and modify the already unstable ones. Consider a real system  $S$  with  $R = 1836$ . The linear analysis can determine the most unstable mode which leads the linear phase. The same analysis for  $S(R < 1836)$  may yield a most unstable of the same nature, or not. If such is not the case, the largest  $R$  for which the dominant mode remains the

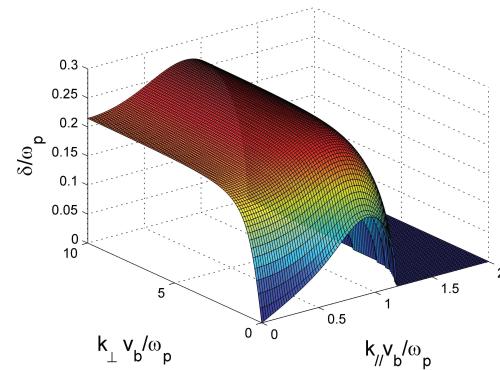


Figure 1: Growth-rate map for  $v_b = 0.9c$  and  $n_b/n_p = 1/10$ . The fastest growing modes are not found aligned with a privileged axis. The background plasma electronic frequency reads  $\omega_p$ .

same defines the critical mass ratio  $R_c$ .

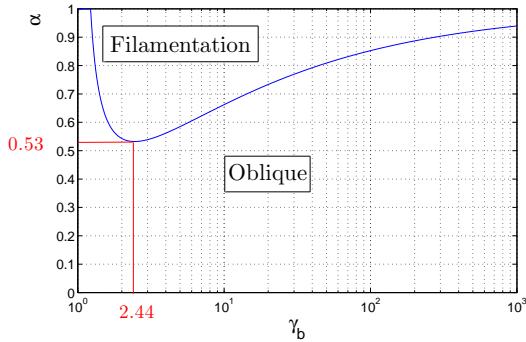


Figure 2: Hierarchy map in the cold regime in terms of the beam Lorentz factor and the beam to plasma density ratio  $\alpha$ . The non-relativistic regime  $\gamma_b = 1$  pertains to two-stream modes.

As an example, consider the system in relation with Figure 1. A finite mass ratio gives rise to an  $R$ -dependant hierarchy map such as the one depicted on Figure 3. Four kinds of unstable modes are here excited, namely Two-stream, Buneman, oblique and filamentation modes (the dominion of two-stream modes is restricted to the line  $\gamma_b = 1$ ). For  $R = 1836$ , the plain black lines picture the frontiers between the regions where the various modes govern the linear phase. For  $R = 30$ , the plain gray lines have the same function. The system represented by a circle switches from an obliquely dominated regime to a filamentation one, when the mass ratio goes from 1836 to 30. By contrast, the system represented by the square is governed by the same kind of mode for  $R = 30$  and 1836. For the circle system, the transition goes from a quasi-electrostatic dominant mode (oblique) to an electromagnetic one (filamentation), which obviously results in different linear and non-linear phases. Note that the criterion is *necessary*, but *not*

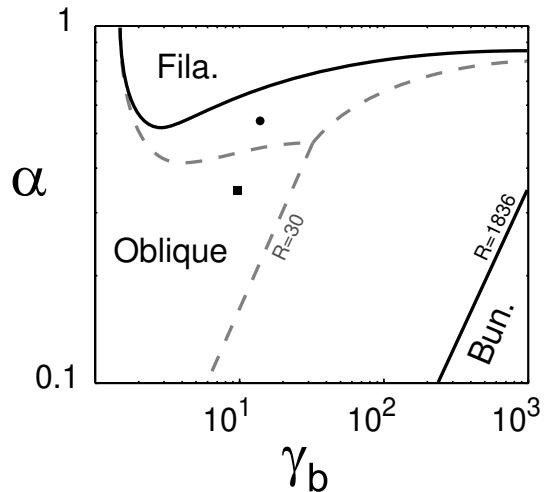


Figure 3: Mass ratio dependant hierarchy map for the present system. The system represented by a circle switches from an obliquely dominated regime to a filamentation one, when the mass ratio goes from 1836 to 30. By contrast, the system represented by the square is governed by the same kind of mode for  $R = 30$  and 1836.

sufficient. Qualitatively different linear phases should result in different non-linear evolutions as well. But similar studies have showed that similar linear phases (with varying  $R$ ) may still result in a different non-linear evolutions [3].

The critical mass ratio can be evaluated numerically and the result is displayed on Figure 4. The most sensitive systems are the ones which representative points in the parameters phase space lie just beneath the oblique/filamentation frontier, or just above the oblique/Buneman one, for  $R = 1/1836$ . Indeed, the slightest decrease of  $R$  will move the frontiers towards them, provoking a switch of dominant mode. In the context of shock formation, where the higher part of the graph is involved (high density ratio), changes in mass ratio are likely to be quite restricted. Things seem to be simpler for the acceleration phase implying the lower graph, with a thin beam-plasma interaction.

The present work is a first step towards a systematic search. The method proposed has been applied to a generic beam-plasma system, evidencing non-trivial values of the critical mass ratio. A similar analysis can be easily conducted varying the set-up: one needs first to evaluate the growth-rate map (the counterpart of Fig. 1) as a function of  $\mathbf{k}$  for the system under scrutiny with  $R = 1/1836$ . The same plot is then evaluated for the desired value of  $R$ . If the dominant mode remains the same, then the present criterion is met.

For example, when dealing with the problem of magnetic field amplification and particles acceleration in Supernova Remnants, a typical PIC setup consists in a non-relativistic beam of *protons* passing through a plasma with a guiding magnetic field [4, 5]. Due to the magnetization, unstable modes such as the Bell's ones [6] enrich the spectrum, and it would be interesting to apply our criterion to these cases.

## References

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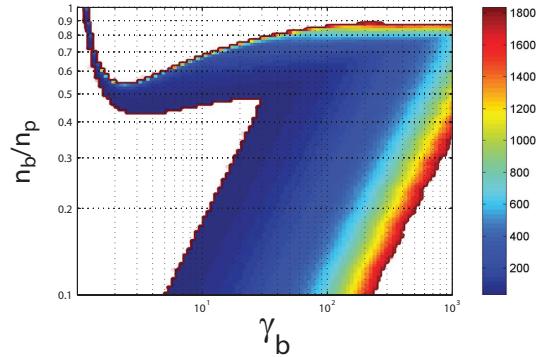


Figure 4: Critical mass ratio for the present system.

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