Nonlinear evolution of drift turbulence: inverse cascade, zonal flows, intermittency

Madalina Vlad, Florin Spineanu

National Institute of Laser, Plasma and Radiation Physics,
Association Euratom-MEdC Romania on Fusion Research,
P. O. Box MG-36, Magurele, Bucharest, Romania

The paper deals with the fundamental problem of strongly nonlinear stage in the evolution of drift type turbulence. We show that trajectory trapping or eddying in the structure of the turbulence potential is the main physical reasons for the strong nonlinear effects that were observed in numerical simulations: inverse cascade, zonal flows, nonlinear damping and intermittent evolution. The conclusion is drawn from a study of test modes on turbulent plasmas, which is based on a new analytical method. This is a Lagrangian approach that describes the effects of trapping on trajectory statistics.

1. Test modes on turbulent plasmas

We consider the drift instability in slab geometry with constant magnetic field (along z axis). We start from the basic gyrokinetic equations for the distribution of electrons and ions. The finite Larmor radius effects combined with the non-adiabatic response of the electrons destabilizes the drift waves. The frequency ω and the growth rate γ of the modes with wave numbers are

$$\gamma = \frac{\sqrt{\pi}}{|k_z| v_{Te}} \frac{k_y^{\,2}(V_* - V_*^{eff}) V_*^{eff}}{2-\Gamma_0} \tag{1}$$

$$\omega = k_y V_*^{eff}, \; V_*^{eff} = V_* \frac{\Gamma_0}{2-\Gamma_0} \tag{2}$$

where $\Gamma_0=\exp(-b)I_0(b)$, $b=k^2\rho^2/2$, ρ is the ion Larmor radius and V_* is the diamagnetic velocity. The wave number components are k_i, i=x,y,z and $k=(k_x^2+k_y^2)^{1/2}$. These are the characteristics of the linear (universal) drift instability on quiescent plasmas.

We consider a turbulent plasma with given statistical characteristics of the stochastic potential and we study linear test modes. The growth rates and the frequencies of the test modes are determined as functions of the statistical characteristics of the background turbulence with potential φ(x,t). The latter is a Gaussian field with the amplitude and the spectrum (or Eulerian correlation) taken as known input parameters. A wave type perturbation of the potential $\delta\varphi(x,z,t)=\varphi_{k,\omega}\exp(ik\cdot x+ik_z z-i\omega t)$ is introduced. It is small ($\delta\varphi\ll\varphi$) and thus it has a negligible influence on particle trajectories. The solutions for the perturbations of electron and ion densities are obtained using the method of the characteristics as integrals along particle trajectories in the background potential of the source terms determined by the density gradient.

The background turbulence produces two modifications in the response. One consists in the stochastic E×B drift that appears in the trajectories and the other is the fluctuation of the diamagnetic velocity due to the fluctuations of the density δn in the background turbulence. Both effects are important for ions while the response of the electrons is approximately the same as in quiescent plasma. They appear in the dispersion relation (quasi-neutrality condition), in the propagator Π and the new term R_{ij}:

$$2+i\sqrt{\pi}\frac{\omega-k_yV_*}{|k_z|v_{T_e}}=i\Pi\Gamma_0\left[\omega+V_*\left(k_y+ik_ik_jR_{ij}\right)\right] \tag{3}$$

$$\Pi=\int_{-\infty}^{t}d\tau\left\langle\exp\left(i\vec{k}\cdot\left(\vec{x}(\tau)-\vec{x}\right)\right)\right\rangle\exp(i\omega(t-\tau)) \tag{4}$$

$$R_{ij}=\int_{\tau}^{t}d\theta'\int_{-\infty}^{\tau-\theta'}d\theta M_{ij}(|\theta|),\quad M_{ij}(|\theta|)=\left\langle v_i(0,0)\frac{\partial}{\partial y}v_j\left(\vec{x}(\theta),\theta\right)\right\rangle \tag{5}$$

The average propagator contains the effects of the stochastic trajectories and the tensor R_{ij} is produced by the fluctuations of the diamagnetic velocity produced by the density fluctuations in the background potential. Both are averaged over the stochastic trajectories using the distribution of displacements P(**x**,t) in the background potential.

We have shown that P(**x**,t) is strongly modified when trapping appears [1,2]. Trapped trajectories form quasi-coherent structures, which determine non-Gaussian P(**x**,t). Trapping combined with the motion of the potential with the diamagnetic velocity V_* determines ion flows when the amplitude of the E×B velocity is larger than V_*. The trapped ions move with the potential while the other ions drift in the opposite direction. Although these opposite (zonal) flows compensate such that the average velocity is zero, they determine the splitting of P(**x**,t). The averages in Eqs. (4) and (5) are estimated using these results and the solution of the dispersion relation (3) is obtained (see [3]):

$$\gamma=\frac{\sqrt{\pi}}{|k_z|v_{Te}}\frac{k_y^2(V_*-V_*^{eff})(V_*^{eff}-nV_*)}{2-\Gamma_0\Im}-k_i^2D_i\frac{2-\Gamma_0\Im n_{tr}}{2-\Gamma_0\Im}+k_ik_jR_{ij}V_*^{eff} \tag{6}$$

$$\omega=k_yV_*^{eff},\quad V_*^{eff}=V_*\frac{\Gamma_0\Im(1-n)+2n}{2-\Gamma_0\Im},\ \Im=\exp\left(-\frac{1}{2}k_i^2S_i^2(V)\right) \tag{7}$$

Comparing to the drift modes on quiescent plasma, Eqs. (1) and (2), one can see that the background turbulence influences both the growth rate and the frequency (through the effective diamagnetic velocity). The turbulent dispersion of ion trajectories determines the second term in the growth rate (6), while the effects of ion trajectory trapping appear in the function $\Im$, in the ratio of the number of trapped and free ions n, and in the tensor R_{ij}. The significance of these terms and their effects are analysed in the next section.

2. Turbulence evolution

The growth rate and the frequency of the drift modes (6), (7) give an image of the turbulence evolution starting from a weak initial perturbation with very broad wave number spectrum. We show that a sequence of processes appear at different stages as transitory effects and that the drift turbulence has an oscillatory (intermittent) evolution.

2.1. Trajectory diffusion and damping of small k modes

At small amplitude of the turbulence only the second term in the growth rate appears and determines a stabilizing contribution due ion diffusion, which leads to the damping of the large k modes and to maximum growth rate at $k \sim 1/\rho$. The well known result of Dupree [4] is reproduced.

2.2. Trajectory structures and large scale correlations

The increase of the turbulence amplitude V above the effective diamagnetic velocity V_*^{eff} determines ion trapping or eddying. As we have shown, this strongly influences the statistics of trajectories. The distribution of the trajectories is not more Gaussian due to trapped trajectories that form quasi-coherent structures. At this stage the trapping is weak in the sense that the fraction of trapped trajectories n_{tr} is much smaller than the fraction n_{fr} of free trajectories and n can be neglected in Eqs. (6), (7). The peaked distribution modifies the average propagator by the factor $\Im$, which is determined by the average size S_i of the trapped trajectory structures. This factor modifies only the effective diamagnetic velocity in a way that is similar with the finite Larmor radius effect. The displacement of the position of the maximum of γ toward small k appears. The maximum of γ moves to smaller k values of the order of $1/S_i$ and the size of the unstable k range decreases. The maximum growth rate decreases. Thus, ion trapping determines the increase of the correlation length of the potential and the decrease of the average frequency (proportional with k_2). In this nonlinear stage, turbulence evolution becomes slower and leads to ordered states (narrower spectra with maximum at smaller k).

2.3. Ion flows and turbulence damping

The evolution of the potential determines the increase of the fraction of trapped ions. This determines a non-negligible average flux of the particles trapped in the moving potential. As the E×B drift has zero divergence, the probability of the Lagrangian velocity is time invariant, i. e. it is the same with the probability of the Eulerian velocity. The average Eulerian velocity is zero and thus the flux of the trapped ions that move with the potential has to be compensated by a flux of the free particles. These particles have an average motion in the opposite direction with a velocity V_{fr} such that $n_{tr}\ V_*^{eff} + n_{fr}\ V_{fr} = 0$. The velocity on structures method that we have recently developed shows that the probability of the displacements splits in two components that move in opposite direction. Thus, opposite ion flows are generated by the moving potential in the presence of trapping. They modify both the effective diamagnetic velocity and the growth rate through the factor n in Eqs. (6), (7). It determines the increase of the effective diamagnetic velocity and consequently the decrease of the growth rate first for small k and as n increases for all values of k.

2.4. Generation of zonal flow modes

The third term in the growth rate (6) in produced by the density fluctuations. The component R_{11} generates modes with $k_2=0$ and $\omega=0$ (zonal flow modes). We have found that R_{11} essentially depends on n. It increases when the ion flows become important up to a positive maximum and then it decreases to zero. It is essentially determined by the anisotropy that is generated by the difference in the average velocity of the trapped ions and the average velocity of the free ions. When n=1, the ion flows are symmetrical and R_{11} vanishes. A clear connection of the zonal flow modes with the ion flows induced by the moving potential appears.

4 Conclusions

A different physical perspective on the nonlinear evolution of drift turbulence is obtained. The main role is played by ion trapping in the stochastic potential. There is no causality connection between the damping of the drift turbulence and the zonal flow modes. Both processes are produced by ion trapping in the moving potential and lead to intermittent evolution of turbulence.

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