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## Trajectory of a charged dust inside magnetised sheath

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In the absence of plasma drag, micron size dust can not be confined to the sheath as the balance between gravity (which is directed to the wall) and electrostatic forces (which is directed away from the wall) is never exact and dust keeps oscillating across the plasma-sheath boundary. The oscillation does not damp in time since forces acting on the dust are conservative in nature. Only when the plasma drag acts on the dust that it can remain confined to the sheath with the dust trajectory mimicking a damped oscillator. It is known that in the absence of gravity, dust performs an inward spiralling motion inside the sheath before continuously oscillating around a small, equilibrium region. Proximity of the oscillating dust to the negatively charged wall is determined by the number of charge on the dust. In the presence of gravity, dust can travel much deeper inside the sheath since part of the repulsive electrostatic force on the dust is offset by the attractive gravitational force. Further, the equilibrium position around which the dust oscillates shifts closer to the wall. Clearly then, the plasma drag on the dust as well as gravity and electrostatic forces plays an important role in determining the dust trajectory inside the sheath.

The primary motivation of this work is to examine the effect of various forces on the dust dynamics in a non-uniform flowing plasma in the presence of an oblique magnetic field. To that end, we shall consider a bounded two-component plasma consisting of electrons and singly charged ions in the presence of an oblique magnetic field with  $\vec{B}/B = (\sin \theta, 0, \cos \theta)$  with the planer sheath boundary located at  $z = 0$  and plasma filling the half space  $z < 0$ . We shall describe the magnetisation of the plasma by the parameter  $\beta_j = \omega_{cj}/v_{jn}$ , which is a ratio of the plasma-cyclotron to the plasma-collision frequencies. The basic set of equations for such a collisional sheath is solved numerically [Pandey et al., 2011].

In Fig. 1(a) – 1(d), the sheath potential, [1(a)], the electric field [1(b)], the plasma densities [1(c)] and the charge on the dust inside the sheath, [1(d)] is shown for the following parameters,  $\theta = \pi/2$ ,  $\beta_i = 0.01$ ,  $v_I = 0.001 \omega_{pi}$ ,  $v_{in} = v_{en} = 0.1 \omega_{pi}$ . The dotted line corresponds to the  $\beta_e = 1$  and bold line to  $\beta_e = 5$ . It is clear that with the increased electron magnetization, expectedly, sheath width decreases (bold line) as the electron motion is increasingly inhibited by the magnetic field. Consequently, there is a less negative wall potential which in turn leads to less

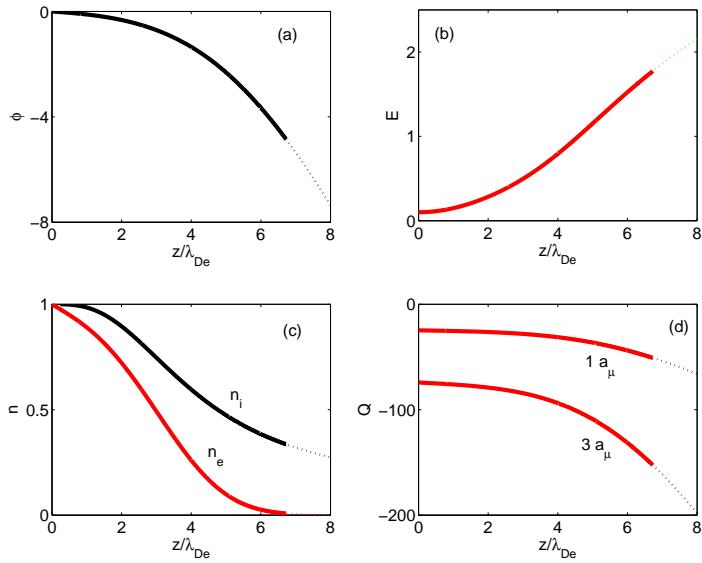


Figure 1: The sheath potential [1(a)], the electric field [1(b)], the plasma densities [1(c)] and charge on the 1 and 3 micron dust inside the sheath is shown in the above figures. The solid and dotted line reflects different values of  $\beta_e$ .

negative charge on the dust grain [1(d)]. Thus, the electron magnetization, although indirectly, plays an important role in the dust dynamics.

The dynamics of the dust grain is determined by numerically solving following set of equations

$$\frac{dz}{dt} = v_d; \quad m_d \frac{dv_d}{dt} = F_{e,i} + F_E + F_g, \quad (1)$$

where the drag forces  $F_{e,i}$  are due to both electrons and ions. The electrostatic force  $F_E = Q\mathbf{E}$  and  $F_g = m_d g$ . Here  $Q$  and  $m_d$  is charge and the mass of the dust respectively. The electron and ion drag forces are [Fortov et al, 2005; Vladimirov et al, 2005]

$$F_e = \pi a^2 n_e m_e v_{te} v_e [(2+z) \exp(-z) + 5z^2], \quad (2)$$

$$F_i = \pi a^2 n_i m_i v_{ti} v_i (2+z\tau + 5z^2\tau^2). \quad (3)$$

Here  $j = \text{electron, ion}$ ,  $v_{tj} = \sqrt{k_B T_j / m_j}$  is the plasma thermal velocity,  $k_B$  is the Boltzmann constant,  $m_j$  is the mass and  $T_j$  is the temperature of the  $j^{\text{th}}$  specie,  $v_j$  is the plasma fluid velocities,  $a$  is the grain radius,  $n_j$  is the plasma number density,  $z = Ze^2/ak_B T_e$  with  $Z = Q/e$  is the number of grain charge and  $\tau = T_e/T_i$  is the ratio of plasma temperatures.

In Fig. 2 (left panel) we see the motion of the micron ( $a = 10^{-6}$  cm) sized dust particle. As can be seen from Fig. 1(d) that the dust is negatively charged. The initial condition chosen for

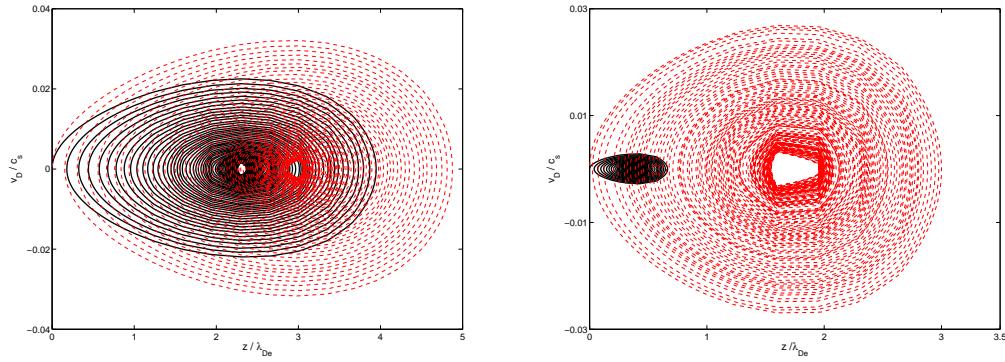


Figure 2: The motion of the micron (left panel) and 3 micron (right panel) size grain (for the sheath parameters of Fig. 1) is shown in the above figures. The solid spirals represents the dust trajectory when only electrostatic and drag forces operate while dotted spiral correspond to the case when gravity is also present.

solving dust Eq. (1) is  $v_D = 0$  at the plasma - sheath boundary  $z = 0$ . Under the influence of the plasma drag and electrostatic forces, dust moves towards the wall. However, as the dust approaches the wall, the repulsive electrostatic force overcomes all other forces. Therefore, almost halfway down the sheath, the dust turns back and almost reaches the plasma - sheath boundary ( $z = 0$ ) and starts the sheath ward journey under the influence of drag forces all over again. The radius of each spiral decreases with each cycle and the dust finally gets confined to a small region inside the sheath. The inward spiralling motion of the dust is indicative of the dissipative nature of the collisional plasma sheath. With each turn, dust loses some amount of energy. The dust motion mimics a driven damped oscillation in time. It is well known that dust can perform the oscillatory motion inside the sheath due to density fluctuations. However, in the present case, we do not introduce any density fluctuations. It would appear that the oscillatory motion of the dust is generic to the plasma sheath.

The role of magnetic field is only indirect to the dust dynamics. It is the sheath size that is directly affected by the magnetic field. The sheath size increases with decreasing angle between the field and the wall. This can also be seen if we recall that in the absence of magnetic field, sheath has maximum potential which can be easily inferred by balancing the electron and ion fluxes at the plasma sheath boundary. This yield [Pandey and Vladimirov, 2011]

$$\Phi \cong \left(1 + \frac{T_i}{T_e}\right) \left(1 + \frac{4}{\pi} \frac{k_B T_i}{m_i v_i^2/2}\right) \frac{M^2}{2}, \quad (4)$$

where  $M = v_i/c_s$  is the Mach number. For typical discharge conditions<sup>4</sup>  $T_i \ll T_e$  and since ions are almost cold,  $k_B T_i \ll m_i v_i^2$  the wall potential becomes  $\Phi \approx M^2$ . Corresponding sheath

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width can be estimated by using Child's law which gives  $d \approx M/2$  in the unit of Debye length.

Therefore, ions response to the sheath electric field determines the sheath potential and its width. The presence of magnetic field inhibits the plasma motion and changes  $M$ . Therefore, depending on the angle between the field and wall, the sheath width can increase or, decrease. There is no physical reason why plasma density or velocity should exhibit any oscillation.

The plasma drag force plays an important role in confining the dust inside the sheath. In the absence of drag, dust can sneak back into the quasineutral plasma region before falling back in the sheath again. Thus, without drag, the confinement of dust inside sheath is difficult. Often the grain charge is inferred by balancing the repulsive sheath field against the gravity. However often plasma drag forces are important and dust speed is 10 – 20 % of the ion acoustic speed, and thus the inference of dust charge from mere balance of the sheath field with the dust gravity should be treated with caution. While investigating the dust dynamics, we have assumed that plasma–sheath provide the passive background.

To summarise, the penetration of the dust in the sheath depends on its charge as the interplay between various forces results in the dust performing an inward spiralling motion causing the dust to move back and forth in a narrow region of the sheath. With the increase in size, dust remains closer to the plasma–sheath boundary as the larger grains become more negatively charged and thus, the repulsive sheath electrostatic field overcomes the flow forces much earlier.

**Acknowledgement** The support of Australian Research Council is gratefully acknowledged for the present work.

## References

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