Thermal Conductivity and Capacity for 2D and 3D Non-ideal Systems

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Dissipative Yukawa systems both 3D and 2D are under concern. These systems are widely used [1] as analytical models for such non-ideal dissipative systems as dusty component of complex plasma.

In this paper the following parameters were used to characterize a system state. One of them is the effective non-ideality parameter Γ^* [2-4]:

$$\Gamma^* = G_1\zeta\cdot(1+\kappa+ \kappa/2)\cdot\exp(-\kappa)/\tau\cdot l_p \quad (1)$$

Here T is temperature in energy units, the "reduced" temperature $\tau \equiv T/M$, where M is a macroparticle's mass, G_1 is dimensionless coefficient: $G_1 = 1.5$ for 2D and 1 for 3D, the coupling parameter $\kappa \equiv l_p/r_D$ where l_p is mean interparticle distance, and r_D is Debye length. This dimensionless parameter describes space scaling properties of the systems concerned. The value of $\zeta \equiv (eZ)^2/M$, where eZ is a macroparticle's charge.

Time scale of the systems studied can be characterized by means of the effective frequency [2-4]:

$$\omega^* = G_2\zeta\cdot(1+\kappa+ \kappa/2)\cdot\exp(-\kappa)/\pi l_p^3 \quad (2)$$

Here $G_2 = 2$ for 2D systems and 1 for 3D systems.

The next dimensionless parameter to describe system's time scaling is the scaling parameter $\xi \equiv \omega^*/\nu_{fr}$. Here ν_{fr} is the value relevant to frequency of collisions between macroparticles and molecules of the background media (i.e. the one that produces plasma). This frequency describes energy dissipation (due to friction) along with energy inflow (due to Brownian collisions of surrounding gas molecules) thus characterizing the thermostat that is responsible for equilibrium temperature T in the system.

In non-ideal systems, where interparticle interaction is significant thermal conductivity depends on pair potential. However in this paper we pay attention to the kinetic part of thermal conductivity χ_K which corresponds to free diffusion of a particle's kinetic energy. It is important to study because this approach allows us to find certain relations as it is described below. Thermal conductivity is measured according to Green – Kubo approach when time autocorrelation function of heat flux fluctuations is studied. Since heat flux fluctuation is represented as

$$\delta \mathbf{J}_K = s^{-1}(MV^2/2 - \langle MV^2/2 \rangle)\mathbf{V} \qquad (3)$$

where $s = 2$ for 2D systems and $s = 3$ for 3D systems, we can derive [5] that the χ_K coefficient is equal to:

$$\chi_K = (n^2 \cdot k_B \cdot \mathrm{V}/s \cdot \tau^2)\lim_{t\to\infty} D_K(t) \qquad (4)$$

where $D_K(t)$ is heat transfer evolution function i.e. integral over time of heat flux fluctuation autocorrelation function $s^{-1}\langle \delta\mathbf{J}_K(0)\delta\mathbf{J}_K(t)\rangle$.

For two dimensional systems it was found that at large values of the effective non-ideality parameter Γ^* the kinetic part of thermal conductivity χ_K decreases with Γ^* growing according law $\sim 1/x^\alpha$, where α is close to 1 (see Fig. 1a). As for weak coupling when $0 < \Gamma^* < 30$ it was found that there is linear dependency $\chi_K \sim \tau$ (see Fig. 1b). It can also be seen that at large values of ξ system has larger values of thermal conductivity χ_K.

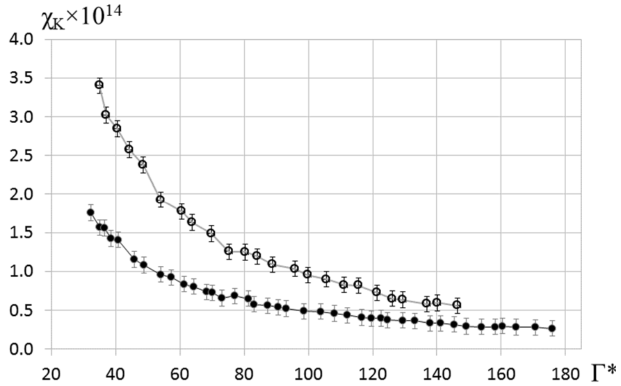

Fig. 1a. The dependence of kinetic thermal conductivity χ_K (in erg/s·K) on the effective nonideality parameter Γ^* for large values $\Gamma^* > 30$ for 2D systems (points: ● – for $\xi = 0.25$ and ○ - for $\xi = 1$).

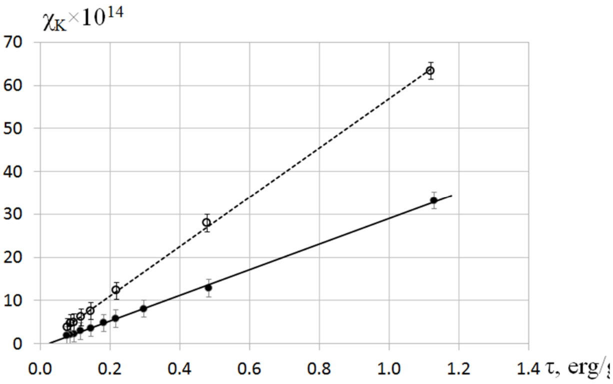

Fig. 1b. The dependence of kinetic thermal conductivity χ_K (in erg/s·K) on the temperature τ for small values $\Gamma^* < 30$ for 2D systems (points: ● – for $\xi = 0.25$ and ○ - for $\xi = 1$; solid line for $\xi = 0.25$ and dotted line for $\xi = 1$ – are linear approximation).

For three-dimensional systems it was found that the kinetic part of thermal conductivity χ_K

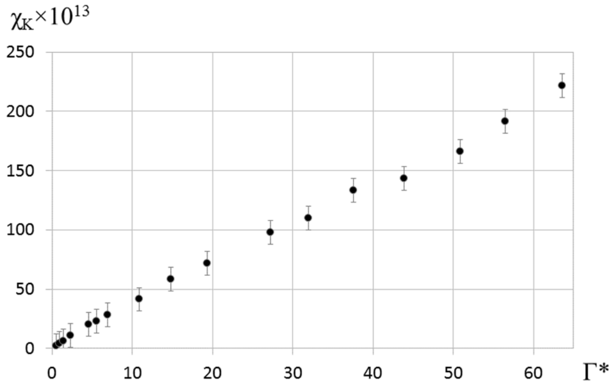

Fig. 2. The dependence of kinetic thermal conductivity χ_K (in erg/cm·s·K) for 3D systems (points: for $\xi = 0.25$) on Γ^*.

has significantly different dependence on non-ideality parameter Γ^* - it increases with growth of the Γ^* value (see Fig. 2).

Two experimental works [6, 7] were carried out to investigate thermal conductivity of dusty component of complex plasma of micron-sized Al_2O_3 particles in RF-discharge plasma. The values obtained in these works are show in comparison with numerically calculated values on Fig. 3. It is seen that at small values of Γ^* exists correspondence between analytical and experimental results.

Thermal capacity at constant volume C_V is calculated according to fluctuation approach [8]:

$$<(\Delta u)^2> = \tau^2 C_V \quad (1)$$

Here C_V is dimensionless thermal capacity: the specific thermal capacity c_V (i.e. per unit mass) can be represented as $c_V = C_V k_B/M$.

Results of thermal capacity of two dimensional system studying are described below (see Fig. 4). It was found that at low Γ^* the value of C_V equals to 1 as it should be for ideal gas asymptotic. For strongly non-ideal systems value of C_V tends to equal 2 in accordance with well-known relation where we have equal thermal capacity 1/2 for each degree of freedom: in strongly coupled systems there are 4 of them.

The dependence $C_V(\Gamma^*)$ proved to be independent of the value of ξ see linear regression approximation (dotted and solid lines) in Fig. 4.

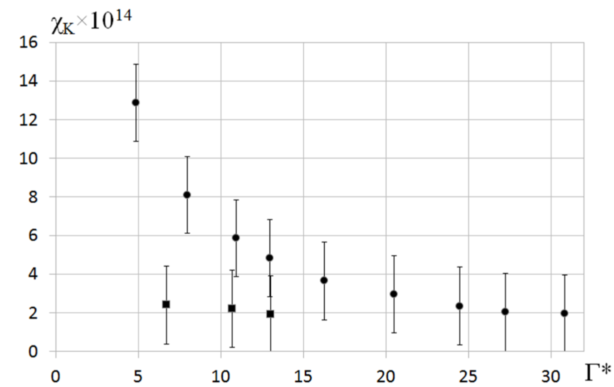

Fig. 3. Comparison between experimentally (■) [6, 7] and numerically (●) measured kinetic thermal conductivity χ_K in dependence on Γ^* for $\xi = 0.25$.

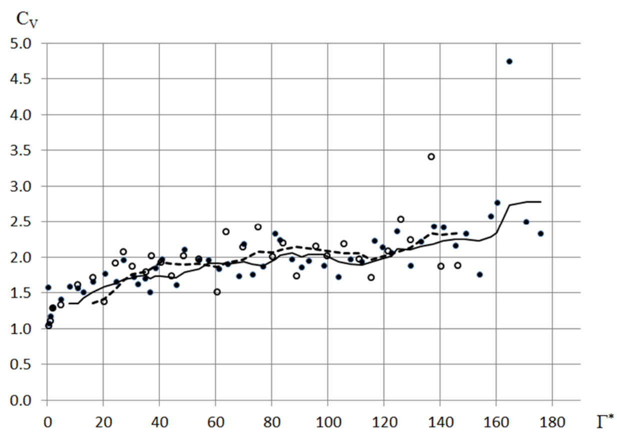

Fig. 4. The dependence of thermal capacity C_V for 2D systems (points: ● – for $\xi = 0.25$ and ○ - for $\xi = 1$) on Γ^*. Lines (solid for $\xi = 0.25$ and dotted for $\xi = 1$) are linear regression approximation.

Another important result of this work is relevant to the value

$$\mu \equiv D(t)/D_K(t), \quad (5)$$

where $D(t)$ is mass transfer evolution function and $D_K(t)$ is heat transfer evolution function (both being integrals over time of corresponding autocorrelation functions). At low Γ^* this quantity proved to be equal to thermal capacity at constant pressure C_P. In Fig. 5 one can see the dependence of μ on Γ^* at high temperatures.

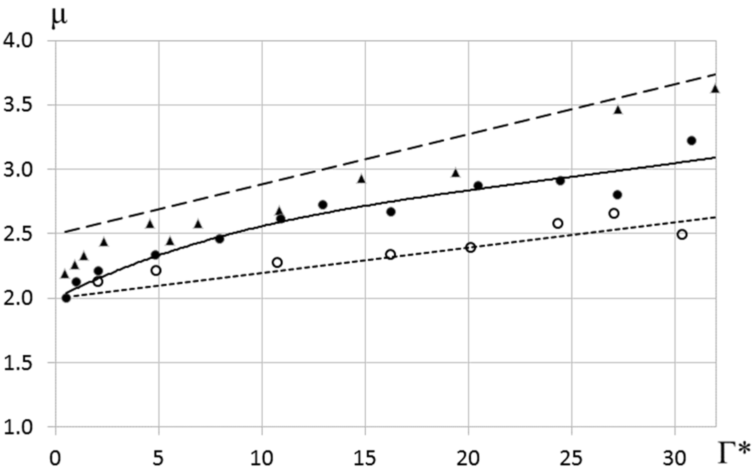

Fig. 5. The dependence of μ parameter on Γ^* for: 3D (▲), 2D for $\xi = 0.25$ (●) and $\xi = 1$ (○). Polynomial approximations are also shown: for: 3D (long dotted line), 2D for $\xi = 0.25$ (solid line) and $\xi = 1$ (short dotted line).

It can be seen that the dependencies $\mu(\Gamma^*)$ for 2D and 3D systems tend to well-known ideal asymptotic at $\Gamma^* \to 0$ which is equal to 2 for 2D system and 2.5 for 3D system.

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