Perpendicular Dynamics of Runaway Electrons in Tokamak Plasmas

I. Fernández-Gómez, J.R. Martín-Solís, R. Sánchez

Universidad Carlos III de Madrid, Avda. de la Universidad 30, 28911-Madrid, Spain.

1. Introduction In this work, a Langevin approach, that includes collisional diffusion in velocity space will be used to analyze the perpendicular runaway dynamics in tokamak plasmas. An investigation of the runaway probability in velocity space will yield a criterion for runaway, which will be shown to be consistent with the results provided by the simpler test particle description [1]. The role played by the perpendicular (to the magnetic field) collisional scattering will be also investigated. Pitch angle scattering increases the perpendicular electron temperature and the electron population in the runaway plateau region. The perpendicular broadening of the runaway distribution function, and the resulting enhancement of the runaway production rate will be discussed.

2. Langevin Equations for Runaway Electrons Langevin equations (LE) constitute a particle approach for studying the electron motion under the stochastic effect of the collisions with the plasma particles, equivalent to the traditional Fokker-Planck (FP) kinetic approach in the infinite particle limit,but more easily generalized to more complex geometry:

$$\mathbf{FP} \equiv \frac{\partial f_e}{\partial t} = -\frac{\partial}{\partial v_i}\left(A_i f_e + \frac{1}{2}\frac{\partial}{\partial v_k}(B_{ik} f_e)\right); \quad \mathbf{LE} \equiv \frac{dv_i}{dt} = F_i(\mathbf{v}) + D_{ik}(\mathbf{v})\xi_k(t) \tag{1}$$

$A_i \equiv \lim_{\Delta t\to 0} \langle \Delta v_i \rangle / \Delta t$ and $B_{ik} \equiv \lim_{\Delta t\to 0} \langle \Delta v_i v_k \rangle / \Delta t$ are the FP coefficients, $\vec{\xi}$ is a gaussian random variable, and the equivalence between both descriptions is given by the relation between the LE and FP coefficients:

$$D_{ij}D_{jk} = B_{ik}; \quad F_i = A_i - \frac{1}{2}D_{jk}\frac{\partial D_{ik}}{\partial v_j} \tag{2}$$

where the Stratonovich algebra has been used.

The Langevin equations for runaway electrons have been obtained following Ref. [2], including the force due to the accelerating electric field, $e\vec{E}_{||}/m_e$, and the ion contribution to the stochastic collision term simplified taking the limit of very cold ions, $m_e/M_i << 1$, resulting in:

$$\begin{aligned} \frac{d\vec{v}}{d\tau} &= \vec{\epsilon} - \frac{\vec{v}}{v^3}\left[-\frac{1}{2}\left(\phi + \psi\left(1 - 3v^2\right)\right) + \left(2\psi(\phi-\psi)\right)^{1/2}\right] + \left(\frac{1}{v}\right)^{1/2}(2\psi)^{1/2}\vec{\xi} \\ &- \left[(\phi-\psi)^{1/2} - (2\psi)^{1/2}\right]\frac{\vec{v}\times(\vec{v}\times\vec{\xi})}{v^2} - \left(\frac{Z_{eff}}{v}\right)^{1/2}\frac{\vec{v}\times(\vec{v}\times\vec{\eta})}{v^2} \end{aligned} \tag{3}$$

Two different gaussian noises, $\vec{\xi}$ and $\vec{\eta}$, are considered for the collisions with the bulk electrons and ions, respectively, Z_{eff} is the effective ion charge, $\vec{v}$ is normalized to the electron thermal velocity, $v_{th} \equiv (T_e/m_e)^{1/2}$, $\epsilon \equiv E_{||}/E_c$, $\tau \equiv t\nu_0$, and $E_c \equiv m_e v_e \nu_0/e$, $\nu_0 \equiv n_e e^4 \ln\Lambda/4\pi\varepsilon_0^2 m_e^2 v_e^3$; $\phi(x), \Psi(x)$ $(x \equiv v/\sqrt{2})$ are the standard error function and the Chandrasekhar function, respectively.

3. Runaway Probability. Conditions for Runaway The Langevin equations for runaway electrons can be used to build up the runaway probability map in $(v_{||}, v_\perp)$ space. Electrons are launched with given initial conditions in velocity space, the fraction of escaping electrons providing the runaway probability. Fig. 1 (left) shows the calculated runaway probability (R) vs the normalized parallel electron velocity, $v_{||}$, assuming $v_\perp = 0$, $\epsilon = 0.04$ and $Z_{eff} = 1, 3, 5, 10$. The runaway probability sharply increases in the direction of the electric field force ($v_{||} > 0$ in the figure) and a critical velocity for runaway generation, v_r, can be introduced using the condition $R(v_r) \equiv 50\%$. An analysis of the dependence of v_r on the plasma conditions in the range $\epsilon = 0.04 - 0.1$ and $Z_{eff} = 1 - 10$, results in $v_r \sim (2 + Z_{eff})^{0.23\pm0.01}\, \epsilon^{-0.475\pm0.005}$ and the width of the runaway generation region [determined by the condition $R(v_{||}) = 25 - 75\%$], on the order $\sim 10\%\, v_r$. This estimate of the critical velocity, v_r, is lower than the Dreicer estimate, $v_D = (2+Z_{eff})^{0.5}\, \epsilon^{-0.5}$, determined by the balance between the acceleration in the toroidal electric field and the collisional friction losses in parallel (to the magnetic field) direction. Such a decrease of the critical velocity, v_r, is the result of considering the perpendicular electron dynamics, which relaxes the conditions for runaway. Perpendicular energy gain due to collisions reduces the net collisional electron energy losses in comparison with the loss of parallel velocity which, as a result, is itself reduced, $dv_{||}/dt \sim -v_{||}/v^3$. Hence, electrons can be found in velocity space initially losing parallel momentum but gaining energy which, if the energy gain is large enough, could finally become runaway.

This Dreicer's criterion for runaway, v_r, is based on the assumption that most of the runaways are produced in the direction of the electric field force ($v_\perp = 0$). More generally, the runaway probability in $(v_{||}, v_\perp)$ space should be considered, as illustrated in right frame of Fig. 1 ($\epsilon = 0.04$, $Z_{eff} = 1$). The runaway region might be defined as the region in velocity space lying above the $R(v_{||}, v_\perp) = 50\%$ line (the runaway separatrix). These criteria for runaway, both v_r and the runaway separatrix, based on the runaway probability analysis, are found to be consistent with the results provided by a single particle description of the runaway dynamics describing average electron trajectories in *absence* of collisional diffusion [1], which yields $v_r \sim (2 + Z_{eff})^{0.25}\, \epsilon^{-0.5}$ and a runaway separatrix indicated by the full line in Fig. 1.

4. Runaway Distribution Function and Production The Langevin equations for runaway electrons can be used to yield the runaway distribution function and, from this, its first and second perpendicular velocity moments, i.e., the parallel runaway distribution function, $F(v_{||}) = \langle f \rangle = 2\pi \int_0^\infty v_\perp\, f\, dv_\perp$, and the effective perpendicular temperature, $T_\perp(v_{||}) = \langle v_\perp^2 f \rangle / 2F$, respectively. The initial electron velocities are randomly distributed over a Maxwellian distribution and evolved in time according to Eq. (3). The advanced distribution function is built by a standard statistical method, until a steady state is reached. Top Fig. 2 (left) shows the resulting $F(v_{||})$ (full line), for $\epsilon = 0.06$ and $Z_{eff} = 1$, together with the thermal Maxwellian distribution (dashed line) for illustration. The

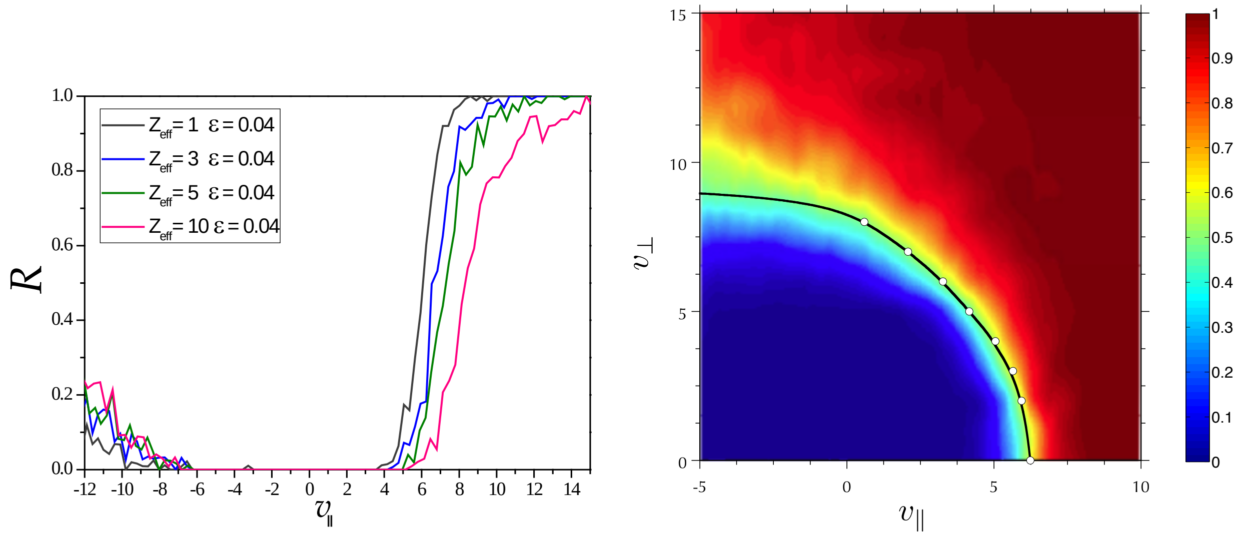

Figure 1: **Left:** Runaway probability R vs $v_{||}$, for $v_\perp = 0$, $\epsilon = 0.04$, and $Z_{eff} = 1, 3, 5, 10$; **Right:** Runaway probability contour map ($\epsilon = 0.04$; $Z_{eff} = 1$). The runaway separatrix provided by the test particle description [1] (full line) is also included.

runaway plateau starts close to the critical velocity, v_r, but the distribution separates from the Maxwellian much before. Bottom Fig. 2 (left) shows $T_\perp(v_{||})$ (normalized to the bulk temperature), providing a local (in $v_{||}$) measure of the perpendicular broadening of the distribution due to pitch angle scattering. Although fast electrons are much less collisional than thermal electrons, pitch angle scattering of fast electrons leads to a perpendicular temperature in the runaway region which is substantially larger than the bulk temperature ($T_\perp > 1$). Moreover, because of the perpendicular scattering, fast electrons are continuously diffused into and scattered out the runaway plateau region. As a result, even for $v_{||} < v_r$, the distribution function is perpendicularly broadened and deviates from a Maxwellian distribution (top figure). On the other hand, the height, $F(v_{||})$, of the runaway plateau region significantly increases and, hence, the runaway production rate, the increase resulting from the larger number of electrons with the enhanced $T_\perp$.

The solution to the steady-state one dimensional (integrated over the perpendicular velocity) Fokker-Planck equation for runaway electrons allows to get an approximation for the runaway production rate, λ, as a function of $T_\perp(v_{||})$. It can be shown [3]:

$$\lambda \propto \left[\int_0^\infty \frac{v_{||}^3}{1 + (1 + Z_{eff}) T_\perp(v_{||})} \exp\left(- \int_0^{v_{||}} \frac{v\,(\epsilon v^2 - 2 - Z_{eff})}{1 + (1 + Z_{eff}) T_\perp(v)}\, dv \right) dv_{||} \right]^{-1} \qquad (4)$$

The effect of the perpendicular broadening on the runaway production rate can then be quantified by the ratio of the runaway production rate including the perpendicular broadening of the distribution function $[T_\perp(v_{||})]$ to the value estimated assuming $T_\perp = 1$ (Maxwellian distribution in perpendicular direction), illustrated in right Fig. 2. The perpendicular broadening of the electron distribution increases the runaway production by several orders of magnitude in comparison with the case of a perpendicular thermal Maxwellian, the effect increasing with the plasma collisionality (larger Z_{eff} and lower ϵ).

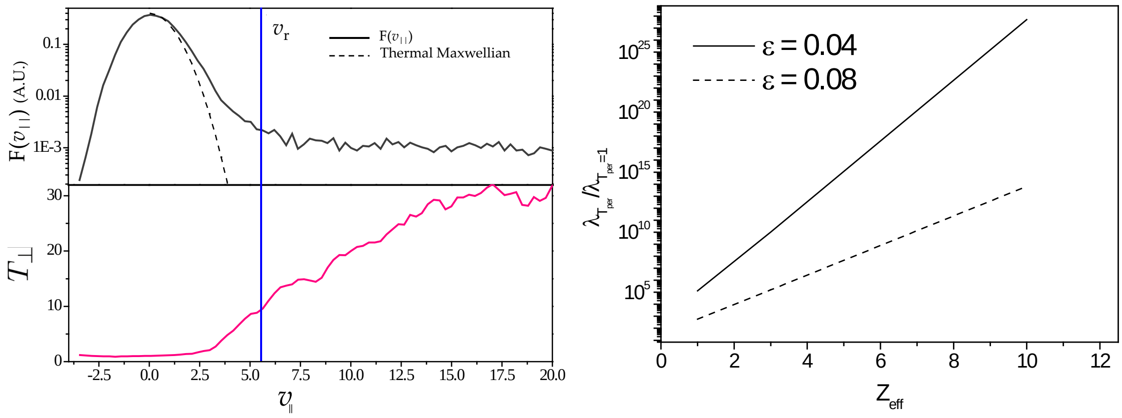

Figure 2: **Left:** $F(v_{||})$ (top) and $T_\perp(v_{||})$ (bottom) for $\epsilon = 0.06$ and $Z_{eff} = 1$. The parallel thermal Maxwellian distribution (dashed line; top) and the critical runaway velocity, v_r are also shown for illustration; **Right:** Ratio of the runaway production rate including the perpendicular broadening of the distribution function $[T_\perp(v_{||})]$ to the value estimated assuming $T_\perp = 1$ vs Z_{eff} (full line: $\epsilon = 0.04$; dashed line: $\epsilon = 0.08$).

It is therefore worthwhile to remark the opposite effects on the runaway population of the collisional dynamics in the parallel and perpendicular directions (to the toroidal magnetic field). Parallel collisional friction balances the energy gain in the toroidal electric field, slowing down the electrons and reducing the runaway growth rate. In contrast, perpendicular energy gain due to collisions reduces the electron energy losses,relaxing the conditions for runaway. Moreover, because of the perpendicular scattering, fast electrons are continuously diffused into the runaway plateau region which, as a result, enhances the runaway production rate. It is well-known that a decrease of the electric field (ϵ) or an increase of Z_{eff}, increasing the plasma collisionality and the parallel friction losses, reduce the runaway production rate, but this reduction, because of the collisional perpendicular dynamics, as illustrated in Fig 2 (right), is much smaller than would be expected from the parallel electron dynamics only.

Acknowledgements: the authors thank D.Ogata and V.Tribaldos for valuable assistance.

References

[**1**] V. Fuchs *et al.*, Phys.Fluids **29**, 2931 (1986).
[**2**] M.G. Cadjan *et al.*, J.Plasma Physics, **61**, *part* 1, 89 (1999).
[**3**] A.V. Gurevich, Zh.Eksp.Teor.Fiz. **39**, 1296 (1960) [Sov.Phys.JETP **12**, 904 (1961)].