

On the theory of dynamics of dust grain in plasma*

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Introduction. In recent years the physics of the interactions of dust grains with plasmas was studied intensively both experimentally and theoretically. However, most of theoretical studies assume that dust grain is made of homogeneous material and have spherical shape. Meanwhile in many cases (e.g. in fusion plasmas [1]) the material dust grain is made of can be inhomogeneous and the shape of the grain can be far from spherical. These features can significantly alter the very basic properties of grain-plasma interactions and result in new phenomena.

For example, in Ref. 2, 3 it was demonstrated that the drag force imposed on the non-spherical grain by plasma flow can have the components perpendicular to plasma velocity. It also was shown that dynamics of rotationally symmetric grain spinning is equivalent to the motion of symmetric top in the gravity field. As a result, the precession of the grain axis could result in significant oscillations of grain trajectory in the direction normal to plasma velocity.

However, in Ref. 3 the effects of the magnetic field on grain dynamics were neglected. Meanwhile, even simple motion of charged non-spherical grain in magnetic field could result in torque, caused by the Lorenz force, acting on the grain. Moreover, grain-plasma interactions in the presence of magnetic field cause additional torques acting on the grain [4, 5]. We also can envision that the effects of magnetic field can alter plasma-grain drug force direction and magnitude. As a result, the synergetic impact of the magnetic field effects and dust-grain interactions can have profound impact on grain dynamics. Here we address some of the issues related to the dynamics of non-spherical dust grain in plasma embedded into magnetic field.

Equations. To describe dynamics of dust grain, which we consider as a rigid body, we need to provide forces, \vec{F} , and torques, \vec{K} . Both forces and torques depend on the shape of the grain, plasma scalar parameters (e.g. density, temperatures, etc.) and such vectors as grain, \vec{V} , and plasma, \vec{V}_p , velocities and magnetic, \vec{B} , and electric, \vec{E} , fields, and angular velocity of the grain, $\vec{\Omega}$. In general case forces and torques can only be found numerically. However,

in Ref. 3 it was shown that the structure of the expressions for forces and torques could be determined from symmetry arguments. In particular, for rotationally symmetric grains it was possible to determine exact expressions for forces and torques. The main argument used in Ref.32 was the following. The expressions for force and torque can be expressed as follows

$$F_\alpha = \Phi_{\alpha\beta}^{(W)} W_\beta + \Phi_{\alpha\beta}^{(\Omega)} \Omega_\beta, \quad (1)$$

$$K_\alpha = T_{\alpha\beta}^{(W)} W_\beta + T_{\alpha\beta}^{(\Omega)} \Omega_\beta, \quad (2)$$

where $\vec{W} = \vec{V}_p - \vec{V}$, $\Phi_{\alpha\beta}$ and $T_{\alpha\beta}$ are some second-order tensors depending on the shape of the grain. Spatial orientation of rotationally symmetric grain can be characterized by unit vector \vec{D} , which is directed along symmetry axis. As a result, tensors $\Phi_{\alpha\beta}$ and $T_{\alpha\beta}$ can be expressed in terms of tensors $\delta_{\alpha\beta}$, $D_\alpha D_\beta$, and pseudo-tensor $\epsilon_{\alpha\beta\gamma} D_\gamma$, where $\delta_{\alpha\beta}$ is the Kronecker delta and $\epsilon_{\alpha\beta\gamma}$ is the Levi-Civita symbol. Then, taking into account that \vec{F} , \vec{W} , and \vec{D} are the vectors, while \vec{K} and $\vec{\Omega}$ are pseudo-vectors, one finds the following expressions

$$\vec{F} = \Phi_1^{(W)} \vec{W} + \Phi_2^{(W)} \vec{D} (\vec{D} \cdot \vec{W}) + \Phi^{(\Omega)} (\vec{\Omega} \times \vec{D}), \quad (3)$$

$$\vec{K} = T^{(W)} (\vec{W} \times \vec{D}) + T_1^{(\Omega)} \vec{\Omega} + T_2^{(\Omega)} \vec{D} (\vec{D} \cdot \vec{\Omega}), \quad (4)$$

where the magnitudes of the scalars $\Phi_1^{(W)}$, $\Phi_2^{(W)}$, $T_1^{(\Omega)}$, $T_2^{(\Omega)}$, $\Phi^{(\Omega)}$, and $T^{(W)}$ can be evaluated from simple physical considerations [3].

In the presence of magnetic field, we still can use the same symmetry arguments, but now we need to take into account that \vec{F} and \vec{K} depend on pseudo-vector \vec{B} (for simplicity we neglect an impact of electric field and assume $\vec{V}_p = 0$).

$$\begin{aligned} \vec{F} = & \Phi_1 \vec{V} + \Phi_2 \vec{D} (\vec{D} \cdot \vec{V}) + \Phi_3 (\vec{\Omega} \times \vec{D}) + \Phi_4 (\vec{V} \times \vec{B}) + \Phi_5 (\vec{V} \times \vec{D}) (\vec{D} \cdot \vec{B}) \\ & + \Phi_6 \vec{\Omega} (\vec{D} \cdot \vec{B}) + \Phi_7 \vec{B} (\vec{D} \cdot \vec{\Omega}) + \Phi_8 \vec{D} (\vec{B} \cdot \vec{\Omega}) \end{aligned}, \quad (5)$$

$$\begin{aligned} \vec{K} = & T_1 (\vec{V} \times \vec{D}) + T_2 \vec{\Omega} + T_3 \vec{D} (\vec{D} \cdot \vec{\Omega}) + T_4 (\vec{\Omega} \times \vec{B}) + T_5 (\vec{\Omega} \times \vec{D}) (\vec{D} \cdot \vec{B}) \\ & + T_6 \vec{V} (\vec{D} \cdot \vec{B}) + T_7 \vec{B} (\vec{D} \cdot \vec{V}) + T_8 \vec{D} (\vec{B} \cdot \vec{V}) + T_9 \vec{B} + T_{10} \vec{D} (\vec{D} \cdot \vec{B}) \end{aligned}, \quad (6)$$

where scalars $\Phi_{...}$ and $T_{...}$ can be found numerically or estimated analytically.

Although Eq. (5, 6) are more cumbersome than Eq. (3, 4) the physical meaning of different terms can be easily interpreted. For example, the terms from T_5 to T_8 describe, in particular, the torque, \vec{T}_L , related to the Lorenz force, acting on the grain charge:

$$\bar{T}_L \Big|_{\alpha} = \frac{1}{c} \int \left\{ \vec{r} \times \left[\vec{V} \times \vec{B} + (\vec{\Omega} \times \vec{r}) \times \vec{B} \right] \right\} dq = \frac{1}{c} \left\{ \vec{D}_q \times (\vec{V} \times \vec{B}) \right\} \Big|_{\alpha} + \frac{1}{c} \varepsilon_{\alpha\beta\gamma} \Omega_{\beta} Q_{\gamma\delta} B_{\delta}, \quad (7)$$

where dq is the differential charge and an integration goes over entire grain; $\vec{D}_q = \int \vec{r} dq$ and $Q_{\alpha\beta} = \int r_{\alpha} r_{\beta} dq$ are dipole and quadrupole components of charge distribution in the grain counted in the center of mass frame. For a grain with rotational symmetry we have $\vec{D}_q \propto \vec{D}$ and $Q_{\alpha\beta} = Q_1 \delta_{\alpha\beta} + Q_2 D_{\alpha} D_{\beta}$, where $Q_1 \sim Q_2 \sim e Z_d R_d^2$, Z_d is the grain charge number and R_d is the grain size. The terms with T_9 and T_{10} can describe the torques, T_B , which are due to gyro-motion of ions impinging the grain and, also, due to $\vec{j} \times \vec{B}$ force caused by electric current which can flow through the grain due to the differences in electron and ion gyro-motion and, correspondingly, in their collection by the grain [4]. For the case where ion gyro-radius, ρ_i , is larger than R_d the estimates give $T_9 \sim T_{10} \sim n T R_d^4 / B \rho_i$, where n and T are the plasma density and temperature [4]. Assuming that $e^2 Z_d \sim R_d T$ and comparing the magnitude of T_B and T_L we find the T_L dominates for relatively small grains

$$R_d < \Omega \lambda_D^2 / V_i, \quad (8)$$

where V_i is the ion thermal speed and λ_D is the Debye length.

Dynamics of large grain spinning in magnetic field. Here we consider the dynamics of spinning of relatively large rotationally symmetric grain where the torque is determined by ion gyro-motion. In this case we have the following equations

$$\frac{d\vec{M}}{dt} = T_9 \vec{B} + T_{10} \vec{D} (\vec{D} \cdot \vec{B}), \quad \frac{d\vec{D}}{dt} = \vec{\Omega} \times \vec{D}, \quad (9)$$

where $\vec{M} = I_0 \vec{\Omega} + I_1 \vec{D} (\vec{D} \cdot \vec{\Omega})$ is the angular momentum, I_0 and I_1 describe the inertial moments of the rotationally symmetric grain. Introducing $\vec{\omega} = \vec{M} / I_0$, $\vec{b} = \vec{B} / B$, $\tau_1 = T_9 B / I_0$, and $\tau_2 = T_{10} B / I_0$ we re-write (9) as follows

$$\frac{d\vec{\omega}}{dt} = \tau_1 \vec{b} + \tau_2 \vec{D} (\vec{D} \cdot \vec{b}), \quad \frac{d\vec{D}}{dt} = \vec{\omega} \times \vec{D}. \quad (10)$$

Although $\vec{\omega}$ is increasing with time, Eq. (10) has two dynamic equilibrium $\vec{\omega} = \vec{\omega}_0(t)$ and $\vec{D} = \vec{D}_0$: (A) where all vectors are parallel, $\vec{\omega}_0 \parallel \vec{b} \parallel \vec{D}_0$, and (B) where $\vec{\omega}_0 \parallel \vec{b} \perp \vec{D}_0$. In what follows we consider the stability of these equilibria by introducing small perturbations $\vec{\omega}_1$ and \vec{D}_1 (we should keep in mind that $|\vec{D}| = 1$).

We start with the case (A). Here we have only $\vec{\omega}_1 \perp \vec{b}$ and after some algebra from Eq. (10) we find equation for $\vec{\omega}_1$:

$$\frac{d^2\vec{\omega}_1}{dt^2} = \tau_2 \vec{\omega}_1 \times \vec{b} - \omega_0(t) \frac{d\vec{\omega}_1}{dt} \times \vec{b}, \quad \frac{d\omega_0}{dt} = \tau_1 + \tau_2. \quad (11)$$

Looking for the solution $\vec{\omega}_1 \propto \exp(S(t))$, where $|S(t)| \gg 1$ we find

$$\vec{\omega}_1 \propto \exp\left\{\pm i \int^t \omega_0(t') dt'\right\} / |\omega_0(t)|^{(\tau_1+\tau_2)/(\tau_1+2\tau_2)}, \quad (12)$$

which shows that dynamic equilibrium (A) is unstable for $-2 < \tau_2/\tau_1 < -1$.

For the case (B) we introduce the component of the vector \vec{D}_1 parallel to the magnetic field, D_{\parallel} , and after some algebra from Eq. (10) we find

$$\frac{d^3 D_{\parallel}}{dt^3} = \tau_2 \omega_0 D_{\parallel} - \omega_0^2 \frac{dD_{\parallel}}{dt}, \quad \frac{d\omega_0}{dt} = \tau_1. \quad (13)$$

Looking for the solution $D_{\parallel} \propto \exp(S(t))$, where $|S(t)| \gg 1$ we find

$$D_{\parallel} \propto \exp\left\{\pm i \int^t \omega_0(t') dt'\right\} / |\omega_0(t)|^{\tau_2/2\tau_0}, \quad (14)$$

which shows that dynamic equilibrium (B) is unstable for $\tau_2/\tau_1 < 0$.

Thus we see that for the case of negative ratio τ_2/τ_1 both dynamic equilibria (A) and (B) can be unstable. From geometric consideration of the torque related to gyro-motion of ions such case can correspond, for example, to oblate spheroid.

Conclusions. By using symmetry arguments we derive expressions for force and torque acting on rotational symmetric grain in plasma embedded into magnetic field. We analyze spinning of rotationally symmetric grain due to torque caused by gyro-motion of ions impinging the grain. We show that for some cases (e.g. oblate spheroid) there is no stable dynamic equilibrium of grain motion.

References

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