

Raman and Brillouin scattering in ultra-relativistic laser-plasma interaction

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Large amplitude plane electromagnetic waves are subject to Raman, Brillouin, and modulational instabilities when propagating in a plasma. In the weakly-relativistic regime one can distinguish between scattering off high-frequency electron Langmuir and low-frequency ion acoustic waves. This leads to the known classification of Raman and Brillouin scattering. In case of a relativistic strong electromagnetic radiation, Raman scattering is known to be modified by the relativistic ponderomotive force and the increase of the effective electron mass. The solutions of the corresponding dispersion relations become notably amplitude sensitive when the pump amplitude increases [1]. In the present work we first extend the theory of plane-wave scattering to the ultra-relativistic regime where the regions of Raman and Brillouin scattering start to overlap.

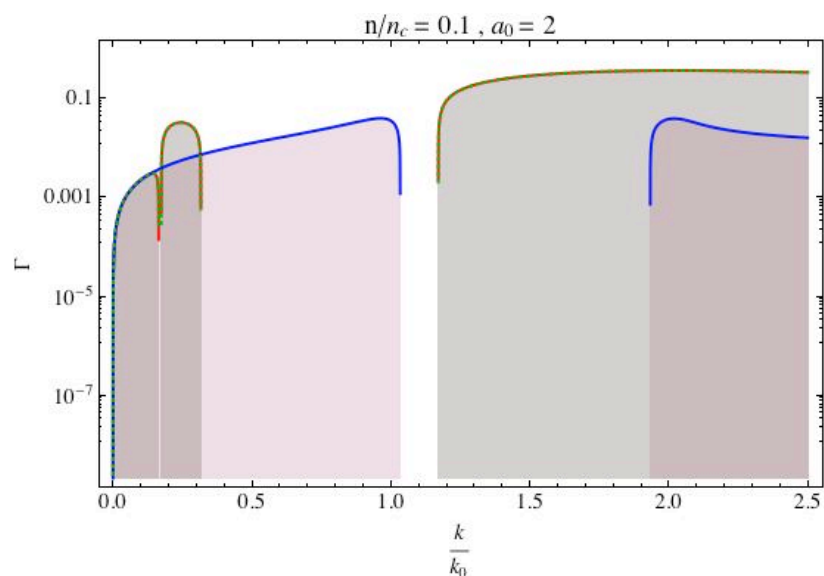
In the non-relativistic regime the growth-rates of Raman and Brillouin scattering are independent of the polarization of the laser. The Raman instability is due to the scattering of the electromagnetic wave off an electron plasma wave and may only exist below a threshold density of $0.25 n_c$. The critical density n_c is defined by the laser frequency and marks the density above which no propagation of the electromagnetic wave is allowed. Scattering of the electromagnetic wave off an ion acoustic mode is referred to as Brillouin scattering. This type of scattering may be observed up to the critical density. Scattering in general may occur at any angle compared to the wave-vector of the laser. We consider only one-dimensional geometry and thus only discuss forward- and backward-scattering. Usually backward scattering has the largest growth-rate.

In the relativistic regime the electrons suffer from mass increase, which then results in a reduced plasma frequency. As a result the growth-rates of Raman scattering are increased and the plasma densities up to which it may take place are increased [1]. The maximum growth-rates for the Raman instability in the relativistic regime are usually larger than in the non-relativistic case. The maximum value of the growth-rate for Raman scattering is of the order of half of the laser-frequency. The exact amplitude at which the maximum is attained depends on the plasma density. An interesting feature of relativistic parametric instability is that the

wave-number regions in which the instabilities are observed change with the amplitude of the wave. This was studied in Ref. [1], where it was found that the modulational instability and Raman forward and backward scattering may overlap and form hybrid instabilities. The analysis in Ref. [1] is based on the dispersion relation for circularly polarized relativistic waves. We extended this analysis to also account for relativistic Brillouin scattering. By analysis of the obtained dispersion relation, we find that the regions in which Brillouin scattering exists are also subject to changes in the relativistic regime. Already at dimensionless amplitudes of $a_0=1$ (measured in units of e/mc), thus still non-relativistic ion motion but already relativistic ions, we observe a broadening of the region which is affected by backward Brillouin scattering. The growth-rates for Brillouin forward scattering are always smaller than for backward scattering. Additionally the forward branch of the Brillouin instability usually merges with the modulational instability.

When we aim for a dispersion relation which takes only Brillouin scattering into account and neglects the scattering off high-frequency Langmuir waves we get a good agreement with results from the dispersion relation which considers Brillouin as well as Raman scattering only in the non-relativistic limit. When we consider relativistic waves we observe differences between the results from the two dispersion relations. The ion acoustic branch is obviously affected by the presence of the Langmuir waves, which now have a reduced frequency in the relativistic regime. In general the predictions by the Brillouin-only calculation show much broader domains of unstable wave-numbers than the combined model, which includes Raman and Brillouin scattering, see Fig. 1.

Figure 1: Growth-rates for Raman, Brillouin and modulational instability for a wave with $a_0=2$ and $n/n_c=0.1$. Results are obtained from dispersion relations for Raman scattering (red line), Brillouin scattering (blue line) and a dispersion relation considering Raman and Brillouin scattering at the same time (green dotted line).



In principle in our model five distinct parametric instabilities could be present in the system, that are modulational instability, Raman forward/backward and Brillouin forward/backward. The real number of distinct instabilities is at maximum these five, but in the relativistic regime, some instabilities start to overlap. The number of distinct instabilities is a function of the plasma density and the wave amplitude. We analyzed our results and present them in Fig. 2 in a similar way the results in Ref. [1] were displayed.

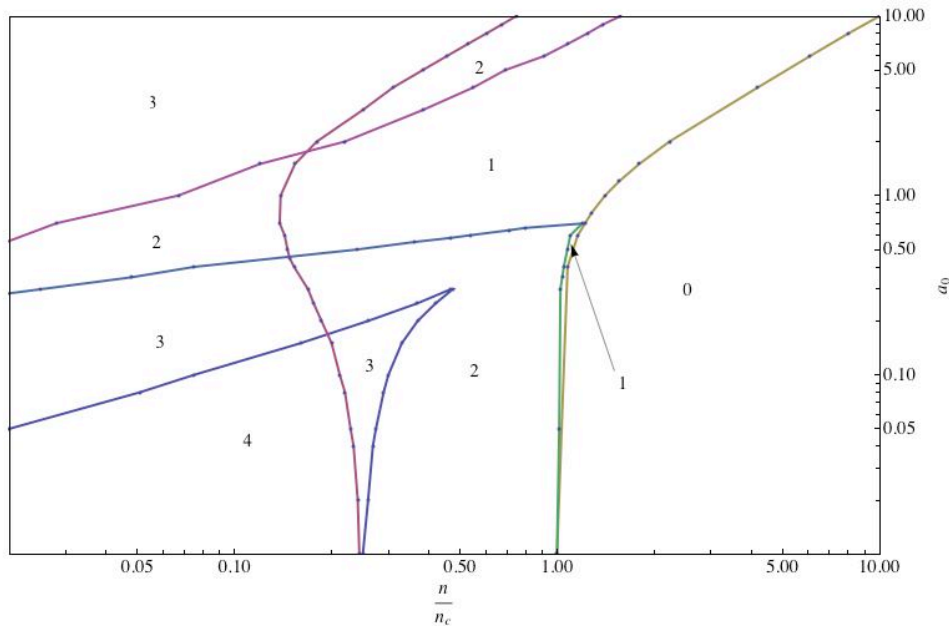


Figure 2: Number of distinct parameter instabilities found at given plasma density n/n_c at wave amplitude a_0 . The numbers inside the regions indicate the number of distinct instabilities.

The maximum number of five distinct instabilities is never attained in the considered parameter region. This is due to the fact, that the Brillouin forward instability always merges with the relativistic modulational instability at very small values of k . Figure 3 shows an example for parameters lying in the region which supports 4 different distinct instabilities.

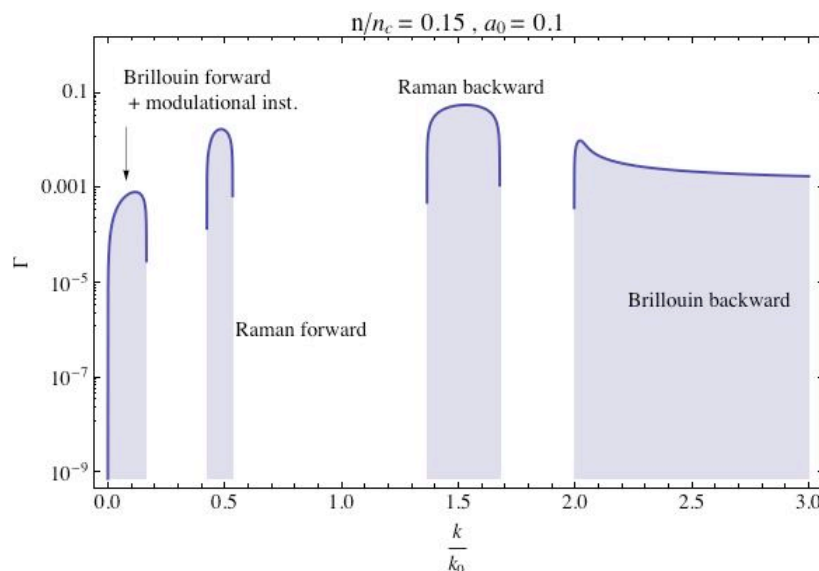


Figure 4: Growth-rates for parametric instabilities, considering density $0.15 n_c$ and laser amplitude $a_0=0.1$.

Investigating scattering processes for linearly polarized waves in the relativistic regime is complicated by the fact, that linearly polarized waves have a different nature than circularly polarized waves in this regime [2]. Circularly polarized waves are purely transversal, while linearly polarized waves are coupled transversal-longitudinal. The wave solutions can only be given analytically in limiting cases. In general a system of coupled differential equations has to be solved to obtain the waves. Depending on the parameter range different types of solutions can be found [2,3]. These waves then can be considered as unperturbed states in a numerical stability analysis. An important difference from the circularly polarized waves is that the linearly polarized waves are a superposition of many uneven harmonics of a fundamental frequency.

We carried out first simulations of the linearized fluid-Maxwell, considering only mobile electrons to this point. The results show scattering signals from all harmonics of the laser field. Similar to the case of circular polarization we observe an increase of the maximum growth-rate with increasing laser intensity. At a certain laser intensity (below $a_0=10$, exact position depending on the plasma density) the Raman instability attains a maximum growth-rate, then decreases again before it saturates and stays almost unchanged over a broad range of intensities. Figure 5 shows an example for the obtained growth rates at a fixed plasma density of $n/n_c=0.3$.

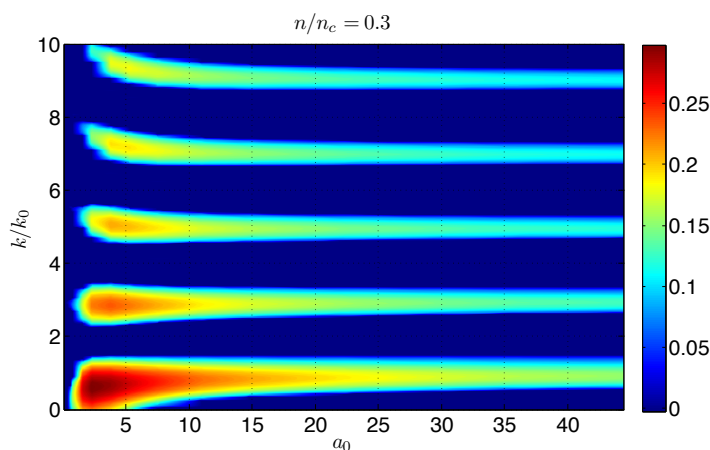


Figure 5: Growth-rate Γ , color coded, for linearly polarized waves with amplitudes a_0 . For each amplitude the growth-rates are displayed as a function of the wave-number of the perturbation. The instabilities from scattering off distinct harmonics of the laser field are clearly visible.

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