Heuristic Drift-Based Model for the Power Scrape-off Width in H-Mode Tokamaks

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Introduction: An heuristic model for the plasma scrape-off width in H-mode plasmas is introduced, in which magnetic drifts into the SOL are balanced against $c_S/2$ parallel flows to the divertor plates. The overall particle flow pattern is a modification for open field lines of Pfirsch-Shlüter flows, including sinks to the divertors. These assumptions result in an estimated SOL width of $\sim 2a\rho_p/R$. They also result in a first-principles calculation of the particle confinement time of H-mode plasmas. It is next assumed that anomalous perpendicular electron thermal diffusivity is the dominant source of heat flux across the separatrix, investing the SOL particle width, derived above, with heat from the main plasma. The separatrix temperature is calculated based on a two-point model balancing power input to the SOL with Spitzer-Härm parallel thermal conduction losses to the divertor. This results in a closed-form prediction for the power scrape-off width that is in reasonable quantitative agreement both in absolute magnitude and scaling with recent experimental data.

Density SOL Width: The model presented here is simple, but appears not to have been directly considered in the literature. It is well known that in the core of a collisional tokamak plasma the grad B and curv B drifts give rise to vertical motion of ion and electron gyro-centers. The divergence of the ion gyro-center flow, resulting from the radial ion pressure gradient, gives rise to an up-down asymmetric accumulation of ions. This causes a parallel pressure gradient that drives a balancing ion flow parallel to the magnetic field. Overall this flow pattern is referred to as Pfirsch-Schlüter flow. This gyro-center drift picture is equivalent to the fluid drift picture, in which the accumulation occurs due to the divergence of the ion and electron diamagnetic flows in a torus.

Consider now the separatrix at the edge of an H-mode tokamak plasma, shown in figure 1. Here the grad B and curv B drifts carry ions across the last closed magnetic surface onto open field lines in the SOL, with Maxwellian-averaged velocity $\langle v_{gradB+curvB} \rangle = 2T/eZBR$. In this region drift flows can be balanced not only by parallel flows that connect the bottom of the plasma to the top, but also by parallel flows that leave the plasma region in the direction of the divertors.

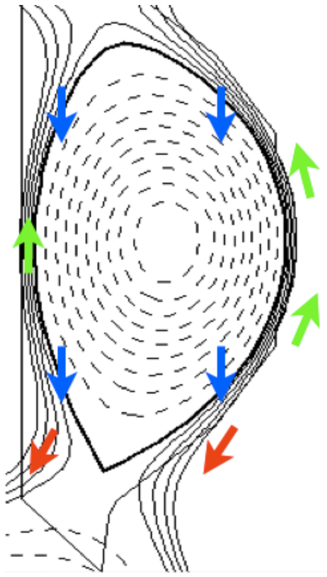

Figure 1: Magnetic drifts (vertical, downwards), Pfirsch-Schüter parallel flows (upwards near midplane), and divertor parallel flows (downwards, towards divertors)

This heuristic picture is generally consistent with measured parallel mass flow patterns [1] (albeit in L-mode), including the reversal of flow in the lower region of the LFS SOL for ion grad B drift down with lower single-null divertor. We estimate the drift-based particle width that should result from these processes by multiplying a typical residence time in the SOL ($\propto L_{//}/c_s$) times the poloidally averaged positive grad B and curv B drift velocities perpendicular to the separatrix. Averaging the radial component of the drift velocity along the magnetic field and assuming an average parallel speed based on experimental data [1] of $c_s/2$, with $T_i = T_e = T_{sep}$, we find:

$$\Delta\psi_p = \int_{MP}^{X-pt} \left(\vec{v}_{gc}\cdot\vec{\nabla}\psi_p\right)\frac{dl_{\|}}{c_s/2} = \frac{2}{c_s}\int_{MP}^{X-pt}\left(\vec{v}_{gc}\cdot\vec{\nabla}\psi_p\right)\frac{B}{B_p}dl_p$$

$$= \frac{2}{c_s}\int_{MP}^{X-pt}\left(\vec{v}_{gc}\cdot\frac{\vec{\nabla}\psi_p}{\left|\vec{\nabla}\psi_p\right|}\right)RBdl_p = \frac{2}{c_s}\int_{MP}^{X-pt}\frac{2T_{sep}}{\bar{Z}eBR}RB\;\hat{z}\cdot\hat{\varphi}\times d\vec{l}_p = \frac{4T_{sep}}{\bar{Z}ec_s}\int_{MP}^{X-pt}\hat{z}\times\hat{\varphi}\cdot d\vec{l}_p = \frac{4T_{sep}a}{\bar{Z}ec_s}$$

which provides a simple formula for the density scrape-off width:

$$\lambda = \frac{\Delta\psi_p}{\left|\nabla\psi_p\right|} = \frac{4T_{sep}a}{\bar{Z}B_pRc_s};\qquad c_s = \left[\frac{\left(1+\bar{Z}\right)T_{sep}}{\bar{A}m_p}\right]^{1/2};\qquad \bar{Z}\equiv\frac{n_e}{\sum_i n_i};\qquad \bar{A}\equiv\frac{\sum_i n_iA_i}{\sum_i n_i}$$

It is not clear whether the ion or electron drift speed will set the radial extent of the SOL through the poloidal electric field they establish, so we evaluate both cases:

$$\lambda = \underbrace{\frac{4a}{\bar{Z}eB_pR}\left(\frac{\bar{A}m_pT_{sep}}{(1+\bar{Z})}\right)^{1/2}}_{ion\ drift}\ or\ \underbrace{\frac{4a}{eB_pR}\left(\frac{\bar{A}m_pT_{sep}}{(1+\bar{Z})}\right)^{1/2}}_{electron\ drift} \approx \frac{2a}{R}\rho_p$$

From the direction of the resulting radial *ExB* drifts, it can be argued that the faster electron drift (for $\bar{Z} > 1$) will set the width on the LFS and the slower ion drift on the HFS. Finally, we note that this calculation is very similar to that of the shift of passing ion orbits, and also to the calculation of the SOL width that gives sonic Pfirsch-Schlüter flows. In a general sense, it can perhaps be considered as the minimum, neoclassical SOL width.

Particle loss current: If we assume that one half of the magnetic drift current is lost to the divertor and one half returns to the plasma as Pfirsch-Schlüter flow, we are immediately given the total particle loss current and particle confinement time:

$$I_{loss} = \frac{4\pi a n_{sep} T_{sep}}{B} \qquad \tau_p = \frac{\pi B R a \kappa n_{core}}{2\left(T_{sep}/e\right) n_{sep}}$$

which are found to be in good agreement with experimental results [2]. In general the assumption of near-neoclassical particle transport at the plasma edge is supported by measurements of near neo-classical ion thermal transport [3, 4] and measurements of turbulent fluxes lower than magnetic drift fluxes [5].

Closed-form Solution: We now assume in this model, consistent with [3, 4], that anomalous electron thermal diffusivity is adequate to "fill" with electron heat the particle channel defined by the flows discussed above. We also assume that electron heat does not flow significantly beyond this channel. Radial turbulent heat flux is limited by falling density even at constant T, through the relation $q_{\perp} = \langle \tilde{p}, \tilde{v} \rangle$. Furthermore at low density outside of the main density channel parallel losses become sheath limited, reducing the heat flux from the plasma in this region.

We next assume that the density channel is emptied of heat by Spitzer-Härm electron thermal conductivity. This corresponds to the usual conduction-limited two-point model along the field lines. Here we assume that the heat flux crossing the separatrix into the SOL is constant along the separatrix surface. This gives

$$P_{SOL} = \frac{4\pi R \lambda B_p \chi_{0,S} T_{sep}^{7/2}}{\left(7/4\right) B L_{\|}}$$

allowing solution for λ in terms of engineering parameters. We find

$$\lambda = 5671 \cdot P_{SOL}^{1/8} \frac{\left(1+\kappa^2\right)^{5/8} a^{17/8} B^{1/4}}{I_p^{9/8} R} \left(\frac{2\bar{A}}{1+\bar{Z}}\right)^{7/16} \left(\frac{Z_{eff}+4}{5}\right)^{1/8} \qquad \textit{all units S.I.}$$

for the electron drift case. For the ion-drift case, a further factor of $\bar{Z}^{-7/8}$ is found.

Discussion: Comparison with experimental results from AUG [6], C-Mod [7], DIII-D [8], JET [9], and NSTX [10] are encouraging both with respect to the absolute magnitude and the scaling of the model. The strong inverse scaling with plasma current and the apparent lack of explicit scaling with size (note that I_p scales as aB for fixed q and plasma shape) are consistent with these data.

The model presented here is heuristic in nature, however, and so should not be expected to be accurate to better than factors of order unity. More work is needed to develop a fully quantitative evaluation of the physics described here. All electric drifts and parallel flows need to be included, based on detailed calculations of the potential distribution in the SOL. It is particularly challenging that no role is assumed in this model for cross-field anomalous particle transport or anomalous viscosity. This means that high-spatial-resolution calculations, with low numerical dissipation, will be required.

The predicted magnitude and scaling of τ_p in low gas-puff H-mode plasmas should be compared more extensively with experiment. Scaling with atomic charge and mass should be examined. Initial JET results [9] for H and He *vs.* D plasmas, appear more consistent with electron, rather than ion, magnetic drifts determining the LFS SOL width. One needs to exercise some caution, however, since edge thermal transport coefficients have not been measured in H or He H-mode plasmas. In the case of He plasmas there is also likely to be higher wall recycling compared with D plasmas. $\bar{Z}$ and $\bar{A}$ must be carefully evaluated.

While this model projects a very narrow power scrape-off layer width for ITER of ~2mm, it also suggests means to increase this width. Gas puffing may not penetrate the high-power SOL of ITER, allowing widening of the particle channel without edge cooling. On the other hand, since the overall projection is that the power scrape-off width varies as $P^{1/8}/B^{7/8}$ for fixed q and plasma shape, this result suggests the need to consider more radical solutions for power handling, such as liquid plasma-facing components, in future devices.

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