Global profile effects on magnetic islands and current sheets formation

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Magnetic islands play are important magnetic structures, both, in astrophysics and tokamaks contexts. For instance, in the solar corona, magnetic islands precede the destruction of flux ropes [1]. In tokamaks, magnetic islands is a major cause of the degradation of plasma confinement and disruptions [2, 3]. In the simplest case, magnetic islands are generated by unfavourable plasma current density gradients. The magnetic island stability is usually characterised by the tearing index stability parameter Δ' . When Δ' is positive, magnetic reconnection occurs and tearing modes grow [4]. The Δ' parameter is determined from the solution in the outer region and depends on the boundary conditions. As a result, the Δ' parameter does depend, in essential way, on global properties of the current profile. After the Δ' parameter has been determined, however, the linear and (to a signficant extent) nonlinear stability of tearing modes is formulated in as a local theory for any given value of Δ'. The existing (analytical) nonlinear theory may include additional local parameters (such as the second derivative of the current), but still remains local. In this paper we demonstarte that a number of essential properties of nonlinear reconnection, such as saturation of magnetic islands and formation of a Y-point singular layer strongly depends on the global features of the current profile.

Effects of global current profiles in the linear stage were studied in [5], where it has been shown that even weak modification of the equilibrium current can significantly affect both the stability and the growth of the tearing modes. In this paper, we focus on the impact of the current profiles on the asymptotic amplitude of the magnetic island in saturation. Analytical theory of nonlinear magnetic islands requires the assumption of a relatively small Δ' [8]. For large Δ', the transition to the $m = 1$ kink-tearing mode regimes [8] occurs as well as the formation of the current sheets which modifie the nature of the dynamics. Experimental values of Δ' are often large and sawtooth regimes can be linked to $\Delta' \gg 1$[7]. In this paper, we will also investigate the role of the equilibrium current profile on the current sheet formation [9, 10].

We use a two field model in the frame of 2D slab reduced MHD approximation in the low β limit[4]. It consists of a set of two coupled equations for the fluctuations of the electrostatic potential ϕ and the magnetic flux ψ. The equilibrium magnetic field is decomposed into a constant toroidal component B_{0z}, and a poloidal component $B_0(x) = \psi_0'(x)$. Noting the parallel equilibrium current $j_0 = \psi_0''(x)$, the time evolution of the fields is given by

$$\partial_t \omega + \{\phi, \omega\} = \{\psi + \psi_0, j + j_0\} + \nu \triangle_\perp \omega \,, \quad (1)$$

$$\partial_t \psi + \{\phi, \psi + \psi_0\} = \eta j \,, \quad (2)$$

where η is the resistivity, ν the viscosity, $\omega = \nabla_\perp^2 \phi$ the vorticity, and $j = \nabla_\perp^2 \psi$ the current density fluctuations. The equations are normalized by $\tau_A = L_\perp / V_A$ for the time, $L_\perp B_{0z}$ for ψ, and $L_\perp V_A$ for ϕ. V_A is the Alfvèn velocity, $L_\perp$ a magnetic shear length and τ_A is the Alfvèn time. Poloidal direction y is periodic. At the radial boundaries $x = \pm L_x/2$, there is no radial plasma flow $\phi = 0$ and the wall is perfectly conducting $\psi = 0$.

The Prandtl number $P_m = \nu/\eta = 0.1$ is kept constant, while η ranges between 10^{-5} and 10^{-3} and $\Delta' \in [1, 100]$. It is worth noting that small scale turbulence effects can lead to the anomalous viscosity coefficient and consequently, the condition $P_m \ll 1$ may become invalid [15]. Linearly, in terms of the mode classification, setting $\eta = 10^{-3}$, it corresponds, to the visco-tearing mode when Δ' is of order 1, to reconnecting mode for $\Delta' \leq 31.4$, and to the resisitive kink otherwise [11]. In this work, we will consider two commonly used equilibrium configurations: the Harris current sheet model $B_0 = \tanh(x/a)$ which will be referred as profile A, and the other profile $B_0 = -2\cosh^{-2}(x/a)\,\tanh(x/a)$, refered below as profile B. Note that the resonance surface is at $x = 0$ in both cases, but the profile B is strongly localized in the vicinity of the resonant surface. Important property of these profiles is that both correspond to symmetric (with respect to $x = 0$) current profiles.

We have studied saturation of the magnetic island growth for a range of positive Δ' with profiles A and B. In figure (1) are drawn the asymptotic island sizes w_s as a function of Δ', both normalized by the characterisitc current gradient length $b = \sqrt{-J_0(0)/J_0''(0)}$. In the thin island approximation, the island size is defined by $w = 4\sqrt{\tilde{\psi}_1(0)/J_0(0)}$ where $\psi_1(x)$ is the m=1 mode amplitude. According to [12, 13], in the limit of thin island and low Δ', it was analytically found $w_s = 2.44 b^2 \Delta'$ (POEM formula as named in [10]). This result is recovered for both profiles in figure (1). However, it was predicted that its validity should be broken when $\hat{w}_s \equiv w_s \Delta' \sim 8$[14]. In figure (1), the island size is numerically computed by identifying

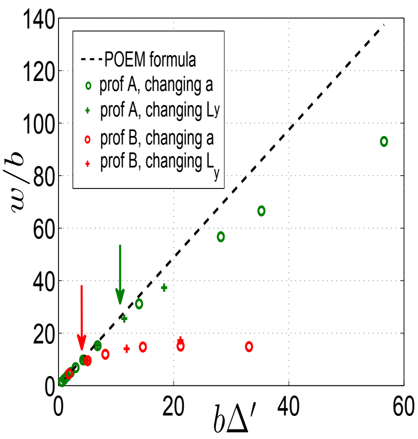

Figure 1: Normalised island size w_s/b for the two profiles as a function of $\Delta_n' \equiv b\Delta'$. $\eta = 10^{-3}$.

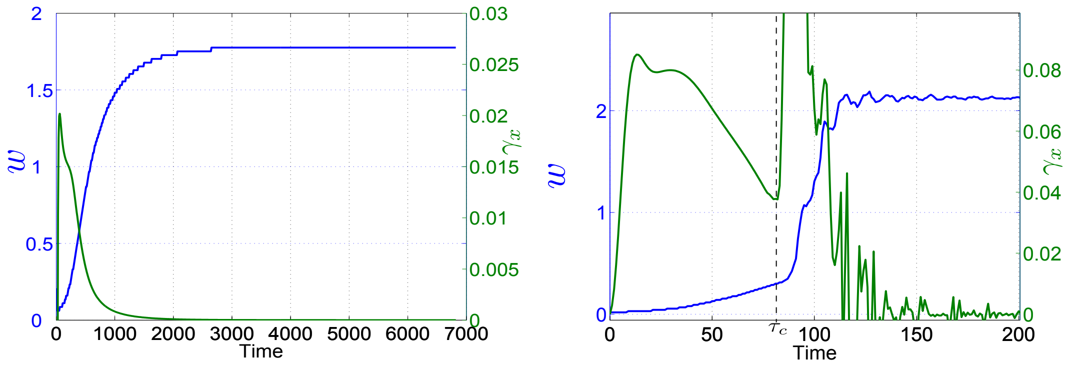

Figure 2: Equilibrium profile A. Time evolution of the magnetic island size (Blue) and of the instantaneous growth rate of the magnetic field fluctuation at the X-point $\gamma_X = d\ln\psi/dt$ (Green). [Left] $\Delta'_n = 1.49$, $\eta = 10^{-3}$. [Right] $\Delta'_n = 13.9$, $\eta = 2.10^{-5}$.

the positions of the separatrix, the X and the 0 points, which allow to quantify the island size even when thin island approximation is broken. We obtain, respectively for profile A and B, that POEM formula is valid when $\hat{w}_s$ is smaller or of the order of 235 and 40, *i.e* in regimes where the constant ψ approximation is not anymore valid. In [14], the critical value $\hat{w}_s = 8$ was obtained neglecting nonlinear viscous effect ($P_m = 0$) and was in agreement with the critical value 8.2 above which current sheets appear in the limit $\eta \to 0$ [10]. It follows that the origin of the high values of $\hat{w}_s$, in addition to profile effects, is linked to viscous effects.

In figure (2) are plotted the time evolution of the width of the magnetic island $w(t)$ at low and high Δ'. Compare to the low Δ' case where the growth of the island is monotonic, the high Δ' case is characterized by an abrupt growth of the island size, followed by a slow decreased. This abrupt growth is linked in fact to the generation of a current sheet. Indeed, as can be seen on the right graph, the instantaneous growth rate of the magnetic field fluctuations at the X-point γ_X strongly increases during this phase and gives rise to a current sheet. Note that once the sheet has appeared, γ_X is measured at the middle of the ribbon. The determination of the time at which current sheets appear by measuring of the length L_{cs} of the current sheet in the limit $L_{cs} \to 0$ is less accurate than by defining it as the time τ_c at which the abrupt growth starts, *i.e* when $d\gamma_X/dt = 0$. In figure (2), $\tau_c = 81\tau_A$ for the case $\Delta'_n = 13.9$. Note that t_c event is in fact a precursor of the abrupt growth. With this criterium, we find that, current sheets originates, roughly, the end of validity of POEM calculations

Following [10], let us investigate the limit $\eta \to 0$, keeping constant the Prandtl number. In figure (3) is plotted $\hat{w}_c(\eta) \equiv \Delta' w_c$ as a function of the resisitivity where $w_c \equiv w(\tau_c)$ is the critical island size above which the current sheet appear. We recover a linear profile for the B and a convergence at roughly the same values when $\eta \to 0$, respectively $\lim_{\eta\to 0}\hat{w}_c \sim 6.8$ when $\Delta'_n = 5.8$ and $\lim_{\eta\to 0}\hat{w}_c \sim 5.7$ when $\Delta'_n = 9$. We obtain a very different behavior for profile A. First, $\hat{w}_c(\eta)$ appear not to be a linear function. Second, $\lim_{\eta\to 0}\hat{w}_c$ lies within the range $[20,25]$, which is accidently close of the value 25 predicted by [8]. It invalidates the finding $\hat{w}_c \simeq 8.2 + f(\Delta')\eta$ [10] as far as $P_m \neq 0$ and noteworthy in the case where the magnetic field is not localized in the vicinity of the resonant surface. The physical origin of such strong profile dependence is however still to be identified.

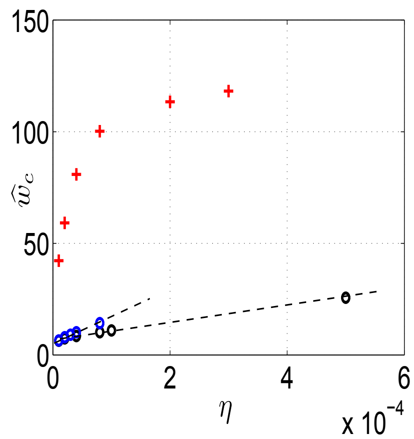

Figure 3: [+] Profile A with $\Delta'_n = 13.9$. [o] Profile B with $\Delta'_n = 9$ (blue) and $\Delta'_n = 5.8$ (black).

In this work, we have shown that the equilibrium current profile strongly affect asymptotic saturation of magnetic islands and change the conditions at which the current sheet appears. Further, our results indicate an important role of the the Prandtl number. For large island width, nonlinear modification of the m=0 becomes important. It is interesting that nonlinear modification of the equilibrium current profile tends to compensate the deviation from analytical theory (strictly speaking valid only for $\Delta' w_s < 1$). In the result, the analytical result for w_s remains in reasonable agreement with full numerical simulations reauts for larger values of Δ'. This , however, depends on the chosen current profile. In this respect, profile A turns out to be more robust.

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