Rotation effect on geodesic modes in tokamak plasmas with isothermal magnetic surfaces

A.G. Elfimov, R.M.O. Galvão, R.J.F. Sgalla

Institute of Physics, University of São Paulo, 05508-900, São Paulo, Brazil

In the past decade, Geodesic Acoustic Modes (GAM) predicted in a theoretical magneto-hydrodynamic (MHD) analysis [1] have attracted great interest due to its relevant role on the H-mode and transport barrier formation to suppress plasma turbulence. The existence of GAMs with M= 1, N=0 poloidal/toroidal mode numbers was experimentally confirmed [2,3] and many theoretical and numerical investigations are currently being pursued to further understand the characteristics of these mode [4-6]. The formation of the H-mode usually occurs during neutral beam or ion cyclotron resonance heating, which are accompanied by poloidal and (or) toroidal rotation of the plasma column [3]. In a non-rotating plasma, the GAM frequency is $\omega_{GAM}^2 = \omega_s^2(\Gamma + 1/q^2)$ where $\omega_s^2 = \gamma P/(\rho R_0^2)$, $q= rh_\zeta/h_\theta R_0$ is safety factor, P is plasma pressure, ρ is the mass density, γ is the adiabatic index, R_0 is the tokamak major radius, Γ=2 in MHD, and Γ=7/2 in kinetic approaches [6]. The oscillations are electrostatic, which depend on the parallel ($\mathbf{h}\cdot\mathbf{V}$) and binormal $(\mathbf{h}\times\mathbf{e}_r)\cdot\mathbf{V}$ velocities where $\mathbf{h}=\mathbf{B}/B$ unit vector of the magnetic field and $\mathbf{e}_r$ is radial unit vector, and they have poloidally symmetric radial electric field E_r. The equilibrium and oscillating field quantities are assumed to be axisymmetric and independent of the toroidal angle ζ. These oscillations do not perturb the magnetic surfaces ($\delta\mathbf{B}$=0), which are assumed to be circular and concentric with the set of the coordinates ($R=R_0+r\cos\theta$, $z= r\sin\theta$, ζ) where r is the plasma surface radius. The zonal flow (ZF) branch $\omega_{ZF}^2 \approx (\gamma-1)V^4/c_s^2R_0^2$ is found in static equilibrium [4] with the isothermal plasma surfaces ($p_{\rm eq} \sim \rho_{\rm eq}$) and toroidal rotation. An ion-sound branch of GAMs may appear in kinetic approach [6] or in plasmas with poloidal rotation [5], $\omega_{\rm GAM2}^2 \approx \omega_s^2(1+3V_{\rm pol}^2/c_s^2)/q^2$.

Here we study modifications of the GAM continuum for different combination of the toroidal and poloidal rotation in plasmas with the isothermal magnetic surfaces using standard MHD approach. The effect of plasma rotation on electrostatic GAM modes is investigated for large aspect ratio tokamaks ($\varepsilon=r/R_0<<1$), taking into account the heat flux. In addition to the standard ideal MHD equations [1-5], we take into account the heat balance equation

$$\frac{3P}{2}\frac{d}{dt}\left(\ln\rho^\gamma - \ln P\right) = \nabla\mathbf{Q}, \quad \mathbf{Q} \approx \frac{5}{2}\frac{\rho}{\omega_{ci}} v_{Ti}^2[\mathbf{h}\times\nabla v_{Ti}^2] \qquad (1)$$

Due to heat flux, we will show that three geodesic modes ($\omega_{\rm GAM1,2}$ and $\omega_{\rm ZF}$) appear in the toroidally and poloidally rotating plasmas. As well, we use reduced Ohm's law

$$cE_r + V_\theta B_\zeta - V_\zeta B_\theta - B\nabla_r P/\omega_{ci}\rho = 0. \qquad (2)$$

The MHD quantities are represented as a sum of time independent equilibrium and perturbed values, $P=p_{eq}+p_0\tilde{p}\exp(\mathrm{i}\omega t)$, $\rho=\rho_{eq}+\rho_0\tilde{\rho}\exp(\mathrm{i}\omega t)$, $\mathbf{V}=\mathbf{u}+\mathbf{v}\cdot\exp(\mathrm{i}\omega t)$, $\mathbf{J}=\mathbf{J}_{eq}+\mathbf{j}\cdot\exp(\mathrm{i}\omega t)$. The simplified equilibrium found for rotating plasmas in [5] is adapted to isothermal concentric magnetic surfaces, $p_{eq}\sim\rho_{eq}=\rho_0(1+\varepsilon\rho_1\cos\theta)$. Then, using the first order ε-corrections in the continuity equation, we obtain the poloidal velocity modulation $u_p=u_0[1-\varepsilon(1+\rho_1)\cos\theta]$. Next, we specify the toroidal rotation $u_t=U+[\varepsilon U-u_0q(2+\rho_1)]\cos\theta$ to equilibrate the poloidal velocity modulation due to the condition of constant electric field over the magnetic surfaces in eq.(2). Finally, taking the scalar and vector products of momentum equation with **h**, we get the radial and poloidal equilibrium conditions, which are valid for low β-plasmas,

$$J_\theta B\approx\nabla_r p_{eq}, \qquad \rho_1=\gamma\left(2M_p^2-2M_pM_t+M_t^2\right)\left(1-\gamma M_p^2\right)^{-1}$$

where $M_p^2=u_0^2/(c_s^2h_\theta^2), M_t^2=U^2/c_s^2\ll 1$ are the poloidal and toroidal Mach numbers and $c_s^2=\gamma P/\rho$. For perturbations, we expand the set of oscillating amplitudes $\{p, v_\parallel, \tilde{\rho}\}$ in Fourier series over poloidal angle, as for density $\tilde{\rho}=\rho_0(\tilde{\rho}_0+\rho_c\cos\theta+\rho_s\sin\theta)$, where the first harmonic is only taken into account. Next, using the vector and scalar products of momentum equation with the **h**-vector in perturbative analyses of MHD equations, we obtain the set of equations for the pressure, density, and parallel velocity oscillations

$$p_s=\gamma\rho_s+\mathrm{i}M_p\,{p_c}/{\Omega}-\mathrm{i}(\gamma-1)q\rho_1h_\zeta\,{v_b}/{\Omega c_s}-\mathrm{i}\gamma(1-M_d)M_p\,{\rho_c}/{\Omega} \qquad (3a)$$

$$p_c=\gamma\rho_c-\mathrm{i}M_p\,{p_s}/{\Omega}+\mathrm{i}\gamma(1-M_d)M_p\,{\rho_s}/{\Omega} \qquad (3b)$$

$$\mathrm{i}\Omega c_s\rho_c=v_s+c_sM_p\rho_s \qquad (3c)$$

$$\mathrm{i}\Omega c_s\rho_s=-2q_Mv_b-v_c-c_sM_p\rho_c \qquad (3d)$$

$$\mathrm{i}\Omega v_s=\left(2q_MM_p-qM_t\right)v_b-M_pv_c-c_sp_c/\gamma \qquad (3e)$$

$$\mathrm{i}\Omega v_c=M_pv_s+c_sp_s/\gamma \qquad (3f)$$

where $q_M=q\left[1+\gamma\left(M_t^2/2-M_pM_t\right)\right]\left(1-\gamma M_p^2\right)^{-1}$, $\Omega=\omega R_0q/c_s$, is the normalized frequency, $v_b=-c\tilde{E}_r/B_0$, and $M_d=(1-1/\gamma)\rho_1\left(2+\rho_1\right)^{-1}$ is the parameter responsible for the heat flux effect where the approximation $|h_\zeta|\approx 1$ is used. Further, to get the geodesic continuum frequency we employ the current continuity equation for the perturbed current $\nabla\cdot\mathbf{j}_\perp=0$ where only $\cos\theta$-pressure perturbations may contribute due to the standard averaging procedure over the magnetic surfaces used to evaluate geodesic modes

$$\oint R(\nabla\cdot\mathbf{j}_\perp)d\theta = R_0\nabla_r < j_r >= \frac{\mathrm{i}\Omega c_s c\rho_0}{B_0 q}\frac{d}{dr}\left[v_b + \mathrm{i}q\frac{c_s p_s}{\gamma\Omega} + \mathrm{i}q\frac{c_s\rho_s}{2\Omega}M_t^2 + \mathrm{i}\frac{qv_s}{\Omega}M_t\right] = 0 \qquad (4)$$

Here, the averaged current may be presented via radial component of the dielectric tensor $< j_r >= -{}^{\mathrm{i}\omega}\!/_{4\pi}\,\varepsilon_0 E_r$. Finally, using combinations of eq. (3a-3f), we calculate the perturbed pressure, density, and velocity, which are introduced into eq. (4) to obtain the geodesic continuum equation through the radial tensor component

$$\varepsilon_0 = \frac{c^2}{c_A^2}\frac{N + \delta N + O(\rho_1^3)}{\left[\Omega^4 - 2\Omega^2\left(1 + M_p^2 + M_p^2 M_d\right) + 1 - 2M_p^2\left(M_p^2 - M_d\right)^2\right]\left(\Omega^2 - \Omega_{d3}^2\right) + \delta D} = 0 \qquad (5)$$

where $c_A = B/\sqrt{4\pi m_i n_i}$ is Alfvén velocity and terms in numerator are

$$N = \Omega^6 - \Omega^4\left[2 + 2q^2\left(1 + M_t^2 + (M_p - M_t)^2\right) + 3M_p^2\right] + \Omega^2\left[1 + 2q^2\left(1 + 2M_t^2 + 5M_p^2 - 6M_pM_t\right)\right] - M_p^2(1 + 2q^2),$$

$$\begin{aligned}\delta N = {} & q^2\gamma\left(2M_t^3M_p + 4M_p^3M_t - 2M_p^4 - 4M_t^2M_p^2 - M_t^4/2\right)\Omega^4 + \left\{3M_p^2\left(M_p^2 - 2M_d\right)\right. \\ & + q^2\left[(4 + 6\gamma)M_p^4 - 3(\gamma + 1)M_t^3M_p - 2M_d\left(M_p^2 + 2M_tM_p\right) + (7\gamma + 13)M_t^2M_p^2\right. \\ & \left.\left. + (\gamma - 1/2)M_t^4 - (8\gamma + 12)M_p^3M_t\right]\right\}\Omega^2 + 2(M_d + M_p^2)M_p^2 - q^2\left[8M_p^4 + (3 + \gamma)M_t^2M_p^2\right. \\ & \left. + (\gamma - 1)(M_t/2 - M_p)M_t^3 - 12M_p^3M_t - 4M_dM_p^2\right]\end{aligned}.$$

Exact expressions in above equation are very complicated and we present them in an approximated form. The expressions for the geodesic modes are found from the numerator roots. Keeping terms of the order of ρ_1 in the numerator of eq.(5), we use the equation N=0 to obtain two high order roots $\Omega_{1,2}^2$, which correspond to the $\mathrm{GAM}_{1,2}$ branches

$$\omega_{\mathrm{GAM1}}^2 = \left[2 + 1/q^2 + \left(2 - 1/q^2 + 2/q^4\right)M_p^2 + 4M_t^2 - 4\left(1 - 1/q^2\right)M_pM_t\right]c_s^2/R_0^2 \ . \qquad (6a)$$

$$\omega_{\mathrm{GAM2}}^2 = \left[1 + \left(3 - 2/q^2\right)M_p^2 - 4M_pM_t\right]{}^{c_s^2}\!\!/_{q^2R_0^2} \ . \qquad (6b)$$

These geodesic modes have been also obtained in Ref 5 and first GAM branch for M_p=0 in eq.(6a) exactly coincides with the one presented in Ref 4. In the denominator, ignoring corrections of $\delta D \approx O(\rho_1^4)$, we found $\Omega_{d1,2}^2 = \left(1 \pm M_p\right)^2 \pm \left(1 \mp M_p\right)M_pM_d + O(\rho_1^4)$,

$\Omega_{d3}^2 = M_p^2\left(1 - M_d\right)^2 + O(\rho_1^4)$. Here, we note that the sound branch in eq.(6b) written in the dimensionless form $\Omega_2^2 \approx 1 + (3 - 2/q^2)M_p^2 - 4M_pM_t$ stays in the frequency band between the denominator roots $\Omega_{d12}^2 \approx 1 \pm 2\sqrt{M_p^2}$ in eq. (7). It means that the branch disappears in the limit $M_p \to 0$ due to the factors ($\Omega\pm1$), which are divided out in the fraction of eq. (5).

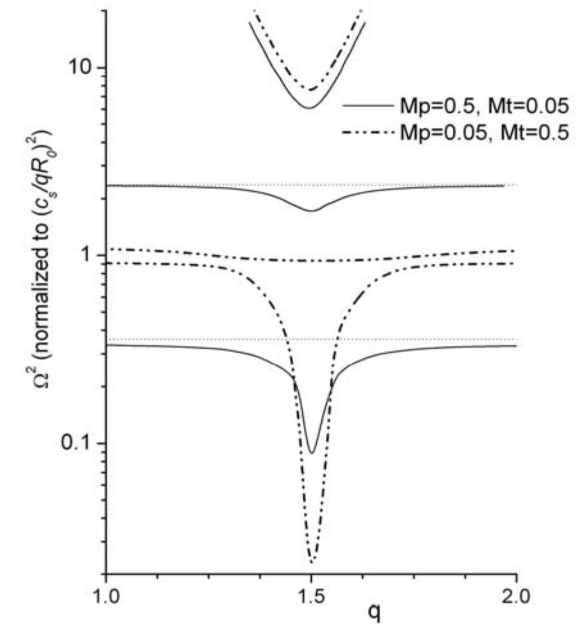
Most interesting is the third new ZF root of equation $N+\delta N=0$ in the numerator of eq.(5)

$$\omega_{ZF}^2 = \left[M_p^2 (1-M_d)^2 + \frac{q^2(\gamma-1)}{2(1+2q^2)} \left(M_t^2 - 2M_t M_p + 2M_p^2 \right) M_t^2 \right] \frac{c_s^2}{q^2 R_0^2} \qquad (7)$$

that is only depend on poloidal/toroidal circulation frequency and it has the frequency, which is small in comparison with the geodesic ones in eq.(6a,b). The mode frequency stays at the poloidal circulation frequency u_0 / r when toroidal rotation $M_t^4 << M_p^2 \sim M_t^2$ is not yet too small. The mode disappears in the formal limit $M_t^2 = 0$ when the factors $\left[\Omega^2 - M_p^2 (1-M_d)^2 \right]$ are divided out in the fraction of eq (5). In the case of preferentially toroidal rotation $M_t^4 >> M_p^2$ in eq.(9), we have result of Ref 4.

It is very important to take into account the Alfvén wave continuum (AWC) to verify transitions of the continuum branches and to know where continuum extremum will occur because the real eigenmodes may only propagate at the maximum or minimum of the continuum [6]. In a quasi-cylindrical approach, the AWC equation may be written in the form:

$$\varepsilon_0 \frac{\omega^2 R^2}{c^2 R_0^2} E_r = \hat{k}_\parallel^2 E_r = \frac{1}{q^2 R^2} \left(nq - \frac{d}{d\theta} \right)^2 E_r$$



Therefore, there is intersection of GAM's with the Alfvén continuum at the rational magnetic surfaces $q=m/n$. To show transitions between the continuum branches, the squared frequency of the continuum is plotted schematically in Fig.1 as a function of q at the rational surface defined by m=3, n=2 poloidal /toroidal mode numbers. Typical tokamak plasma parameters $r=0.1R_0$, $(c_A/c_s)=10$, and $c/c_A=100$ are chosen for the preferentially poloidal ($M_p=0.5$, $M_t=0.05$) or for preferentially toroidal ($M_p=0.05$, $M_t=0.5$) rotation.

[1]Windsor N, Jonson J L and Dowson J M, 1968 *Phys. Fluids*, **11**, 2448.

[2] Krämer-Flecken A, *et al*, 2006 *Phys. Rev. Lett.* **97**, 045006.

[3] Conway G D *et al*, 2005 *Plasma Phys. Control. Fusion*, **47**, 055009.

[4] Wahlberg C, 2008 *Phys Rev. Lett*. **101**, 115003.

[5] Ilgisonis VI *et al*, 2011 *Plasma Phys. Control. Fusion*, **53,** 065008.

[6] Elfimov AG, 2010 *Physics of Plasmas* **17**, 022102.