Poloidal and toroidal plasma rotation and resistive wall modes in tokamaks

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Abstract

Contrary to wide spread belief, determination of the frequencies and growth rates of MHD waves and instabilities in rotating plasmas does *not* require non-self adjoint operators. The physics involves the generalized force operator and the Doppler–Coriolis shift operator, which are both self-adjoint, but they occur in a quadratic eigenvalue problem [1] with complex eigenvalues. Enclosing the system with a resistive wall yields a cubic eigenvalue problem [2], where the dissipation of the wall permits the additional class of resistive wall modes. Since these modes grow on a much longer time scale than the ideal MHD ones, they may be feedback stabilized [3]. To accomplish that, knowledge of the full spectrum of modes of the system is essential. A general method to achieve this for the quadratic (ideal) eigenvalue problem has been developed recently by constructing *the solution paths* in the complex ω-plane [4]. These are obtained by taking away the outer boundary and solving the open boundary value problem, but restricting the solutions to have no energy flow into or out of the system. This yields curves in the complex ω plane on which the eigenvalues must be situated. They are determined by imposing the missing boundary condition. Here, the method is generalized to the cubic (dissipative) eigenvalue problem by accounting for the energy dissipation in the resistive wall. The obtained topologies of the solution paths yield important new insights into the coupling of the resistive wall modes with the co- and counter-rotating external kink modes. Stability regimes obtained depend on the details of the profiles of the safety factor q, the toroidal velocity v_φ, the poloidal velocity v_p, the wall position w and the dissipative time scale τ_D of the wall.

1 Solution path method

Moving from the static MHD spectral problem of Bernstein *et al.* (1958),

$$\mathbf{F}(\boldsymbol{\xi}) = -\rho\omega^2\boldsymbol{\xi}\,, \tag{1}$$

to the stationary one described by Frieman and Rotenberg [1],

$$\mathbf{G}(\boldsymbol{\xi}) - 2\omega U\boldsymbol{\xi} + \rho\omega^2\boldsymbol{\xi} = 0\,, \qquad U \equiv -\mathrm{i}\rho\mathbf{v}\cdot\nabla\,, \tag{2}$$

the widely spread conviction has been that "the latter problem is non-self-adjoint". However, energy is conserved and both operators, the generalized force operator $\mathbf{G}$ and the

Doppler–Coriolis shift operator U, are self-adjoint! A new approach to the MHD spectral problem of stationary flow, exploiting these properties, is described and extensively exploited in the first chapters of the new textbook on advanced MHD [5].

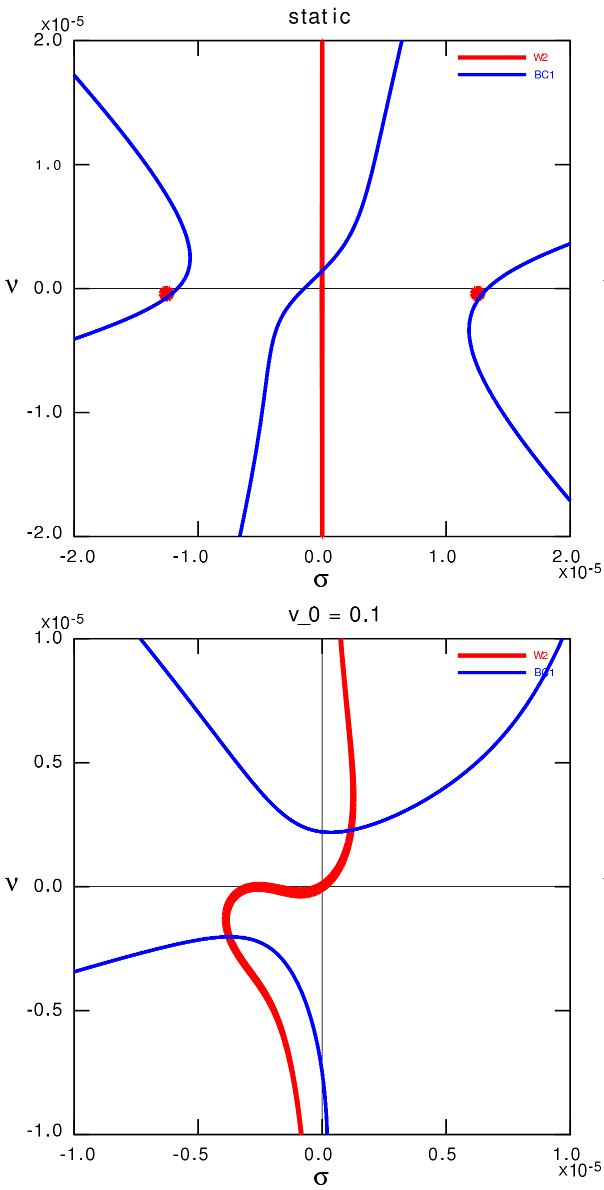

Figure 1: Eigenvalues for a toroidally rotating plasma surrounded by a resistive wall occur at the intersection of the solution path (red) and the real part of the RWM boundary condition (blue); $r_{\rm wall} = 1.05 r_{\rm pl}$, $\tau_{\rm D} = 10^8$.

The method consists of considering the open system, obtained by removing one of the boundaries, and computing the imaginary energy W_2 that has to be injected or extracted in order to get harmonic time dependence $\exp(-\mathrm{i}\omega t)$. Closing the system again by demanding that this energy vanishes yields curves in the complex ω-plane:

$$W_2 \equiv \mathrm{Im}\big(W[\boldsymbol{\xi}(\mathbf{r};\omega)]\big) = 0 \quad \Rightarrow \quad \textit{solution path} \text{ of unstable solutions}\,. \tag{3}$$

The complex eigenvalues $\omega = \sigma + \mathrm{i}\nu$ have to lie on this path. They are found by demanding that the real part of the *alternator* R (the alternating ratio of solutions of the differential equations) vanishes:

$$R_1 \equiv \mathrm{Re}\big(\xi(x_\mathrm{e})/\Pi(x_\mathrm{e})\big) = 0 \quad \Rightarrow \quad \text{eigenvalues}\,. \tag{4}$$

The solution path and alternator yield a powerful new way of computing the complex eigenvalues of stationary moving plasmas.

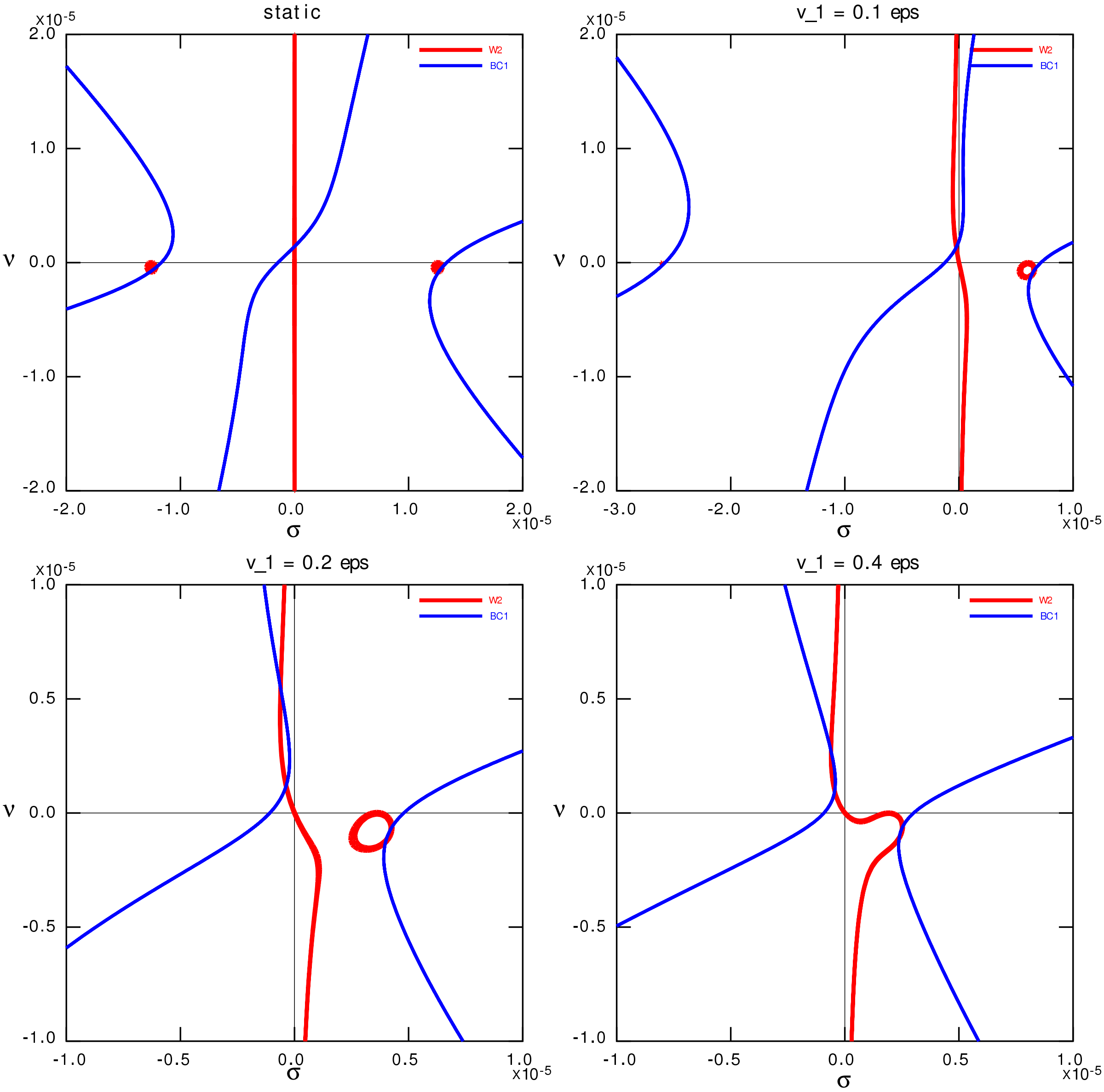

Figure 2: Eigenvalues for a poloidally rotating plasma surrounded by a resistive wall occur at the intersection of the solution path (red) and the real part of the RWM boundary condition (blue); $r_\mathrm{wall} = 1.05\, r_\mathrm{pl}$, $\tau_\mathrm{D} = 10^8$.

2 Resistive wall modes in rotating plasmas

The generalization to the analysis of the resistive wall mode (RWM) is straightforward. Again, the solution path is obtained from energy conservation in the open system, but now *accounting for dissipation in the resistive wall*:

$$W_{2,\mathrm{tot}} \equiv W_{2,\mathrm{ideal}} + D_{\mathrm{RW}} = 0\,. \tag{5}$$

The requisite expression $W_{2,\mathrm{total}}$ now contains an ideal part $W_{2,\mathrm{ideal}}$, following from self-adjointness of the force operator $\mathbf{G}$ in the plasma, and an additional dissipative part D_{RW} of the dissipation in the wall. Again, the eigenvalues have to lie on this solution path. They are found by a similar dissipative adaptation of the alternator concept, involving the real part of the boundary condition at the resistive wall.

Solutions are shown in Figs. 1 and 2 for toroidally, resp. poloidally, rotating plasmas in a low-β tokamak. The two figures show how the three mode picture of two damped external kink modes (just below the real axis) and the single resistive wall mode (just above the origin on the imaginary axis), first found in [2] for the static case, changes for increasing rotation velocities. The three modes are situated on solution paths (in red) that merge and split again, depending on the magnitude of the rotation speed (measured in units of the Alfvén speed). Note that poloidal rotation, though an order of magnitude $\epsilon \equiv B_{\mathrm{pol}}/B_{\mathrm{tor}}$ smaller than toroidal rotation, has similar effects on the spectrum of the dissipative three mode interaction. In particular, a more sizeable stabilization occurs for rather moderate rotation speeds. Since the results shown are for the worst possible case of a tokamak with a constant current profile, a range of stable operation regimes may be obtained. A detailed investigation of these is in progress.

3 Conclusions

- *The solution path method* has been generalized to the resistive wall mode problem.
- Both toroidal and poloidal rotation yield *an intricate interaction between the two stable kink modes and the unstable RWM:* stabilizing or destabilizing, depending on details of the velocity profiles.
- The method is perfectly parallelizable, may be extended to toroidal systems, and thus *provides a powerful new tool for the study of the interaction of MHD modes with the external environment.*

References

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