

Stabilization of the resistive wall mode instability by trapped energetic particles

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A theoretical model for investigating the effect of the trapped energetic particles (EPs) on the resistive wall mode (RWM) instability is proposed. The results demonstrate that EPs have a dramatic stabilizing effect on the RWM because of resonance between the mode and the precession motion of EPs. The plasma rotation can enhance this stabilization. In addition, when the EPs beta exceeds a critical value, a fishbone-like bursting mode (FLM) branch with external kink eigenstructure can exist. The characteristics of FLM are qualitatively consistent with the experimental observations [Phys. Rev. Lett. 103, 045001 (2009)].

1. Theory model

To steadily achieve high- β (ratio of plasma to magnetic pressures) plasma in advanced tokamak(AT), one kind of slowly growing macroscopic MHD instabilities, resistive wall mode (RWM), must be suppressed [1]. Recent experiments and theories indicate that the kinetic effects of the particles on the RWM are important [2-5]. However, the effects of the trapped energetic particles (EPs) on the RWM have not been fully resolved, and could be crucial for the future AT [6]. A theoretical model for investigating the effect of EPs on the RWM is proposed in this work.

The extended dispersion relation [2] of the RWM, neglecting the inertial term but taking into account the contribution of the trapped EPs, is written as

$$D = -i\omega\tau_w^* + \frac{\delta W^\infty + \delta W_k + \delta W_{MHD,h}}{\delta W^b + \delta W_k + \delta W_{MHD,h}} = 0, \quad (1)$$

where $\omega = \omega_r + i\gamma$ is the eigenvalue of the RWM instability, with ω_r and γ being the real frequency and the growth rate of the mode, respectively. δW^∞ and δW^b refer to the perturbed fluid potential energy without and with an ideal wall, respectively. The fluid potential energy includes both the plasma and vacuum contributions. δW_k and $\delta W_{MHD,h}$ denote the kinetic and fluid components, respectively, of the trapped EPs. The factor $\tau_w^* = \mu_0\sigma bd(1 - a^{2m}/b^{2m})/(2m)$ is defined as the typical wall eddy current decay time, with

a , b , d , m , σ , and μ_0 being the plasma minor radius, the wall minor radius, the wall thickness, the poloidal mode number, the wall conductivity, and the permeability of free space, respectively.

For simplicity, the external kink mode eigenfunction for a cylindrical equilibrium [2] is used to calculate the energies in Eq.(1), which is taken as $\xi_{\perp} = am\hat{r}^{m-1}(\mathbf{e}_r + i\mathbf{e}_{\theta})e^{i(m\theta-n\phi)} / F_0$ with n being the toroidal mode number, $F_0 = (m-nq)a / (Rq)$ and $\hat{r} = r / a$. We also assume a slowing-down distribution function for EPs induced by neutral beam injection. The perturbed energy induced by EPs can be obtained as[7]

$$\begin{aligned} \delta\hat{W}_{K0} \equiv \delta\hat{W}_k + \delta\hat{W}_{MHD,h} &= 12\pi(1 - \frac{\alpha_0 B_0}{2})^2 \frac{\beta_h R}{Ka} \{(\hat{A} - \hat{B}) \frac{2}{7} \Omega \ln(1 - \frac{1}{\Omega}) \\ &- \frac{2}{7}(\hat{A} + \frac{5}{2}\hat{B})\Omega [2(\frac{1}{5\Omega} + \frac{1}{3\Omega^2} + \frac{1}{\Omega^3}) - \frac{1}{\Omega^3} \frac{1}{\sqrt{\Omega}} \ln(\frac{1+\sqrt{\Omega}}{1-\sqrt{\Omega}})]\} + M \end{aligned} \quad (2)$$

where $\delta\hat{W} = 2R\mu_0 F_0^2 \delta W / (\pi B_0^2 a^4 m^2)$, and $\Omega = (i\gamma + \omega_r - \omega_0) / \omega_{ds} \equiv i\gamma / \omega_{ds} + \Omega_r - \Omega_0$.

The coefficients are

$$\begin{aligned} \hat{A} &= (2m-2)(2E-K) / q + (2E-K) / (2q) \\ &- (2E-K) / (\alpha B_0)(E/K)' - [E/(\alpha B_0 K) + k_t / q - 1 / (2\alpha B_0) - 1 / (2q)](2E-K)' \end{aligned} \quad (3)$$

$$\hat{B} = [(1-2k_t) / q - (2E/K-1) / (\alpha_0 B_0)]K(E/K)' - (2E-K) / q, \quad (4)$$

$$M = -12\pi(1 - 0.5\alpha_0 B_0)^2 \beta_h R (1 - 2k_t) (1 - 1/q) (2E-K)' / [Ka(4m-3)], \quad (5)$$

and $\omega_{ds} = K_2(a)E_m q / [K_b(a)m_h a \omega_c R]$ denotes the precession frequency of the trapped EPs at the plasma edge, with E_m being the birth energy of the trapped EPs. The quantity q states the value of the safety factor. The prime in Eqs.(3)-(5) denotes the derivation of the complete elliptic integrals to $k_t = (1/\alpha B_0 + \epsilon - 1) / (2\epsilon)$ with $\epsilon = r / R$ (the ratio of the radial variable to the major radius of the torus). αB_0 is defined as the pitch angle. The normalized forms of the fluid potential energies are given, respectively, as [2]

$$\delta\hat{W}^{\infty} = -4\pi(m-nq)^2 [1 / (m-nq) - 1] / (mq^2) \quad (6)$$

and

$$\delta\hat{W}^b = -4\pi(m-nq)^2 \{1 / (m-nq) - 1 / [1 - (b/a)^{-2m}]\} / (mq^2). \quad (7)$$

2. Numerical results

In the following calculations, the parameters are chosen as: $m = 2$, $n = 1$, $a = 1\text{ m}$, $R = 3\text{ m}$, $B_0 = 2.3\text{ T}$, $E_b = 85\text{ KeV}$, $q = 1.42$, $\beta = 0.055$, $\sigma = 10^6 \Omega^{-1} \text{ m}^{-1}$, $d = 0.01a$,

$\alpha_0 B_0 = 0.98$ and the density $n_0 = 10^{20} \text{ m}^{-3}$. The choice of the harmonic numbers and the (flat) q value yields an RWM regime (i.e., $\delta W^\infty < 0$ and $\delta W^b > 0$).

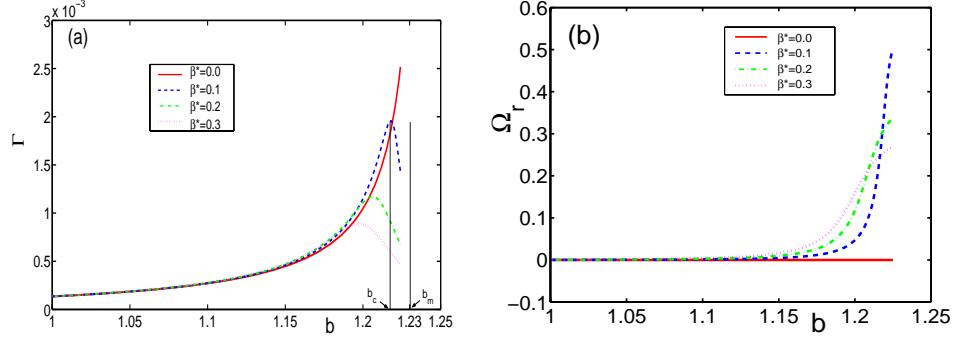


Fig. 1. The (a) growth rate Γ (normalized to Alfvén frequency) and (b) mode frequency (normalized to precession frequency of EPs ω_{ds}) Ω_r as functions of the wall position b for different choices of β^* ($\equiv \beta_h / \beta$), with $\Omega_0 = 0$. Here, b_c denotes the wall position at which the value of Γ achieves its maximum value. At $b = b_m$, $\delta W^b = 0$.

Fig. 1(a) shows that, for the cases of $\beta^* = 0.1$, $\beta^* = 0.2$ and $\beta^* = 0.3$, the Γ increases with increasing b initially, and then decreases gradually after reaching a maximum at certain value b_c which is inversely proportional to β^* . The reason for kinetic stabilizing effect appearing near the ideal-wall marginal point (b_m), is mainly due to the fact that the denominator in Eq. (1) is easily modified by the kinetic contribution when δW_b approaches zero. The stabilization mechanism is due to the energy dissipation resulting from the resonance between the EP-induced finite mode frequency ($\Omega_r - \Omega_0$) [shown by Fig. 1(b)] and the EP's precession drift frequency ($\Omega_d = \omega_d / \omega_{ds}$), that is satisfied for EPs at particular radial locations and with particular energy [8].

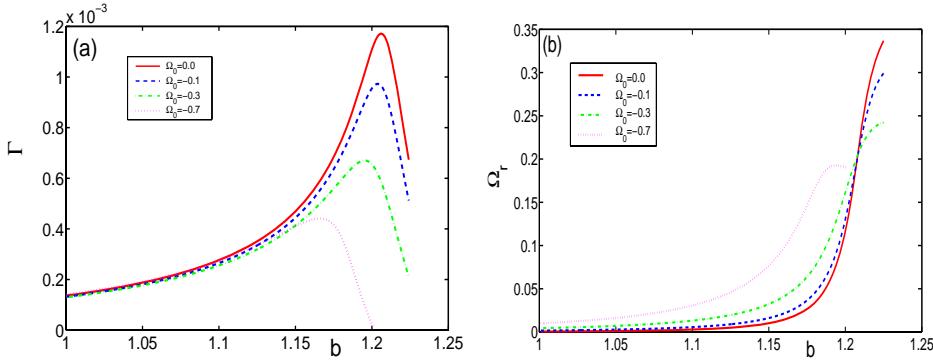


Fig. 2. The (a) growth rate Γ and (b) mode frequency Ω_r as functions of the wall position b for different choices of the plasma rotation Ω_0 , whilst the $\beta^* = 0.2$ is kept constant. Here, the minus sign of Ω_0 implies that the direction of the plasma rotation is opposite to that of the precession frequency of EPs.

Fig. 2(a) shows that the plasma rotation can enhance stabilization of trapped EPs on the RWM; Fig. 2(b) indicates that, when the wall position b is close to 1, the mode also

possesses a finite real frequency for the cases of finite plasma rotation. That is, Ω_0 can affect the mode real frequency “globally”.

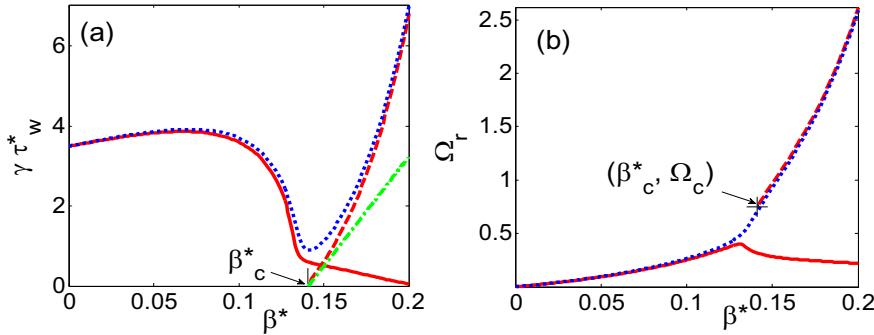


Fig. 3 The (a) plots the normalized growth rate of the mode and (b) normalized real frequency of the mode versus β^* , for the pitch angle $\alpha_0 B_0 = 0.95$ and the plasma rotation $\Omega_0 = -0.73$. The dotted curves corresponds to the case of $\Omega_0 = -0.76$. The dash-dotted curve in (a) shows the growth rate from a perturbative analysis.

Figure 3(a) shows that when β^* is less than a critical value $\beta_c^* = 0.141$, only one branch of instability (the unstable RWM with the damping effect of the trapped EPs) exists. However, when $\beta^* > \beta_c^*$, there are two unstable branches: one branch (solid curve) is completely suppressed when $\beta^* > 0.2$. This is the conventional RWM branch. The other branch (dashed curve) is a bursting mode (i.e., FLM), with initial real mode frequency $\Omega_c = 0.75$ [Fig. 3(b)], and the mode growth rate γ^* being roughly a linear function of β^* ($> \beta_c^*$). At $\beta^* = 0.2$, the real frequency of the FLM is $\omega_r = 2.6\omega_{ds} = 1.5 \times 10^4$ rad/s⁻¹, or $f_r = 2.5$ kHz, as shown by Fig. 3(b), and the mode growth rate $\gamma = 2.8 \times 10^3$ s⁻¹. These values are qualitatively comparable with the experimental results of $f_r^{\text{exp}} \approx 3.0$ kHz and $\gamma^{\text{exp}} \approx 1.0 \times 10^3$ s⁻¹ [11]. The dotted curve in Fig. 3(a) also shows a possible mode conversion between the RWM and the FLM. The direct mode conversion can approximately occur in a region of $0.12 < \beta^* < 0.14$, where the mode possesses both the RWM character (growth rate decreasing with β^*) and the FLM character (real frequency increasing with β^*).[9]

This work was supported by the National Natural Science Foundation of China under Grant No.10775040 and by National Magnetic Confinement Fusion Science Program under Grant No. 2009GB101002.

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