Momentum conservation and gyrokinetic simulations of toroidal rotation

J. Abiteboul, X. Garbet, V. Grandgirard, S.J. Allfrey, G. Dif-Pradalier,

Ph. Ghendrih, G. Latu, C. Passeron, Y. Sarazin and A. Strugarek

CEA, IRFM, F-13108 Saint-Paul-lez-Durance, France.

The global behavior of fusion plasmas in magnetic confinement devices is critically associated to large-scale mean flows. In particular, toroidal flows impact both turbulent transport, through the saturation of turbulence by sheared flows; and MHD stability by increasing the threshold for the onset of resistive wall modes. In view of predicting the toroidal rotation in future fusion devices such as ITER, where external momentum input will be small, accurate simulations of momentum transport are crucial.

We investigate the turbulent transport of toroidal momentum using gyrokinetic simulations. In order to validate this approach, we first show that the gyrokinetic model, as formulated by Brizard and Hahm[1] and implemented in gyrokinetic codes, provides a local conservation equation for toroidal angular momentum. This equation is verified numerically using the full-f, global gyrokinetic code GYSELA in the flux-driven regime[2]. Thus gyrokinetic simulations provide an appropriate description of toroidal momentum transport. This allows us to study the turbulent generation of toroidal rotation. This process is shown to be dominated by the turbulent Reynolds stress, and is strongly correlated to heat transport. Finally, the role of boundary conditions is investigated, in order to study the effect of scrape-off-layer and edge flows on core toroidal rotation.

Conservation of toroidal angular momentum in gyrokinetics

We consider the gyro-averaged guiding-center distribution function $\bar{F}(\mathbf{z})$ in the coordinate system $\mathbf{z} = (\chi, \theta, \varphi, v_{G\parallel}, \mu)$. χ is the opposite of the poloidal magnetic flux, θ and φ are the poloidal and toroidal angles, $v_{G\parallel}$ is the parallel velocity and μ is the magnetic moment, which is an adiabatic invariant. The equilibrium magnetic field is axisymmetric: $\mathbf{B} = I\nabla\varphi + \nabla\varphi \times \nabla\chi$. The gyrokinetic equation for each species s can be written in its conservative form[1]:

$$\partial_t \bar{F} + \frac{1}{B_{\parallel}^*} \nabla_{\mathbf{z}} \cdot \left(\dot{\mathbf{z}} B_{\parallel}^* \bar{F} \right) = \mathscr{C}(\bar{F}) \tag{1}$$

where $\dot{\mathbf{z}} = \mathrm{d}_t \mathbf{z}$. $B_{\parallel}^* = B + m v_{G\parallel}/e \mathbf{b} \cdot (\nabla \times \mathbf{b})$ is the Jacobian of the gyrocenter transformation. Details of the collision operator $\mathscr{C}(\bar{F})$ are not critical as long as it verifies Boltzmann's H-theorem and the conservation of particles, momentum and energy. We consider only electrostatic turbulence. The self-consistent model is obtained by coupling Eq. (1) to the gyrokinetic

quasi-neutrality equation

$$-\sum_s \nabla \cdot \left\{ \frac{n_{eq}m_s}{B^2} \nabla_\perp \phi \right\} = \sum_s e_s \int 2\pi B_\parallel^* \mathrm{d}\mu \mathrm{d}v_{G\parallel} J \cdot \bar{F} \tag{2}$$

where J is the gyro-averaging operator and n_{eq} is the equilibrium density of guiding-centers. In the case considered here, electrons are taken into account in the adiabatic asymptotic limit.

In the gyrokinetic ordering, the toroidal canonical momentum is $P_\varphi = m_s u_\varphi - e\chi$ where we define $u_\varphi = \frac{I}{B} v_{G\parallel}$. If the system is axisymmetric, P_φ is an exact motion invariant. When axisymmetry is broken by turbulence, $\mathrm{d}_t P_\varphi = -e\partial_\varphi \bar{\phi}$. Considering the expression of P_φ, it is consistent to define the local gyrocenter toroidal momentum as $\mathscr{L}_\varphi = \sum_s m_s \int \mathrm{d}\tau^* u_\varphi \bar{F}$ where $\int \mathrm{d}\tau^*$ corresponds to the integration over all phase-space variables other than χ. As P_φ differs from the canonical momentum of particles only by terms of order $\mathscr{O}(\rho_*^2)$, where ρ_* is the ion thermal gyroradius normalized to the minor radius of the tokamak, the *gyrocenter* momentum $\mathscr{L}_\varphi$ is equivalent to the particle (and thus "physical") toroidal momentum at the ρ_* ordering of the gyrokinetic model. From Eq.(1), we obtain

$$\partial_t \mathscr{L}_\varphi + \partial_\chi \Pi_\varphi^\chi + \partial_\chi \mathrm{T}_\varphi^\chi = \mathscr{J} \tag{3}$$

where

$$\Pi_\varphi^\chi = \sum_s m_s \int \mathrm{d}\tau^* \bar{F} u_\varphi v_G^\chi \quad ; \quad \mathrm{T}_\varphi^\chi = \sum_s e_s \int^\chi \mathrm{d}\chi \int \mathrm{d}\tau^* \bar{F} \partial_\varphi \bar{\phi} \quad ; \quad \mathscr{J} = \sum_s e_s \int \mathrm{d}\tau^* v_G^\chi \bar{F} \tag{4}$$

where $v_G^\chi = \dot{\mathbf{z}} \cdot \nabla \chi$ is the guiding-center toroidal velocity in conventional contravariant notations, which contains contributions from both the $E \times B$ drift and the magnetic drifts. Eq. (3) is an exact equation for gyrocenter momentum in the sense that, once the gyrokinetic model is given, no additional approximation is required.

The tensor Π_φ^χ is the off-diagonal $(\varphi\chi)$ component of the conventional Reynolds stress. The interpretation of the second term, which can be written as the divergence of a flux by using the Hermitian property of the gyroaverage, is less straightforward. Using a Padé approximation of the gyroaverage operator, one can show[3] that this term contains the polarization stress identified by McDevitt *et al.*[4], corrected by finite Larmor radius (FLR) effects.

The term on the right-hand-side in Eq. (3) corresponds to a radial current of gyrocenters. It appears as a source in this equation. It actually describes the exchange of momentum between gyrocenters and the electromagnetic field. An evolution equation can be derived for the polarization σ, which corresponds to the field momentum in the Minkowski formulation. This is simply $\partial_t \sigma = -\mathscr{J}$. Thus the conservation equation for the total toroidal momentum, i.e.

including field and gyrocenter contributions, reads $\partial_t(\mathscr{L}_\varphi + \sigma) + \partial_\chi(\Pi_\varphi^\chi + T_\varphi^\chi) = 0$ and does not contain any volume source term, as it should. This formulation is consistent with previous results by Scott[5] and Brizard[6].

Gyrokinetic simulations of turbulent toroidal momentum transport

The conservation equation derived in the previous section has been tested numerically using the gyrokinetic code GYSELA in the flux-driven regime[2]. For a simulation with the normalized radius $\rho_* = 1/512$, which is close to the value expected for ITER, a precision of approximately 1% was obtained for the momentum balance[7]. This result demonstrates that gyrokinetic codes are capable of accurately describing the transport of toroidal momentum.

Considering more precisely the relative amplitude of the terms in Eq. (3), we find that the Reynolds stress (Π_φ^χ) is the dominant contribution to the local balance of toroidal angular momentum. The polarization stress (T_φ^χ) also contributes significantly, while the radial current of gyrocenters ($\mathscr{J}$) is negligible (approximately 0.1% of the total magnitude), as expected for simulations with adiabatic electron response.

These important results allows us to investigate the generation of *intrinsic* rotation by turbulence in gyrokinetic simulations. We present the result of a simulation for $\rho_* = 1/512$. No momentum source was prescribed in the system. The time trace of both the parallel flow and
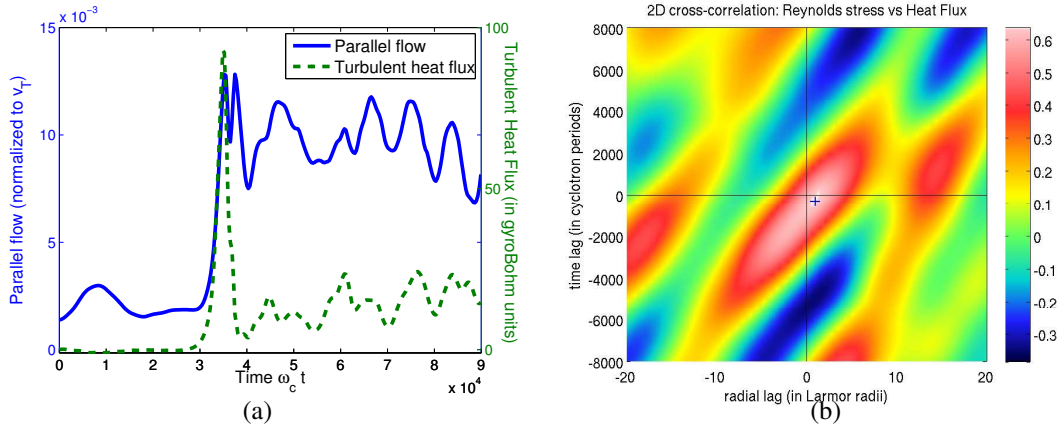

Figure 1: Nonlinear simulation with $\rho_* = 1/512$ and $\nu_* = 0.1$. (a) Time evolution of the flux-surface averaged toroidal flow and turbulent heat flux at mid-radius; (b) Cross-correlation in two dimensions (radius and time) of turbulent heat flux and Reynolds stress

turbulent heat flux at mid-radius is shown on Fig.1(a) From a near-zero initial value, the velocity grows exponentially during the linear growth phase of the instability and then fluctuates during the saturated turbulence regime. The exponential growth of the toroidal velocity generates a dipolar structure for toroidal momentum[3], reflecting the global conservation of momentum in the system. During the steady-state regime after the saturation of turbulence, one can investigate the statistical behavior of turbulent momentum transport. As presented in Fig.1(b), the cross-correlation of turbulent heat flux and Reynolds stress reaches values above 0.6, highlight-

ing a strong correlation between heat and momentum transport. Moreover, the clear elongated structures in Fig. 1(b) indicate that the avalanche-like events governing radial heat transport in flux-driven gyrokinetic simulations[2] also transport toroidal momentum.

Because the dynamics of toroidal momentum transport are governed by a local conservation equation, boundary conditions are the only possible source of *net* momentum generation. Experimentally, the issue of the impact on core rotation of scrape-off-layer (SOL) flows, which correspond to boundary conditions in core plasma simulations, is an active area of research[8].

In the simulations described above, boundary conditions are such that the toroidal flow is vanishing at the outer boundary ("no-slip") of the simulation domain and has a vanishing radial gradient at the inner boundary. These boundary conditions can be modified in GYSELA to assess the penetration of SOL and edge flows. We present here simulations at $\rho_* = 1/64$ where different values were imposed for the toroidal flow at the outer boundary, -0.1, 0 and 0.1 as apparent in Fig.2. For the three simulations, the initial profiles are plotted in dashed lines while the solid lines correspond to the steady-state profiles. It appears that, once steady-state has been reached, the three rotation profiles have different values but roughly the same shape inside $r/a \simeq 0.55$. In particular, this implies that the shear of the rotation profile in the core is comparable for all three simulations.
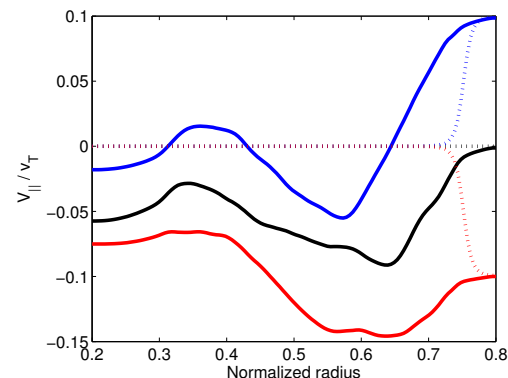

Figure 2: Final (solid) and initial (dashed) toroidal velocity profiles for three simulations with various boundary conditions ($\rho_* = 1/64$)

In conclusion, a local conservation equation for toroidal angular momentum is derived for the gyrokinetic model. This equation is verified numerically with the gyrokinetic full-f code GYSELA. Turbulent generation of intrinsic toroidal rotation, via the turbulent Reynolds stress, is observed in simulations. Toroidal momentum transport is found to be strongly correlated to heat transport and exhibits intermittent avalanche-like events. Finally, preliminary results suggest that edge flows set the magnitude of the toroidal rotation but have a reduced impact on its shear in the core.

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