Free Energy Transfers in Gyrokinetic Turbulence

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Abstract

Spectral analysis of plasma turbulence is of prime importance for assessing comparisons between experiments and numerical simulations [1]. Indeed, plasma turbulence is strongly connected to the energy confinement time, a key issue in thermonuclear fusion research. A major difficulty in the analysis of turbulence comes from the wide spectrum of scales that are dynamically active. In fusion plasmas, fluctuations appear at scales as small as the particle gyroradius up to scales of the order of the device size. Turbulent phenomena at different scales usually usually experience nonlinear interactions. As a consequence, microturbulence in a magnetized plasma, as described by the gyrokinetic formalism, is characterized [4] by a cascade of free energy in the plane perpendicular to the background magnetic field from the largest to the smallest scale. This cascade is similar to the direct cascade of kinetic energy in fluid turbulence [2, 3]. In the present study, recent gyrokinetic simulations are studied in details through the analysis of the free energy balance in Ion Temperature Gradient (ITG) driven turbulence. The spectra of both free energy injection due to the temperature gradient and free energy dissipation are presented. The nonlinear transfers due to the ExB drift term are analyzed in terms of free energy balance and are interpreted as a cascade of free energy from the largest scales dominated by the background temperature gradient to the smallest scales at which most of the dissipation takes place. Moreover, this cascade appears to be essentially local in the wave vector space. The nonlinear exchanges of free energy are indeed strongly dominated by mode-to-mode transfers between wave vectors with similar amplitudes. Finally, a limited self-similarity range has been identified in which the free-energy exchanges depend only on the ratio of the wave-vector amplitudes and not on the absolute values of these wave-vectors.

The Gyrokinetic Formalism

Gyrokinetic simulations take advantage of the helical motion of charged particles in presence of intense magnetic fields to simplify the numerical study of magnetized plasmas. In particular, in the limit of low frequencies compared to the gyrofrequencies related to this helical motion, a five dimensional (instead of a six-dimensional) velocity-position distribution function [5] can be used to describe the plasma. The reduction of the phase space dimension as well as the use of larger time steps are the major advantages of the gyrokinetic formalism in terms of numerical simulations.

The evolution equation for the distribution function f can be formally expressed as

$$\partial_t f = L[f] + N[f,f] + D[f], \tag{1}$$

where the linear term represents the influence of the fixed ion density (ω_{ni}) and temperature (ω_{Ti}) gradients, the effects due to magnetic curvature and the parallel dynamics. The linear dissipation term D represents the effects of the collisions and/or numerical hyperdiffusions. The nonlinear term N represents the effect of the self-consistent electric field in the $\vec{E} \times \vec{B}$ drift of charged particles.

Free energy balance equation

The nonlinear term in the gyrokinetic equation (1) has the property to conserve the free energy $\mathscr{E}$ [2]. It is proportional to

$$\mathscr{E} \propto \int \mathrm{d}\Lambda f^2, \tag{2}$$

where the integration over Λ has to be understood as a phase-space integration.

The evolution equation for the free energy is given by

$$\frac{\partial \mathscr{E}}{\partial t} \propto \int d\Lambda f \frac{\partial f}{\partial t} = \mathscr{G} + \underbrace{\mathscr{N}}_{0} - \mathscr{D} \tag{3}$$

in terms of the source and dissipative terms, given respectively by

$$\mathscr{G} \propto \int d\Lambda f L[f] \qquad \mathscr{D} \propto -\int d\Lambda f D[f]. \tag{4}$$

The free energy plays the same role in gyrokinetic turbulence as the kinetic energy in fluid turbulence.

Nonlinear transfer function

The transfer of free energy between different modes in the saturated turbulent state is induced by the nonlinear term. Indeed, even if it does not influence the global free energy balance equation, it can change, e.g., the value of this quantity associated with particular perpendicular wavenumbers. Following the procedure used for studying energy transfer in Navier-Stokes and in MHD turbulence [8], we decompose the perpendicular wavevector plane into domains and measure the free energy transfer between these domains. The set of domains $\{d_\ell\}$ is assumed to be a partition and all domains together cover the entire plane. The distribution function can then be written as a sum over all contributions for which the perpendicular wavevectors lie in the domain d_ℓ. As a consequence of the Parseval theorem, the free energy can also be split into parts which are associated to the domains d_ℓ: $\mathscr{E} = \sum_\ell \mathscr{E}^\ell$. The evolution of $\mathscr{E}^\ell$ due to the nonlinear term can be expressed as

$$\frac{\partial \mathscr{E}^\ell}{\partial t}\bigg|_{\mathscr{N}} \propto \int d\Lambda f^\ell \frac{\partial f}{\partial t}\bigg|_{\mathscr{N}} = \sum_{\ell'} T^{\ell,\ell'}, \tag{5}$$

where the two-domain interaction terms is defined as

$$T^{\ell,\ell'} \propto \int d\Lambda f^\ell N[f, f^{\ell'}]. \tag{6}$$

This two-domain interaction terms will be interpreted as the free energy transfers between the domains d_ℓ and $d_{\ell'}$. As a consequence of the nonlinear conservation of the free energy, it is easy to show that $T_f^{\ell,\ell'} = -T_f^{\ell',\ell}$, which reinforces the interpretation in terms of free energy exchanges. Indeed, if the domain d_ℓ is considered to receive a certain amount of free energy per unit of time $T_f^{\ell,\ell'}$ from the domain $d_{\ell'}$, then the domain $d_{\ell'}$ is seen as loosing exactly the same amount of free energy per unit of time in profit of the domain d_ℓ. The complete dynamical equation for $\mathscr{E}^\ell$ then reads

$$\frac{\partial \mathscr{E}^\ell}{\partial t} = \sum_{\ell'} T^{\ell,\ell'} + \mathscr{G}^\ell - \mathscr{D}^\ell. \tag{7}$$

Numerical results

The free energy transfer term defined above is now evaluated from a numerical simulation using GENE. The physical parameters employed in this context correspond to a widely used case of collisionless ion temperature gradient (ITG) turbulence known as the Cyclone Base Case [6]. The simulation domain is about 125 ion gyroradii wide in the perpendicular directions, and $256 \times 128 \times 16 \times 48 \times 16$ grid points are used in $(x, y, z, v_\parallel, \mu)$ space.

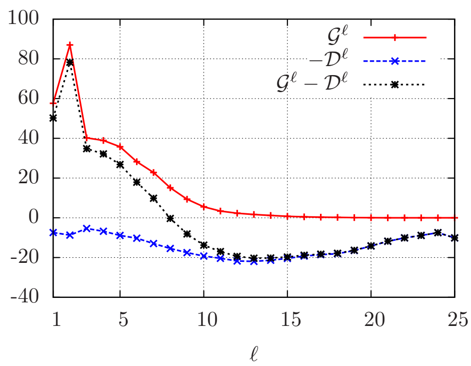

Figure 1: Shell decompositions in perpendicular wavenumber space of the drive ($\mathscr{G}^\ell$) and dissipation ($-\mathscr{D}^\ell$) terms (as well as their sum).

Fig. 1 shows the numerical results for the source and dissipation terms (averaged over time during the saturated phase of the simulation). As expected, the injection of free energy is well localized at low $k_\perp$. However, the dissipative terms are not just active in the high $k_\perp$ range, but throughout the entire $k_\perp$ spectrum, including the drive range. There is a net source of free energy up to shell $\ell = 8$ and a net dissipation beyond that.

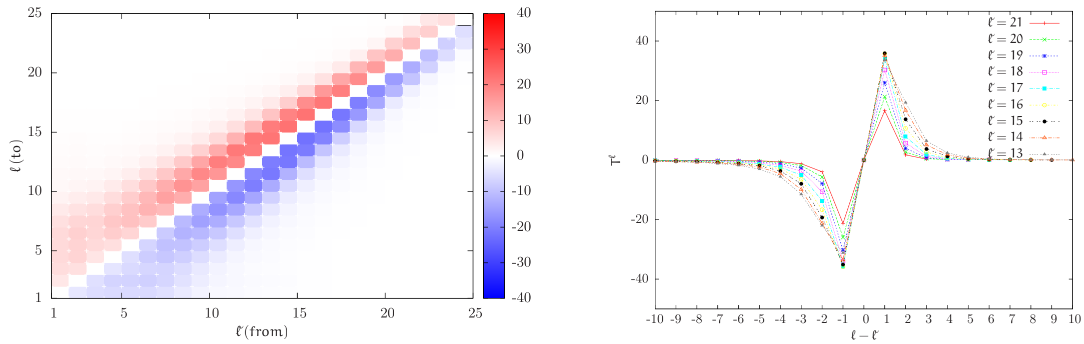

Figure 2: Shell-to-shell transfer in perpendicular wavenumber space of free energy (left). Total transfer T^ℓ(right) for various ℓ' as function of $(\ell - \ell')$ (right).

The corresponding shell-to-shell free energy transfer terms are shown in fig. 2(left), and various interesting features can be observed there. First, the free energy transfer is from the large scales to the small ones; the transfer is systematically negative for $\ell' > \ell$ and, due to the antisymmetry property, systematically positive otherwise. Second, the free energy transfer is very local in wavenumber space. Indeed, only values of $T^{\ell,\ell'}$ with ℓ close to ℓ' are significantly different from zero. In practice, for $|\ell - \ell'| > 5$ the free energy transfers almost vanish. This corresponds to a ratio of wave numbers between the two shells of the order of two. Third, a limited self-similarity range can be identified for ℓ between 13 and 20. Indeed, in this range, the transfer $T^{\ell,\ell'}$ seems to depend on $\ell - \ell'$ only, and not on the two indices separately. This is analyzed in detailed in fig. 2(right) where profiles of $T^\ell = T^{\ell,\ell'}$ for various ℓ' as function of $\ell - \ell'$ are shown. It can be observed that these functions collapse in a range of ℓ', which suggest the existence of a self similarity range.

References

[1] Ö. Gürcan *et al.*, Phys. Rev. Lett. **102**, 255002 (2009).
[2] A. A. Schekochihin *et al.*, Plasma Phys. Control. Fusion **50**, 124024 (2008).
[3] T. Tatsuno *et al.*, Phys. Rev. Lett. **103**, 015003 (2009).
[4] A. Bañón Navarro *et al.*, Phys. Rev. Lett. **106**, 055001 (2011).
[5] A. J. Brizard and T. S. Hahm, Rev. Mod. Phys. **79**, 421 (2007).
[6] A. M. Dimits, G., Phys. Plasmas **7**, 969 (2000).
[7] F. Jenko *et al.*, Phys. Plasmas **7**, No. 5, 1904-1910 (2000).
[8] A. Alexakis *et al.*, Phys. Rev. E **72**, 046301 (2005)