Impact of the Boussinesq approximation in tokamak scrape–off layer turbulence

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Introduction

Turbulent transport in the edge/SOL region is important regarding the confinement profiles of tokamak plasma. SOL turbulence is typically modeled by electrostatic fluctuations, consisting of fluctuations in the particle density of the charged species and electrostatic potential. Particle and charge conservation relations are used to determine the fluctuating flow–fields in most computational models of SOL turbulence [1, 2]. In gyrokinetic and fluid representations, extraction of the potential field from this equation is simplified by assuming that the background density gradient length scales are much larger than fluctuation scales, the so-called Boussinesq approximation. However, in practice there is no significant scale separation between the fluctuation and background gradient length–scales.

It has been reported that the Boussinesq approximation results in reduced velocities for isolated mesoscale coherent structures evolving in a quiescent and uniform background plasma [3]. Such an idealized situation is not directly applicable to the turbulent scrape–off layer (SOL) where structures propagate through fluctuating fields of magnitudes comparable to that of the structure.

Beginning with a linear analysis, we use nonlinear simulations, with the fluid code TOKAM–2D [1], to investigate the effect of Boussinesq approximation on isolated coherent structures and on SOL turbulent transport. It is found that in most cases of interest the Boussinesq approximation has a relatively weak effect on the overall turbulence behaviour. However, when the contrast between the density in the blob structure and that of the background plasma is large, hence governing strong density gradients, one observes a slowing down of the electrostatic potential evolution, in agreement with our analytical predictions.

Equations & Boussinesq approximation

We consider the growth rate of perturbations in SOL plasma in the presence of drift–waves, interchange drive and a parallel sheath closure described as follows

$$\begin{aligned}\left\{\frac{\partial}{\partial t}+[\phi,\cdot]-D_n\nabla^2\right\}n &= -\sigma n e^{\Lambda-\phi}+S_o(x)\\ \left\{\frac{\partial}{\partial t}+[\phi,\cdot]-\nu\,\nabla^2\right\}\Omega &= \ \sigma n\left(1-e^{\Lambda-\phi}\right)-g\frac{\partial n}{\partial y}\end{aligned} \tag{1}$$

where, $[\phi,\,n]=\hat{b}\cdot(\nabla\phi\times\nabla n)$ is the Poisson bracket, $\Omega=\nabla\cdot(n\nabla\phi)$ is a modified vorticity, and $S_o(x)$ is a particle source term. The effect of density fluctuations on the polarization current is commonly assumed to be negligible in comparison to the potential fluctuations; this is the Boussineq approximation that results in the following form of the vorticity equation,

$$\left\{\frac{\partial}{\partial t}+[\phi,\cdot]-\nu\nabla^2\right\}\nabla^2\phi=\ \sigma\left(1-e^{\Lambda-\phi}\right)-g\frac{1}{n}\frac{\partial n}{\partial y} \tag{2}$$

Linear Analysis

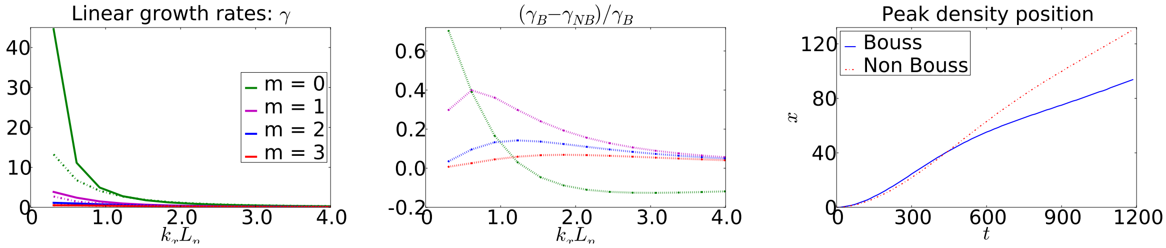
We consider the linear growth rate of perturbations over an equilibrium described as, $n_{eq}=n_0e^{-x/L_n}$, $\phi_{eq}=\Lambda$. Assuming scale separation between equilibrium and fluctuations (of the form, $\tilde{f}=\sum_{\vec{k}}f_{\vec{k}}\ \exp\left\{\gamma t+i\vec{k}\cdot\vec{x}\right\}$),we have the dispersion relation,

$$\left(\gamma+D_nk^2\right)\left\{\left(\gamma+\nu k^2\right)\left(1+il_b\right)+\sigma\right\}=\left(k_yL_n^{-1}+i\sigma\right)gk_y \tag{3}$$

where, $l_b=\ (k_xL_n)\,(kL_n)^{-2}$. The effect of the Boussinesq approximation on the above linear dispersion relation amounts to neglecting l_b, which only acts as minor modification under the regime of validity of our analysis ($kL_n\gg1$).

For equal coefficients for viscosity and diffusivity ($\nu=D_n$), Eq. 3, has the following roots,

$$\gamma=-\nu k^2-\frac{1}{2}\frac{\sigma}{k^2\left(1+il_b\right)}\left\{1\mp\left[1+\left(1+i\frac{\sigma L_n}{k_y}\right)\left(1+il_b\right)k_y^2k^2\frac{4g}{\sigma^2L_n}\right]^{1/2}\right\} \tag{4}$$



(a) (b)

Figure 1: Fig. 1(a): Perturbation growth rates from linear analysis for Boussinesq and Non–Boussinesq system of equations are shown as solid and dashed lines respectively in the plot on left. Fig. 1(b): Position of peak density in computations with an initial condition of an imposed density perturbation on uniform density.

In Figure 1(a) we compare the growth rates (Eq. 4) for the dimensionless parameters used in TOKAM turbulence simulations, $\sigma = 9.0810^{-4}$, $g = 1.14510^{-3}$, $D_n = \nu = 10^{-2}$, and $L_n = 100$. Clearly, from Fig. 1(a), the linear effect is maximum at low wavenumbers: $k_x \leq L_n^{-1}$ where our analysis is not valid. The poloidal mode, $k_y = 0$, appears to be affected even at large radial wavenumbers where the growth rates themselves are very small.

Comparison with blob studies

From Fig. 1(b) one confirms that the potential relaxation rate is faster under the Boussinesq approximation, resulting in the faster development of the dipole that propels the blob, and also a faster deformation of the blob. Here, we consider a density perturbation in a quiescent plasma that propagates radially under the influence of the curvature term. Our computations agree with previous studies, which reported that predicted blob velocities are lower under the Boussinesq approximation [3].

SOL Turbulence computation

We consider a 2-D periodic domain, and evaluate all spatial gradients using a spectral scheme, time evolution of density and vorticity is done using II-order predictor-corrector scheme [1]. The polarization current contribution to the charge conservation relation gives us the time evolution of a nonlinear vorticitiy ($\Omega = \nabla \cdot [n \nabla \phi]$) in Eq. 1. We use an iterative scheme to obtain the potential from the instantaneous density and vorticity fields,

$$\phi^{\text{new}} = \Delta^{-1} \left(\frac{\omega_{rel}}{n} - \frac{1}{n} \nabla n \cdot \nabla \phi^{\text{old}} \right) \tag{5}$$

An under–relaxation scheme is used in the present work, with the relaxation parameter, $\omega_{rel} = 0.8$. Since the terms on the right hand side are products of filtered quanities, there is some error due to filtering. The amplitude of the high–wavenumber terms is a thousand times smaller than the low–wavenumber terms, but the residual can not be eliminated. The iteration scheme is considered to be converged for $\left|\phi^{\text{new}} - \phi^{\text{old}}\right|_{max} \leq 10^{-4}$.

To verify whether the range of densities is not sufficiently large to allow for significant differences in density of structures with respect to the surrounding plasma, we consider a domain that is elongated along the radial direction with the sink term amplified eight times the expected values from fluid modeling ($\sigma = 9.081 \cdot 10^{-5}$), allowing for a larger background density gradient ($L_n \sim 100$).

Fig. 2 shows the mean values of poloidally averaged profiles for the fields after the simulation has progressed for $750,000$ since the perturbed initial state. Our region of interest is the region of exponential variation of density in the domain; being far from the source region, it allows us to observe the effect of Boussinesq approximation on steady-state turbulence in SOL. We find that the while density profile is not significantly affected, electrostatic potential appears to be weakly modified. However, fluctuation levels ($\langle\widetilde{n}\rangle_{rms}$, $\langle\widetilde{\phi}\rangle_{rms}$) are not significantly affected.

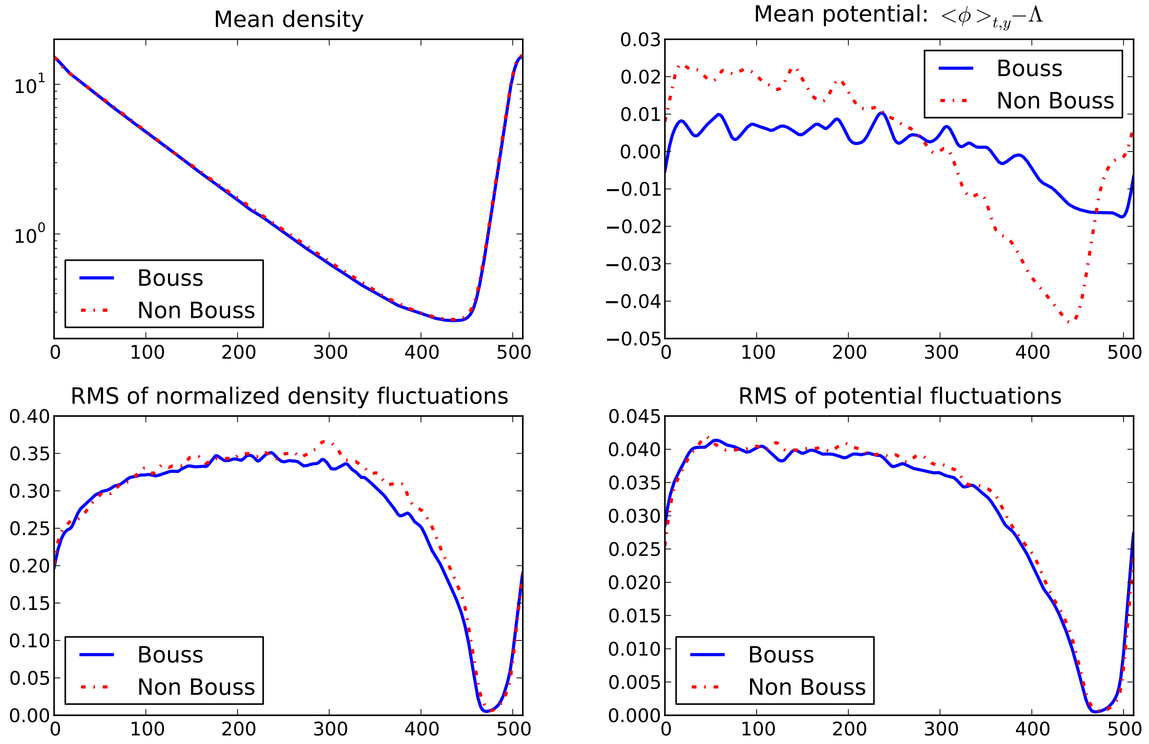


Figure 2: Profiles of poloidally and temporally averaged density and potential profiles, and their normalized fluctuation levels, plotted against the radial coordinate. The averages are computed from the fields obtained from 10,000 snapshots over 150,000 time steps.

Conclusions

Boussinesq approximation, as used in most 2-D fluid computations of SOL turbulence, is not strictly valid due to the presence of large density fluctuations. Even–though the evolution of isolated convecting structures is affected by the Boussinesq approximation, this effect is not significant for structures observed in the turbulent SOL. Consistent with the ordering procedure that is introduced to obtain the vorticity equation in both fluid and kinetic models, one finds that Boussinesq approximation has a weak impact on turbulence simulations.

References

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