Consequences of profile shearing on toroidal momentum transport

Y Camenen[1], Y Idomura[2], S Jolliet[2], A G Peeters[3]

[1] *IIFS/PIIM, CNRS - Université de Provence, Marseille, France*

[2] *Japan Atomic Energy Agency, Tokyo, Japan*

[3] *Universität Bayreuth, Bayreuth, Germany*

Introduction

Turbulent momentum transport in tokamaks is intimately linked to broken parallel symmetry (symmetry with respect to the midplane, following a magnetic field line). Amongst other mechanisms, radial inhomogeneity in the equilibrium can induce parallel symmetry breaking and, as a consequence, momentum transport. The residual stress (non-diagonal non-pinch momentum flux) resulting from background $\mathbf{E}\times\mathbf{B}$ sheared flows is a well known example of this effect. It is however not the only one: radial variation of the plasma profiles (temperature and density) and magnetic equilibrium, refered to as "profile shearing" in the following, will also result into parallel symmetry breaking. The impact of profile shearing on turbulent momentum transport has recently been investigated [1] and the main results are summarised here. The study is conducted in the linear regime, without background $\mathbf{E}\times\mathbf{B}$ flows, to focus on the physical mechanism description in preparation of a future quantitative study in the non-linear regime.

Impact of profile shearing on the linear eigenmode

The starting point of the present study is the impact of profile shearing on the global structure of ion-scale turbulence. It was demonstrated in the 90s that, to lowest order in $1/n$, where n is the instability toroidal mode number, and after expansion of the (slowly varying) equilibrium quantities around a rational flux surface, the linear eigenvalue problem reduces to a 1D differential equation. The most unstable mode is ballooned at the low field side (LFS) midplane and poloidally symmetric. To next order in $1/n$, however, the mode structure needs to accommodate the radial variation of the equilibrium (temperature, density, flows, magnetic field) and the 2D envelope results from the interplay between the shear in the mode frequency, the advection by plasma flows and the constraint given by the magnetic field (magnetic shear). As a consequence, the most unstable mode is in general no longer aligned with the radial direction but poloidally tilted and the radial envelope modified accordingly. Analytical expressions of the radial envelope and poloidal tilt have been derived in various limits. For instance, as shown in [2], assuming a linear pressure profile and no background flows, the radial envelope is a Gaussian whose width is given by $\Delta r = [2\gamma_0 \sin\theta_0/(k_\theta \hat{s}\omega_r')]^{1/2}$ with r the radial coordinate, $\hat{s}$ the magnetic shear at the flux surface r_0 under consideration and θ_0 the Bloch shift parameter related to the poloidal tilt of the mode. The Bloch shift parameter is defined as $\theta_0 = -k_r/(\hat{s}k_\theta)$ where k_r and k_θ are the radial and poloidal wave vectors at the LFS midplane in standard (r,θ) coordinates. The usual approximation $k_\theta \sim nq(r_0)/r_0$ for circular concentric flux surfaces is used. The eigenfrequency and the mode growth rate are assumed to have a form typical of electrostatic drift waves: $\omega_r = \omega_r(r_0) + \omega_r'(r-r_0)$ and $\gamma = \gamma_0 \cos\theta_0$ with γ_0 the growth rate of the lowest order solution. In the 2D envelope problem, the most unstable mode is then characterised by a finite poloidal tilt (or equivalently finite k_r):

$$\theta_0|_{\gamma_{max}} = -\mathrm{sign}(\hat{s}\omega_r') \left[\frac{\omega_r'}{2k_\theta \gamma_0 \hat{s}}\right]^{\frac{1}{3}}. \quad (1)$$

Interestingly, the 2D mode structure (poloidal tilt and radial width of the Gaussian) can be directly inferred from the dependencies of the lowest order eigenvalues. Eq. (1) shows that the poloidal tilt arises from the radial shear in the mode frequency ω_r', which for toroidal ion temperature gradient (ITG) mode is roughly proportional to the radial shear in the magnetic drift frequency $\omega_d = k_\theta T/(RZeB)$ with T the temperature, R the major radius, Ze the ion charge and B the magnetic field. Note that a temperature profile with constant gradient is sufficient to tilt the eigenmode, no curvature in the profile is required.

Local simulations: effect of the mode tilting on momentum transport

To investigate the effect of mode tilting on toroidal momentum transport, local gyrokinetic simulations are then performed with the flux-tube code GKW [3] for the Cyclone base case. The poloidal tilt θ_0 is varied by changing the input k_r (both quantities are related by $k_r = -\hat{s}\theta_0 k_\perp \rho_{\mathrm{ref}}$ with ρ_{ref} the reference ion Larmor radius). In the local approximation, the most unstable mode is at $\theta_0 = 0$ and the growth rate decreases when $|\theta_0|$ increases, as shown in the left plot of Fig. 1. To address momentum transport, the toroidal angular momentum flux is decomposed into diagonal, pinch and residual stress parts, as is customary, and the normalised ratio of the residual stress to the momentum diffusivity C^*/χ_φ is used to characterise the residual stress part (see [1] for more details). The most important impact of

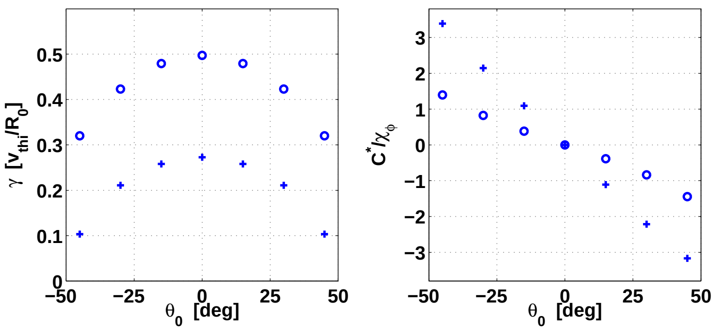

Figure 1: Mode growth rate and residual stress coefficient as a function of θ_0 in local simulations with adiabatic (crosses) and kinetic (circles) electrons.

a finite θ_0 in the local simulations is the generation of a residual stress whose sign depends on the sign of θ_0, right plot in Fig. 1. The residual stress is found smaller with kinetic electrons than with adiabatic electrons, but in both cases represents a significant contribution to the total momentum flux. The origin of the residual stress lies in the radial component of the vertical curvature and ∇B drift that is antisymmetric with respect to the midplane and leads to parallel symmetry breaking provided $\theta_0 \propto k_r$ is finite.

Global simulations: self-consistent mode structure and momentum transport

Motivated by these results, global simulations are performed with the code GT5D [4] to compute the poloidal tilt and the corresponding momentum flux self-consistently. A first set of simulations is performed for the global version of the Cyclone base case with adiabatic electrons at $\rho_* = \rho_{\rm ref}/a = 0.518 \times 10^{-2}$ and $n = 30$ (most unstable mode). The poloidal tilt of the electrostatic perturbations is found to increase with the normalised temperature gradient R/L_{Ti} (Fig. 2) and, as in the local simulations, it induces a significant residual stress (Fig. 3)

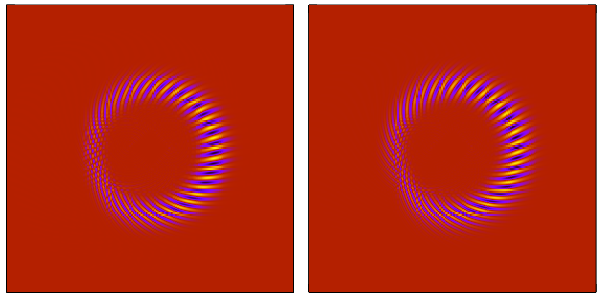

Figure 2: Electrostatic potential perturbations in the poloidal plane for $R/L_{Ti} = 6.6$ (left) and $R/L_{Ti} = 9.85$ (right).

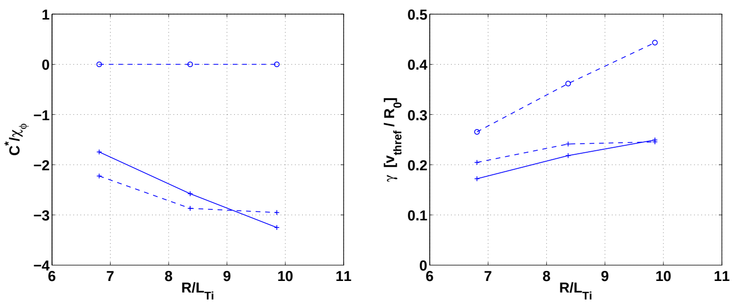

Figure 3: Residual stress C^*/χ_φ (left) obtained in global simulations (full line) and in local simulations (dashed line) at $\theta_0 = 0$ (circles) and at finite θ_0 (crosses), with the value taken from global simulations. The mode growth rates γ are shown in the right plot.

Comparable values of residual stress are obained by performing local simulations with the θ_0 values that were obtained in the global simulations, Fig. 3. This confirms that the residual stress obtained in the global simulations is mainly due to the mode structure modification by profile shearing. A second set of global simulations is performed with kinetic electrons at $\rho_* = 1.0359 \times 10^{-2}$ and $n = 15$, varying the normalised temperature to density gradient $\eta_i = L_n/L_{Ti}$. The simulations are performed with a reduced ion to electron mass ratio $m_i/m_e = 100$ to minimise the computational costs. For $\eta_i < 2$, the most unstable mode is a Trapped Electron Mode (TEM) while for $\eta_i > 2.5$ it is an Ion Temperature Gradient mode (ITG). The poloidal tilt of the electrostatic potential perturbations changes sign at the TEM/ITG transition, following the sign of the mode frequency shear.

The ratio of the momentum flux to the total heat flux (with zero rotation and the convective contributions subtracted) is shown as a function of η_i in Fig. 4. The direction of the residual stress changes from outward to inward at the TEM/ITG transition in close relationship with the change of θ_0. The global mode structure modification (poloidal tilt) by profile shearing is again the main mechanism generating the residual stress in the global simulations, as demonstrated by imposing a finite value of θ_0 in the local simulations, with the value of θ_0 taken from the global simulations. It is checked with GKW that using the actual ion to electron mass ratio does not significantly change the picture.

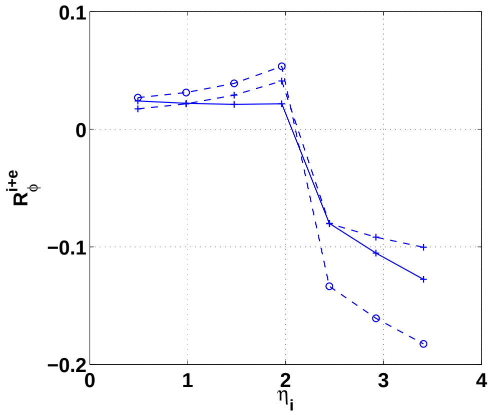

Figure 4: Momentum to heat flux ratio as a function of η_i for global simulations (full line) and local simulations (dashed line) with $m_i/m_e = 100$ (crosses) and the real m_i/m_e (circles).

Conclusions

The impact of profile shearing on momentum transport has been investigated in linear gyrokinetic simulations. The radial profile inhomogeneity modifies the global structure of the linear eigenmode and induces a poloidal tilt of the electrostatic potential perturbations. This effect is related to the shear in the mode frequency and is therefore also observed for linear profiles (constant gradient). The resulting parallel symmetry breaking induces a finite residual stress. Profile shearing residual stress changes sign at the TEM/ITG transition, following the sign of the mode frequency shear. It enhances co-current rotation in ITG turbulence and counter-current rotation in TEM turbulence. The magnitude of the effect (quasi-linear estimate) is very large (of the order of a couple of NBI beams!). The study now needs to be extended to the non-linear regime to provide a definite prediction of the magnitude.

References

[1] Y. Camenen, Y. Idomura, S. Jolliet, A.G. Peeters, Nucl. Fusion **51**, 073039 (2011)

[2] Y. Kishimoto *et al.*, Plasma Phys. Control. Fusion **41**, A663 (1999)

[3] A.G. Peeters *et al.*, Comput. Phys. Comm. **180**, 2650 (2009)

[4] Y. Idomura, M. Ida, T. Kano, N. Aiba, S. Tokuda, Comput. Phys. Comm. **179**, 391 (2008)