Probing the linear structure of toroidal drift modes.

D. Dickinson[1,2], C.M. Roach[2], H.R. Wilson[1]

[1] *York Plasma Institute, Dept. Physics, University of York, Heslington, York, YO105DD, UK*

[2] *Euratom/CCFE Fusion Association, Culham Science Centre, Abingdon, OX143DB, UK*

Introduction

Ballooning theory [1] has proven to be a useful tool in the study of high toroidal mode number, n, instabilities. The separation of scales between rational surface spacing and equilibrium length scales that exists at high n is exploited to perform an expansion in the small parameter, $1/nq'$, where q' is related to magnetic shear. In leading order ballooning theory, captured by local gyrokinetic flux tube simulations, the effects of radial profile variations are neglected, resulting in a reduction from the full 2D eigenmode problem to a 1D problem in ballooning space (distance along field line). This leading order theory introduces the ballooning angle, θ_0, as a free parameter which is typically selected to maximise the growth rate, usually at $\theta_0 = 0$ corresponding to modes ballooning on the outboard midplane. Global gyrokinetic simulations, which retain profile variations, have shown that linearly unstable toroidal drift modes, such as the ion temperature gradient (ITG) mode do not always peak at the outboard midplane [2], indicating that profile variations are sometimes important and $\theta_0 = 0$ isn't always the correct choice.

In this work the role of profile variations in determining the global mode structure is investigated. Full global (2D) solutions of a reduced (fluid) gyrokinetic ITG model are compared with solutions of the local (1D) ballooning transformed model. Higher order theory [3] indicates two classes of mode are expected, depending on equilibrium profiles, and both cases are found in global calculations. A procedure for obtaining the global mode structure and growth rate using only solutions of the local model is discussed and demonstrated. Finally the effect of linearly sheared flows is briefly investigated in the limit of low shearing rates.

Model equations

A simple gyrokinetic ITG model, used in the literature (e.g. see [4]), is given in Eq. 1

$$\Bigg[\underbrace{k_\theta^2\rho_s^2 + \rho_s^2\frac{\partial^2}{\partial x^2}}_{\text{F.L.R.}} - \underbrace{\frac{L_n^2}{R^2q^2k_\theta^2\rho_s^2\Omega^2}\left(\frac{\partial}{\partial\theta} + inq'x\right)^2}_{\text{Ion sound terms}} + \underbrace{\frac{\Omega-1}{\Omega+\eta_i}}_{\text{Eigenvalue}} - \underbrace{\frac{2L_n}{R\Omega}\left(\cos(\theta) + \frac{i\sin(\theta)}{k_\theta}\frac{\partial}{\partial x}\right)}_{\text{Coupling term}}\Bigg]\phi(x,\theta) = 0 \tag{1}$$

with ϕ the perturbed potential, k_θ the poloidal wavenumber, ρ_s the ion Larmor radius at sound velocity, safety factor q, R the major radius, Ω the complex mode frequency normalised to the electron diamagnetic frequency and $\eta_i = L_n/L_T$ the ratio of density and temperature length scales. It is advantageous to Fourier decompose in poloidal mode number, m;

$$\phi(x,\theta) = \sum_m u_m(x)\exp(im\theta) \quad (2)$$

The global 2D model in Eq. 1 reduces to a set of coupled equations for the radial functions, $u_m(x)$. In the limit $n \to \infty$ the rational surface spacing reduces to 0 whilst equilibrium scale lengths remain unchanged. The solution on each rational surface will then be identical, meaning each u_m can be represented in terms of a generic function u_0:

$$u_m(x) = u_0\left(x - m/nq'\right)\exp(im\theta_0) \quad (3)$$

The ballooning parameter, θ_0, contains amplitude and phase information due to finite radial variations. Fourier transforming u_0 using:

$$u_0(x) = (2\pi)^{-1}\int_{-\infty}^{\infty} v(\eta)\exp\left(-inq'x\eta\right)d\eta \quad (4)$$

and substituting into Eq. 1 (equivalent to applying the ballooning transform) leads to the lowest order (local) ballooning representation of the model as given in Eq. 5, with magnetic shear s, which can be solved for the local mode frequency $\Omega_0(x,\theta_0)$ and the eigenfunction $v(\eta)$ for a given x and θ_0.

$$\left[\rho_s^2\eta^2 + \frac{L_n^2}{R^2q^2k_\theta^2\rho_s^2\Omega_0^2}\frac{d^2}{d\eta^2} + \frac{2L_n}{R\Omega_0}\left(\cos[\eta-\theta_0] + s\eta\sin[\eta-\theta_0]\right) + \frac{\Omega_0 - 1}{\Omega_0 + \eta_i}\right]v(\eta) = 0 \quad (5)$$

Two codes have been developed: a 2D code solves eqn. 1 (for ϕ and Ω), and a 1D code solves eqn. 5 (for $v(\eta)$ and Ω_0). The x dependence of Ω_0 arises from profile variations.

Higher order theory and global mode structure

In higher order ballooning theory profile effects are retained. These restrict the choice of x and θ_0 allowed in Eq. 5. Two distinct classes of mode are then found, known as *isolated* and *general* modes [3]. *Isolated* modes are only valid at a stationary point in Ω_0 whereas *general* modes can be found everywhere else.

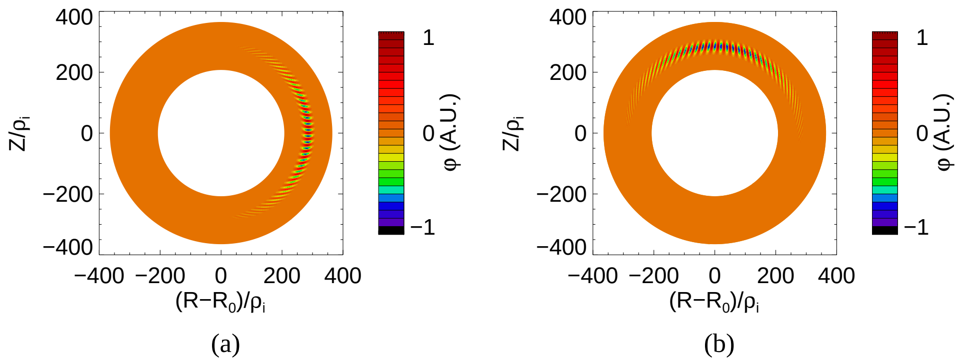

Figure 1: Contour of perturbed potential for *isolated* mode (a) and for *general* mode (b) obtained using quadratic and linear η_i profiles respectively.

In the simple model used here the profile shape of Ω_0 matches that of the profile of η_i, which is the ITG drive term. By selecting a peaked quadratic η_i profile, a stationary point in Ω_0 is obtained and *isolated* modes are found by the 2D code. Switching to a linear η_i profile gives only *general* modes, as there is no longer a stationary point in Ω_0.

Calculations have been performed with $k_\theta \rho_s = 0.3$, $n = 50$, $65 \leq m \leq 115$, $s = 2$ and η_i given by $5 - 312.5x^2$ or $5 - 24x$. Figure 1 shows the global mode structure for both classes of mode. The *isolated* mode peaks at the outboard midplane and has the largest possible growth rate, γ, whereas the *general* mode peaks at the top of the plasma with a reduced γ. The 1D code's γ_0 agrees with that found by the 2D code if $\theta_0 = 0$ is used for the *isolated* case and if one averages over θ_0 for the *general* mode (as predicted in [3]).

Converting from 1D to 2D

The local model of eqn. 5 neglects profile effects but if an expression for θ_0 is known $u_0(x)$ can be obtained from the solution of eqns. 4 and 5 and the full 2D global mode structure, ϕ, can be determined. θ_0 can be obtained by using higher order ballooning theory to invert model expressions for $\Omega_0(x, \theta_0)$ [3]. Expressions for *isolated* and *general* modes are given in eqns. 6 and 7 respectively, obtained by inverting the simple models in eqns. 8 and eqn. 9. For given equilibrium profiles the parameters λ and ε can be obtained by fitting to $\Omega_0(x, \theta_0)$ derived from the 1D code. Figure 2 shows $\Omega_0(x, \theta_0)$ for the quadratic and linear η_i profiles described above. The 1D solution was obtained at each x and θ_0 value from eqn. 5 to provide $\Omega_0(x, \theta_0)$.

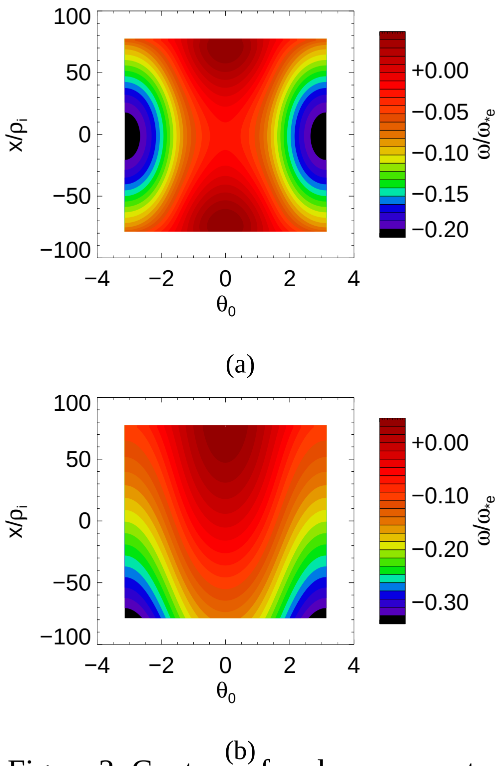

Figure 2: Contour of real component of $\Omega_0(x, \theta_0)$ for quadratic (a) and linear (b) η_i profiles.

$$\theta_0 \approx \cos^{-1}\left(1 - x^2\lambda/\varepsilon\right) \quad (6) \qquad \Omega_0(x, \theta_0) = \Omega_{00} + \lambda x^2 + \varepsilon \cos(\theta_0) \quad (8)$$

$$\theta_0 \approx \cos^{-1}\left(-x\lambda/\varepsilon\right) \quad (7) \qquad \Omega_0(x, \theta_0) = \Omega_{00} + \lambda x + \varepsilon \cos(\theta_0) \quad (9)$$

The global mode structure reconstructed following this procedure using only results from the 1D code shows excellent agreement with the mode structures obtained from the global code shown in figure 1.

The effect of sheared flows

The effect of sheared plasma flows can be incorporated into the global model by introducing a radially dependent Doppler shift to Ω. This introduces an additional equilibrium profile. Figure 3 shows γ for both classes of mode, with and without sheared flows, as the strength of the coupling term in eqn. 1 is increased (equivalent to going from a cylindrical system to the large aspect ratio torus used above). The growth rate of the *isolated* mode is reduced by flow shear to the cylindrical value whilst the *general* mode's growth rate is always at the cylindrical value. This effect on the *isolated* mode can be attributed to the removal of the stationary point in Ω_0 by the linear Doppler shift, keeping a *general* mode and prohibiting the formation of an *isolated* mode.

Conclusions

The global mode structure of a simple ITG mode has been investigated for different equilibrium radial profiles. Two classes of mode are obtained, in agreement with higher order ballooning theory [3]. In our model equilibrium *isolated* and *general* modes have similar global mode structure, but *isolated* modes peak at the outboard midplane and *general* modes peak at $\theta = \pi/2$. Solutions to the corresponding local model where also investigated. Agreement is found between the global and local growth rates for both classes of mode provided the correct ballooning angle, θ_0, is used in each case. A procedure for obtaining the full global mode structure only from solutions of the local model was illustrated, giving excellent agreement with the full global results. Weak linearly sheared flows were found to have a strong stablising effect on the *isolated* mode growth rate but no effect on the *general* mode. This can be attributed to sheared flow destroying the conditions required to obtain *isolated* modes. The proper consideration of profile effects, neglected in local analyses, can be important in determining the expected linear growth rates. The effect for non-linear studies remains an area of further work.

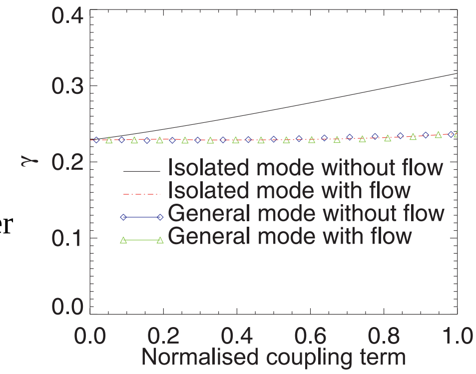

Figure 3: Growth rate as a function of normalised coupling term, χ, (where $2L_n/R \rightarrow 2\chi L_n/R$ in eqn. 1) for *isolated* and *general* modes with and without sheared flow.

References

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This work was part-funded by the RCUK Energy Programme under grant EP/I501045 and the European Communities under the contract of Association between EURATOM and CCFE. The views and opinions expressed herein do not necessarily reflect those of the European Commission.