Estimation of the electron temperature in the plasma of a Hall thruster

J. Cavalier[1], N. Lemoine[1], S. Tsikata[2], C. Honoré[2], D. Grésillon[2], G. Bonhomme[1]
[1]*Institut Jean Lamour, UHP, 54506, Vandœuvre Les Nancy, France*
[2]*LPP, Ecole Polytechnique, route de Saclay, Palaiseau, France*

An instability known as the beam-cyclotron instability [1] leads to the growth of a longitudinal wave. This occurs when a beam is incident on a magnetized plasma in a direction orthogonal to the magnetic field. Similarly, such instability can be found in Hall thrusters [2], where the electrons drift azimuthally due to an axial electric field and a crossed radial magnetic field.

A study of the 2D dispersion relation (wave vector orthogonal to the magnetic field) has shown a mode which exists only for a discrete number of wave vectors [3]. However, recent measurements by collective light scattering have pointed out that the wave vector has a component in the magnetic field direction and that the associated dispersion relation is continuous. Accordingly, we propose here to study the dispersion relation in the 3D case.

I. The dispersion relation

The plasma at the exit of a Hall thruster may be approximated as collisionless, subject to a uniform magnetic field $\vec{B}_0$ parallel to the z-axis and an uniform electric field $\vec{E}_0$ parallel to the x-axis, in a slab geometry. The electrons are magnetized while the ions are not. In the case of a Maxwellian distribution for both ions and electrons, the kinetic theory leads to:

$$1+\frac{1}{k^2\lambda_D^2}\left[1+g\left(\omega,k,V_d,v_{the},\omega_{ce}\right)\right]-\frac{1}{2k^2\lambda_{Di}^2}Z'\left(\frac{\omega-k_x v_p}{\sqrt{2}k v_{thi}}\right)=0 \quad (1)$$

where $g\left(\omega,k,V_d,v_{the},\omega_{ce}\right)$ is the Gordeev function [4]:

$$g\left(\omega,k,V_d,v_{the},\omega_{ce}\right)=\frac{\omega-k_y V_d}{k_z v_{the}\sqrt{2}}e^{-\left(\frac{k_\perp v_{the}}{\omega_{ce}}\right)^2}\sum_{m=-\infty}^{+\infty}Z\left(\frac{\omega-k_y V_d-m\omega_{ce}}{k_z v_{the}\sqrt{2}}\right)I_m\left(\left(\frac{k_\perp v_{the}}{\omega_{ce}}\right)^2\right) \quad (2)$$

and $Z(x)$ is the plasma dispersion function, $Z'(x)$ its derivative and $I_m(x)$ the modified Bessel functions of first kind. ω, k and $k_\perp=\sqrt{k_x^2+k_y^2}$ are the pulsation, the wave vector and the orthogonal wave vector respectively. $V_d=\frac{E_0}{B_0}$, v_p, v_{the} and v_{thi} are the drift, the beam the electron thermal and the ion thermal velocities. λ_D and λ_{Di} are the electron and ion Debye length.

In order to find the parameters of interest, equations (1) and (2) are normalized to λ_D, ω_{pi} the ion plasma pulsation, and $c_{s,0} = \sqrt{\frac{k_B T_e}{M_i}}$ the sound speed. $\hat{M} = \frac{M_i}{m_e}$ (=2.4e^4 for Xenon) and $\hat{T} = \frac{T_i}{T_e}$ are introduced and the parameters become $\hat{k} = k\lambda_D$, $\hat{\omega} = \frac{\omega}{\omega_{pi}}$, $\hat{V}_d = \frac{V_d}{c_{s,0}}$, $\hat{v}_p = \frac{v_p}{c_{s,0}}$, $\hat{\omega}_{ce} = \frac{\omega_{ce}}{\omega_{pi}}$.

Thus, equations (1) and (2) transform to:

$$1 + \frac{1}{\hat{k}^2}\left[1 + g\left(\hat{\omega}, \hat{k}, \hat{V}_d, \hat{\omega}_{ce}, \hat{M}\right)\right] = \frac{1}{2\hat{k}^2\hat{T}} Z'\left(\frac{\hat{\omega} - \hat{k}_x \hat{v}_p}{\sqrt{2\hat{T}}\hat{k}}\right) \quad (3)$$

$$g\left(\hat{\omega}, \hat{k}, \hat{V}_d, \hat{\omega}_{ce}, \hat{M}\right) = \frac{\hat{\omega} - \hat{k}_y \hat{V}_d}{\hat{k}_z\sqrt{2\hat{M}}} e^{-\hat{M}\left(\frac{\hat{k}_\perp}{\hat{\omega}_{ce}}\right)^2} \sum_{m=-\infty}^{+\infty} Z\left(\frac{\hat{\omega} - \hat{k}_y \hat{V}_d - m\hat{\omega}_{ce}}{\hat{k}_z\sqrt{2\hat{M}}}\right) I_m\left(\hat{M}\left(\frac{\hat{k}_\perp}{\hat{\omega}_{ce}}\right)^2\right) \quad (4)$$

It is important from equations (3) and (4) to notice that $\hat{v}_p$, $\hat{V}_d$, $\hat{\omega}_{ce}$ and $\hat{T}$ are the parameters of interest. Their consequence on ω will be studied in the next section after solving equation (3).

II. Numerical solving of the dispersion relation

Equation (3) is a rather complicated differential equation, not easily solved. However, an approximation can be made using results from collective scattering [2]. In fact, the pulsation has found to be $\omega = 6.10^7 rad/s$ at maximum for a corresponding wave vector of $k_y = 12000 rad/m$. On the other hand, the drift velocity and the electron cyclotron frequency are estimated to be $V_d = 7.10^5 m/s$ and $\omega_{ce} = 3.10^9 rad/s$. From these results, it is clear that $k_y V_d \gg \omega$ and $|k_y V_d + m\omega_{ce}| \gg \omega$ for $m>0$. Whereas for $m<0$, there is some ambiguity depending on the exact value of k_y, V_d and ω_{ce} yet the approximation is still considered. Thus, the Gordeev function (4) is now constant in ω and equation (3) reduces to $Z'(x) = \text{cst}$. The inverse Z' function can be found, by using a method which is not described here, allowing to calculate ω. Once calculating, ω is reintroduced in equation (4) and recalculated in order to increase the accuracy.

Collective scattering measurements indicate a linear dependence of ω with k leading to a group velocity equal to *3410m/s*, not far from the expected sound speed $c_{s,0}$. In order to get such a curve, a large value of $\hat{k}_z$ is required and *0.03* will be taken. $\hat{k}_x$ can be set as null while $\hat{k}_y$ is the variable and usually lies between *0* and *1*. The contribution of v_p to the pulsation is a

Doppler shift and has a small effect on the growth rate. Consequently, this quantity does not affect the group velocity and is considered null.

The dependence of the other parameters on the group velocity is studied by linearly fitting $\hat{\omega}(\hat{k}_y)$ for $\hat{k}_y \in [0.05; 0.4]$. The fitted slope called v_g is plotted versus the drift velocity $V_{d,num}$ and for different ω_{ce} in Fig. 1. The ratio of the temperature T was fixed to *0.06*.

From Fig. 1, it is clear that v_g is quite constant and close to unity with V_d and ω_{ce}. However, as the dispersion relation is less linear when ω increases to highest values of ω_{ce}=*45-55*, v_g is seen to differ from one. Because of the linearity of the dispersion relation observed by collective scattering, values higher than *45* will be eliminated. Accordingly, the slope is found to be constant with the V_d and ω_{ce} and to only depend on T.

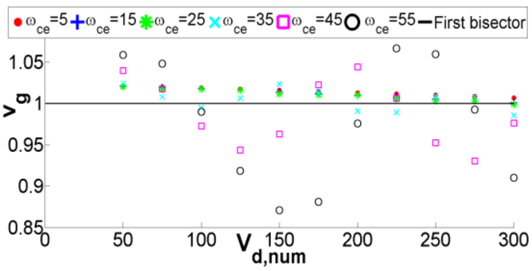

Fig. 1. Variation of the group velocity v_g with the drift velocity V_d for different electron cyclotron pulsation ω_{ce}. v_g is constant with V_d and only differs from *1* for high ω_{ce}.

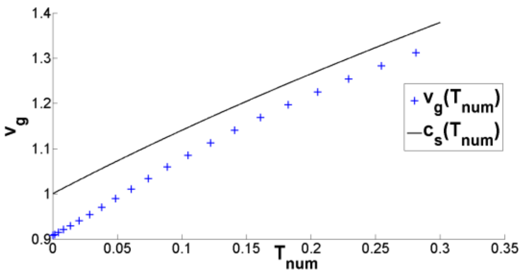

Fig. 2. Variation of the group velocity v_g with the parameter T_{num}. In the thruster, T_{num} is expected to be small, so v_g=0.9c_s.

The dependence of v_g as a function of T is shown on Fig. 2. The ion temperature in the azimuthal direction was found [5] to be low (*0.06eV*) compare to the electron temperature (*15eV*). Thus, the ratio T is believed to be small and v_g can be equal to $0.9 \times c_s$. Consequently, the ratio of the temperature can be calculated, knowing the experimental group velocity. In the next section, a model will be developed and should presumably allow determining the electron density n_e.

III. The analytical model: evanescent magnetic field and cold ions

In the limit of a zero magnetic field, Schmitt [6] has demonstrated that equation (3) should be replaced by ξ Z(ξ), where $\xi = -\frac{V_d}{v_{the}}$. The mandatory conditions are that $-\frac{kV_d}{\omega_{ce}}$ and $\frac{kv_{the}}{\omega_{ce}}$ tend to infinity while ξ remains constant. These limits are not fully justified but are a starting point. In addition, a more justified limit can be taken: the cold ions limit. In that case, the right hand side of equation (3) can be re-written as $\frac{2\hat{T}\hat{k}^2}{\left(\hat{\omega} - \hat{k}_x \hat{v}_p\right)^2}$. Taking these two limits into account, ω can be obtained analytically depending on the parameters T_e, n_e and V_d.

Using this model, equation (3) can be fitted with V_d being the fitted parameter. T and v_p are taken as null. k_x and k_z are set as null and *0.03* respectively while k_y vary from *0.1* to *1*. The fitted value $V_{d,model}$ is plotted versus the input values $V_{d,num}$ on Fig. 3 for differen values of ω_{ce}. Except for small values of ω_{ce}, $V_{d,model}$ is not far from the first bisector, indicating that the model fits quite well equation (3). Hence, by un-normalizing the model, a fit of the experimental dispersion relation would allow to determinate T_e and n_e knowing V_d.

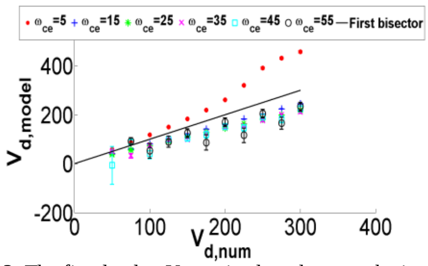

Fig. 3. The fitted value $V_{d,model}$ is plotted versus the input value $V_{d,num}$ for different ω_{ce}. $V_{d,model}$ is similar than $V_{d,num}$ for most of ω_{ce}.

IV. Comparison to experiment

A dispersion relation obtained by collective scattering for the azimuthal mode is presented on Fig. 4. Its fit gives a group velocity of *3410* m/s. Therefore, using the Xenon mass (2.175×10^{-25} *kg*), a value of *20eV* is found for the T_e using results from part II. Unfortunately, the electron density cannot be estimated with the model from part III because no curvature is visible. However, recent measurements which are not yet fully understood have shown a saturation of the frequency and would allow measuring n_e.

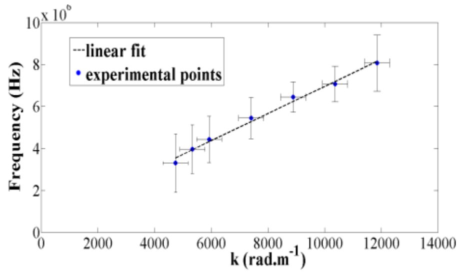

Fig. 4. Dispersion relation obtained by collective scattering for the azimuthal mode and its fit.

V. Conclusion

The 3D dispersion relation has been solved numerically and its initial slope is shown to be equal to $0.9\times c_s$. A model was developed to fit equation (3) in order to find T_e and n_e. Collective scattering measurements allow measuring group velocity and T_e is estimated to be equal to *20eV* neglecting T_i. Unfortunately, n_e cannot be found because no curvature is visible.

The authors would like to thank CNES and Snecma for their financial support.

References

[1] M. Lampe et al, Phys. Rev. Lett., Vol. 26, N°20, pp 1221-1225 (1971)
[2] S. Tsikata et al, Phys. Plasmas, Vol. 16, 2009, pp. 033506-0335006(10)
[3] A. Ducrocq et al, Phys. Plasmas, Vol. 13, 2006, pp. 102111-102111(8)
[4] G.V. Gordeev, JEPT (USSR), 6, 660 (1952)
[5] G. Bourgeois et al, Phys. Plasmas, Vol. 17, 2010, pp113502-113502(6)
[6] Schmitt et al, Phys. Plasmas, Vol. 12, 1974, pp. 51-59