Fluctuation-induced electron mobility in the observed turbulent E-field of the Hall effect thruster

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In Hall thruster plasmas[1] the $E_o \times B$ fluctuations are now well documented [2,3]. A model is investigated of the cross B-field particle motion that results from these fluctuations. The diffusion and mobility coefficients are calculated from the fluctuation spectrum characteristics, and the mobility variations with the fluctuation wave number and the electron velocity are shown. This model is applied to the actual fluctuation characteristics that were experimentally found by collective scattering[4]. The mobility coefficient thus obtained is consistent with the so-called « anomalous » mobility expected from discharge fit models.

1. Trajectory and geometry for a single wave

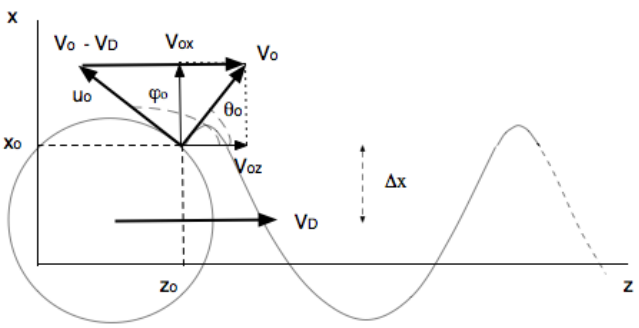

Figure 1: Zero-order ExB drift trajectory

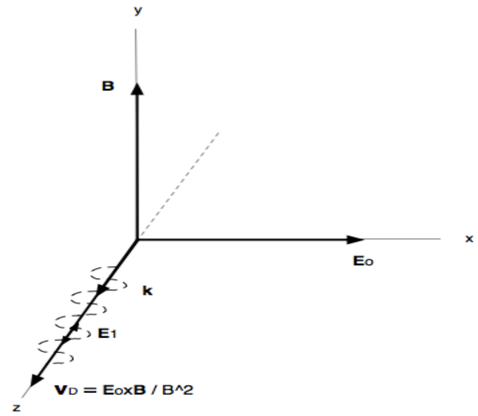

Figure 2: Wave geometry

A perturbing longitudinal wave $\bar{E}_1$ propagates in the direction of the $\bar{E}_o \times \bar{B}$ drift. The motion is

$$m\frac{d\bar{u}}{dt} = q[\Re\{E_1 e^{i(kz-\omega_1 t)}\}\vec{e}_z + \bar{u} \times \bar{B}] \tag{1}$$

where $\bar{u}$ is the velocity in the drift-reference frame. Eq. 1 is non linear, but can be solved by a perturbation expansion (in term of power of E_1). It induces an oscillating motion along $\bar{E}_o$.

2. Electron mobility in a continuous spectrum of waves

If instead of a single wave, a continuous spectrum of such $E_1(k,\omega)$ waves is added, random motion is obtained along the E_o direction as well as a second order constant velocity along E_o. The resulting mobility coefficient of an electron of cyclotron radius ρ_c $(\rho_c = v_\perp/\omega_c)$ is a function of the drift cyclotron wave radius $(\rho_D = v_D/\omega_c)$

$$\mu_{tu} = -\frac{\pi}{4B^2 E_o}\Sigma_N \int dkkS_E(k,\omega = kv_d + N\omega_c)\Sigma_n J_n^2(k\rho_c)[\frac{dJ_{n-N}^2(z)}{dz}]_{z=k\rho_D} \quad (2)$$

The mobility of a distribution of Maxwell electrons with thermal velocity v_{th} ($\rho_{cth} = v_{th}/\omega_c$) can be obtained from the diffusion coefficient (first order expansion) and the Einstein relation between diffusion and mobility coefficients. This velocity averaged mobility is

$$\mu_m = \frac{\pi e}{2KT_e B^2}\Sigma_N \int dkS_E(k,\omega = kv_D + N\omega_c)e^{-(k\rho_{cth})^2}\Sigma_n I_n(k\rho_{cth})J_{n-N}^2(k\rho_D) \quad (3)$$

3. Transport fine structure

The sum of Bessel functions products in the integrands of Eq. 2 and 3 are shown below, as a function of the wave number (normalized to $k_D = \rho_D^{-1}$) and of the velocity (normalized to v_D)

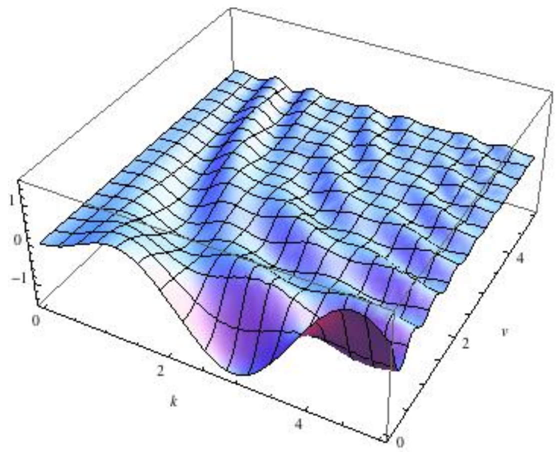

Figure 3

Mobility factor μ_{tu} of a single electron of velocity v

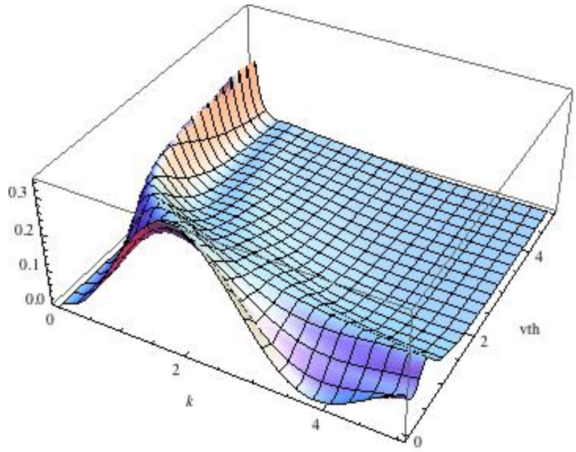

Figure 4

Mobility factor μ_m of a Maxwell electrons population

The single electron mobility μ_{tu} as a function of v and k (Fig 3) is strongly modulated. Its amplitude is largest when v is small. It depends mostly on $k\rho_c$. The mobility can be of either sign. The velocity averaged mobility μ_m (Fig. 4) is more regular, positive, also maximum for small thermal velocity and along a $k\rho_c \sim 1$ line.

4. Experimental form factor measurements

Fluctuation spectrum quantitative data have been obtained from two campaigns of collective scattering experiments on the PPS-X000 thruster in the PIVOINE facility[4]. The spectrum characteristics are threefold: the form factor $\bar{k}$-vector-space distribution $S(\bar{k})$, the k-wave number spectrum $S(k)$, and the dynamic form factor $S(k,\omega)$. These three structure factors are successively shown below (Figures 5-7). The largest fluctuation form factor amplitude lies near to the $E_o \times B$ direction ($k_x = k_z = 0$) but its direction is significantly biased off this $E_o \times B$ axis by ~10° towards the anode and ~4° along the B-field direction (Fig. 5).

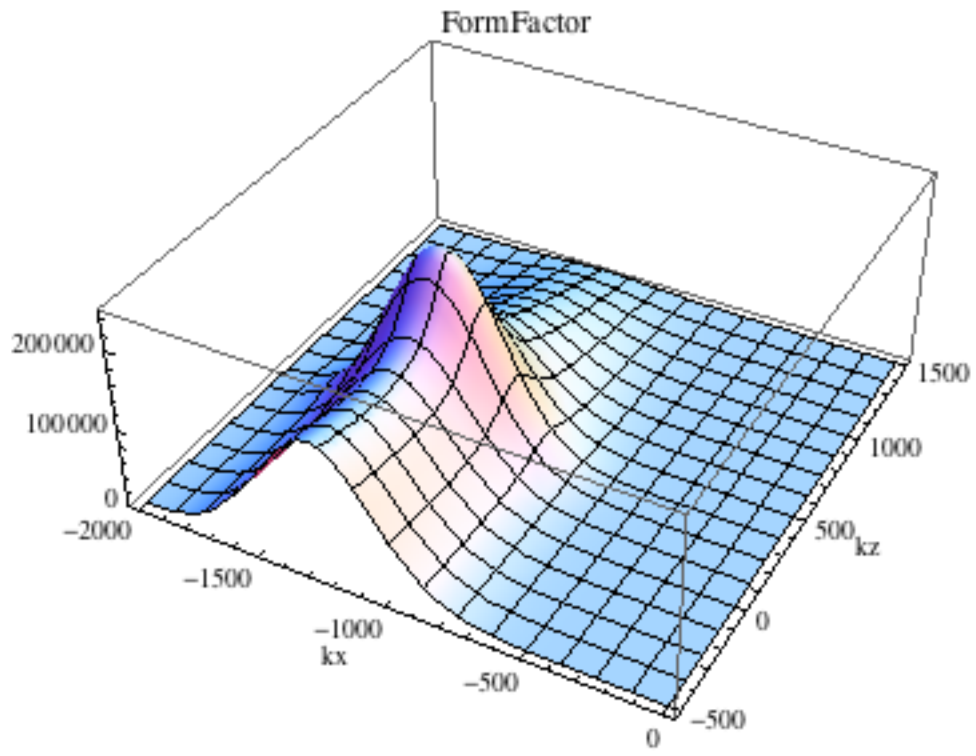

Figure 5

The form factor space distribution as a function of k components k_x (along E-field) and k_z (along B-field), when k=5190 rad/m.

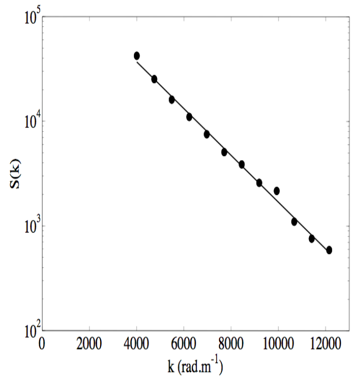

Figure 6: Form factor k-distribution

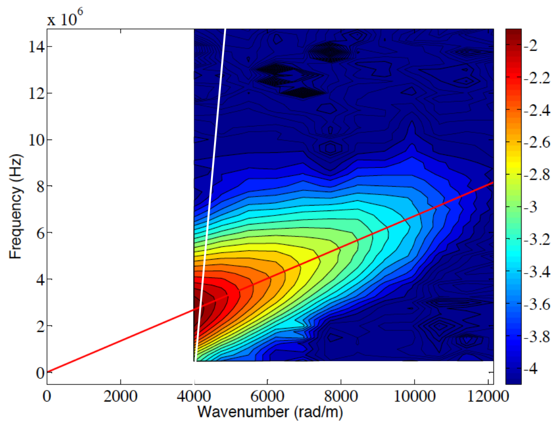

Figure 7: Dynamic form factor

The form factor amplitude decays nearly exponentially as a function of k (Fig. 6). The decay rate is of the order of the cyclotron radius and the maximum amplitude at low k's can be as large as 2.10^5. The dynamic form factor S(k,ω) (Fig. 7), at each wave number, is a broad frequency line, its mean frequency increases with k at a group velocity of the order of the ion acoustic speed. The white, almost vertical line is a sketch of the relation *($\omega = k\, v_D - \omega_c$).*

5. Absolute "anomalous" mobility measurement

The experimental conditions are such that

$KT_{the}/e \sim 20$ eV ; $v_D \sim 7.10^5$ m/s ; $n_o = 3.10^{17}$ m^{-3} ; $k_D \sim 4.10^3$ rad/m

Results shown in §4 provide the necessary data for the mobility of Eq. 3:

from Fig. 5, $S(k_D) = 200\,000$; $\Delta k_x \sim \Delta k_z \sim k_y/10$

from Fig. 7, the integration along the $(\omega = k\, v_D - \omega_c)$ path is seen as a frequency integral providing the form factor: $\int dk\, S_E(k, \omega = k\, v_D - \omega_c) \sim S_E(k_D)/v_D$ (4)

The mobility factor (Fig. 4) is taken at $v_{the}/v_D \sim 3$ on the maximum when $k/k_D \sim 0.3$ (i.e. $k/k_{ce} \sim 1$). The mobility factor is there ~0.2.

In addition the Boltzmann equilibrium is also assumed to provide a relation between the measured *electron density* S_n - and the required *E-field* S_E - spectral densities,

$S_E(k) \sim (k\, KT_e/e)^2\, S_n(k)/n_o$ (5)

These lead to the following Gauss-distribution averaged mobility and diffusion coefficient:

$\mu = 0.2\ m^2/V.s$; and $D_{xx} = (KT/en_o)\, \mu = 4\ m^2/s$,

6. Comparison with global equilibrium mobility measurement

The electron mobility space distribution in Hall thruster discharge was estimated from fit models[5],[6]. The electron mobility in front of the channel (where collisions are scarce) was found to be $\mu > 0.1\ m^2/V.s$, and qualified as "anomalous" since it is one or two order of magnitude above the collision induced mobility. The present microscopic model, together with collective scattering measurements, quantitatively identify the so-called "anomalous" transport to the fluctuation induced transport.

Conclusions

The fluctuation-induced mobility was shown to be of the expected magnitude. It is also shown to be a sensitive function of electron temperature and wavelength that is worth investigating.

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