Dynamics of mode number bicoherence computed from two-dimensional data of a magnetized plasma column

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Mode coupling processes and nonlinear energy transfers play an important role in the organization of plasma turbulence. To detect mode combinations that can couple nonlinearly, the matching conditions for the different fluctuating components must be tested. This can be done by computing higher order spectra, like the bispectrum and its normalized form, the bicoherence. Those quantities are primarily computed in the frequency domain but methods of computation in the wave number domain have been developed. One of those methods uses wavelet analysis on a single set of spatial data to obtain a bicoherence related quantity with a high temporal resolution[1]. Another method computes the bicoherence for combinations of mode and frequency by applying Fourier analysis to spatio-temporal data[2].

In experimental devices, spatio-temporal data of fluctuations are usually provided by multiple probe arrays and especially azimuthal probe arrays. Recently, fast camera imaging emerged as an alternative diagnostic to obtain two-dimensional time-resolved measurements of electron plasma density fluctuations without perturbing the plasma[3]. Alternatively, the fluctuating quantities can be provided by numerical simulations. A comparative study of the bicoherence for different data sets and different analysis methods is necessary to understand the restrictions and possibilities of bicoherence analysis. The application to a plasma with a restricted number of modes helps to gain a better insight into general properties of low frequency instability development and its relation to third order spectra.

This contribution presents bicoherence computations on data provided by the numerical simulation code CYTO [4]. Spectra and bispectra are computed using Fourier-based methods as well as wavelet based methods.

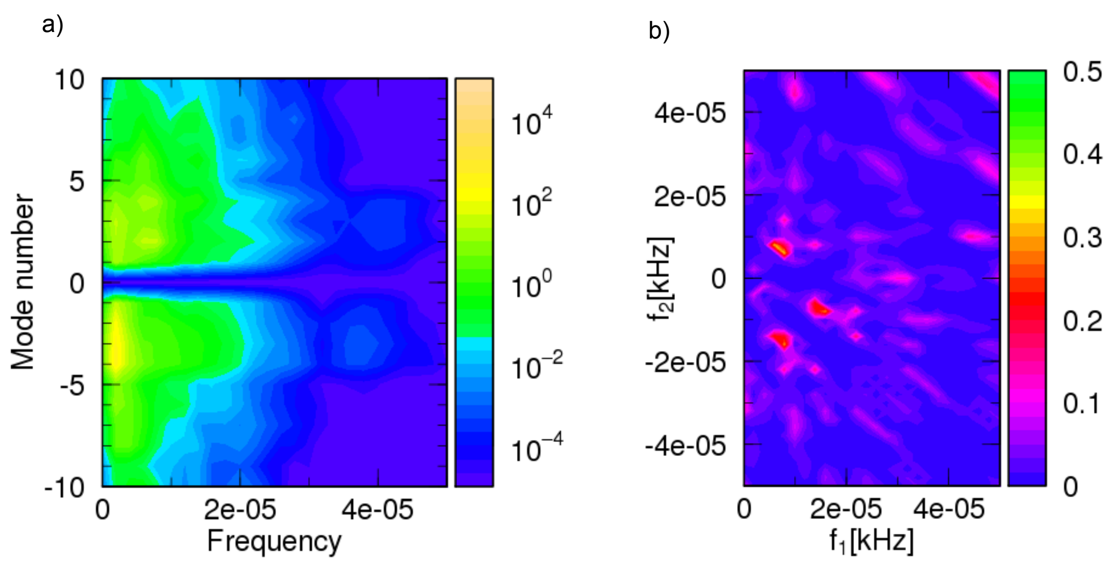

Figure 1: a) 2D spectrum of mode number and frequency combinations computed for the radius of $r = 8$. b) Frequency bicoherence for the same radial position.

Description of fluctuation data

Simulation data were obtained using the 2D version of the CYTO fluid code, developed by Volker Naulin [4]. The simulations reproduce well the unstable regimes with nonlinearly interacting global modes in cylindrical geometry. The code provides data of plasma potential and density on a grid of 32 radial and 64 poloidal positions.

Analysis using Fourier decomposition methods

First, to gain an insight in the different modes and frequencies that compose the fluctuations, the two-dimensional power spectrum in mode number and frequency is computed for the complete time series of spatio-temporal data. The definition of the two-dimensional Fourier decomposition is:$Z_{mk} = \frac{1}{MN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1} z_{ij}\exp\left[2\pi I\left(\frac{im}{M}-\frac{jk}{N}\right)\right]$, where I is the imaginary unit, m is the poloidal mode number and k is the index for the frequency [5]. The frequency is calculated as $f = k\Delta f$ where Δf is the frequency resolution determined by the inverse of the time window length. In the actual computation procedure, the data is first decomposed in the time direction to obtain M sets of complex Fourier components. Then the complex Fourier components are decomposed in the space direction to extract the complex phase distribution in the poloidal space. Finally, the two-dimensional power spectrum is computed from the two-dimensional Fourier decomposed signal as $S_{mk} = |Zmk|^2/\Delta f$.

The resulting spectrum is shown in figure 1a) for a radius where the mode $m = 4$ is visible in the spatio-temporal plot of the raw data. The $m = 4$ mode is indeed visible at a frequency of $0.8 \cdot 10^{-5}$, along with the mode $m = 2$ at almost the same frequency and a broad band of

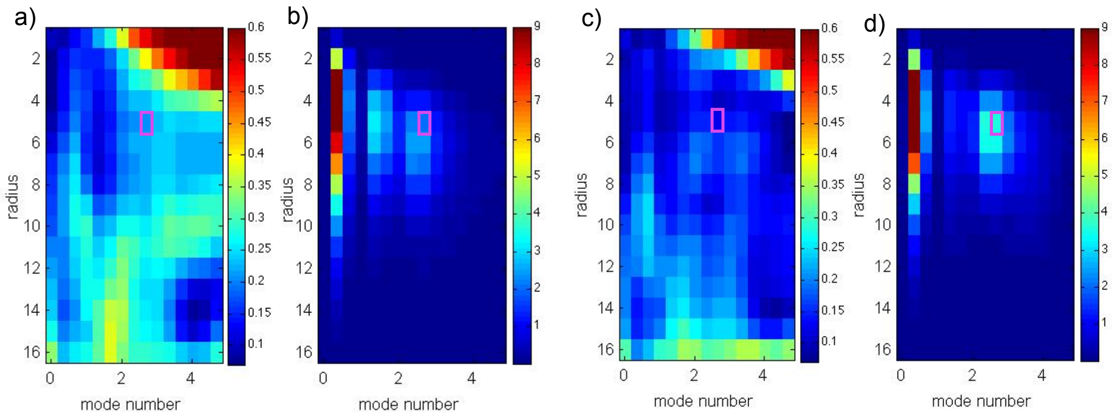

Figure 2: Normalized $B^{\star}$ (a) and power spectrum (b) at the moment $t = 405$ and bicoherence (c) and mode spectrum (d) at the moment $t = 452$.

negative mode numbers, propagating in the inverse direction, at low frequencies.

Using Fourier decomposition, the frequency bicoherence can be computed for components a, b and b satisfying the matching condition of $f_a + f_b = f_c$ as $b^2 = \frac{|\langle Z_a Z_b Z_c \rangle|^2}{\langle |Z_a Z_b|^2 \rangle \langle |Z_c|^2 \rangle}$. The result of this computation is shown in figure 1b). Combinations with a strong bicoherence involve the frequencies $0.8 \cdot 10^{-5}$ and $1.6 \cdot 10^{-5}$ that correspond to the mode $m = 4$ and its first harmonic. It also reveals a pattern of lines showing the possible coupling with a broad frequency range, as it could appear in case of a change of the modes frequencies with time or in case of an additional coupling with low frequency components.

Analysis using wavelet mode number bicoherence

When wavelet coefficients $X(m, \theta)$ calculated from 2D data are used instead of Fourier coefficients the quantity $B^{\star}(m_1, m_2) = \int X(m_1, \theta) X(m_2, \theta) X^{*}(m_1 + m_2, \theta) \mathrm{d}\theta$ can be calculated [6] for each time and normalized by the power spectra of the mode triplet. this normalized $B^{\star}$ combines information about the spatial phase matching of the modes and the modes power. The definition of this quantity is very similar to a wave number bicoherence, but its interpretation is not straightforward, as the prerequisite of averaging over independent ensembles is not fulfilled.

Figure 2 shows the wavelet computed normalized $B^{\star}$, denoted $b^{\star}$, and wavelet computed mode power spectra for two successive moments in time. In figure a), the $b^{\star}$ is relatively high for a large range of mode number values and radii and involving in particular mode numbers $m = 2$ to $m = 4$. At the next moment, the power of the $m = 3$ mode is increased. (The position of $m = 3$ is marked on each of the figures a) to d) by a red square to ease the reading of the figures). In figure 3 a similar sequence can be observed for the mode $m = 4$. The value of $b^{\star}$ is high on $m = 4$ in a) and then decreases while the power of $m = 4$ increases in figure d). A peak of $B^{\star}$ followed by an increase of the mode's power had also been observed with camera

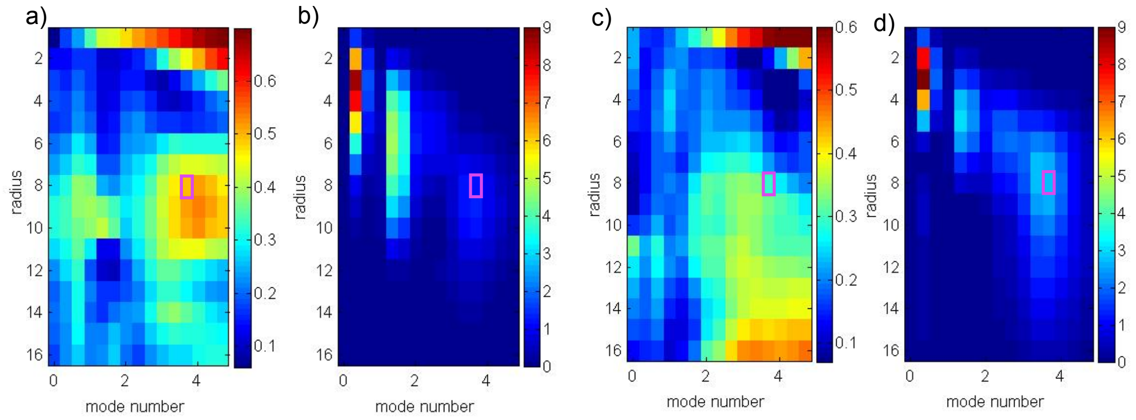

Figure 3: Normalized $B^{\star}$ (a) and power spectrum (b) at the moment $t = 1007$ and bicoherence (c) and mode spectrum (d) at the moment $t = 1244$.

data obtained on the linear device Mirabelle [6], although the limited temporal resolution of the measurement did not allow a systematic observation of such sequences. It can also be noticed that in many cases the simulation results indicate that the maxima of $b^{\star}$ travel in a preferred direction, towards higher radii.

Conclusions

The dynamics of nonlinear couplings have been investigated by applying mode number bicoherence computation to two dimensional data of plasma density fluctuations. By applying wavelet methods the temporal dynamics of power spectra and the bispectrum related quantity $B^{\star}$ have been studied. This study revealed a typical spectrum sequence, that had already been observed on the Vineta [1] and Mirabelle [6] devices. This third observation in numerical simulations of short-time three mode couplings preceding an energy increase is an important confirmation that calls for further studies aiming at understanding the causality between changes in power spectrum and bispectra or $B^{\star}$.

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