On possibility of turbulence wave number spectra reconstruction using radial correlation reflectometry in Tore Supra and FT-2 tokamaks

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Radial correlation reflectometry (RCR) is a microwave technique for measuring the properties of electron density fluctuations in tokamaks and more precisely its use is devoted to tokamak turbulence characterization. The principle of this method is to launch simultaneously two frequencies into the plasma, which are reflected at two different cut-off layers, and then to process the coherence decay of two scattering signals with increasing difference of probing frequencies in order to determine the correlation length [1]. It is often supposed that the distance between cut offs at which the correlation of two reflectometry signals is suppressed is equal to the turbulence correlation length. Unfortunately according to both numerical and analytical studies [2-3], this assumption is incorrect. As it was shown already in 1D numerical computations performed in the Born approximation [2], the scattering signal cross correlation function (CCF) decrease is much slower than the decay of turbulence CCF. This slow decay of RCR CCF was attributed to the contribution of small angle scattering off very long scale fluctuations. Later this observation was confirmed also in full-wave 1D numerical modeling for small level of turbulent density fluctuations and at the same time it was shown that non-linear effects induce a reduction of the correlation length [4]. Recently the comprehensive analysis has been performed in 1D model [5] showing the possibility to overcome this difficulty and determine the turbulence correlation length and even the spectrum.

In the present paper the results of numerical modeling of micro turbulence wave number spectra reconstruction using RCR data in one dimensional model for the actual profiles of tokamaks Tore Supra (France) and FT-2 (Russian Federation) are presented.

The turbulence wave number spectrum reconstruction background

Supposing probing wave propagation strictly in the direction of plasma density gradient we use 1D model to treat the RCR problem describing the O-mode probing by equation

$$\left\{\frac{d^2}{dx^2}+\frac{\omega^2}{c^2}-\frac{4\pi e^2\left[n(x)+\delta n(x)\right]}{m_e c^2}\right\}E_z(x,\omega)=0 \qquad (1)$$

where $n(x) = n_c\, x/L$ is the background linear density profile; $\delta n(x) = \frac{1}{2\pi}\int \delta n_\kappa e^{-i\kappa x} d\kappa$ stands for turbulent density perturbations assumed statistically homogeneous, κ is a radial wave number; ω is a probing frequency; E_z is the total field of the probing wave. Under the assumption that density fluctuation level is small $\delta n(x)/n_c \ll 1$ the solution could be found using the perturbation theory methods (Born approximation) and the scattering signal amplitude in 1D model can be obtained with the help of straightforward approach based upon the reciprocity theorem [6]: $A_s(\omega) = \frac{i\omega\sqrt{S_i}}{16\pi}\int_0^\infty \frac{\delta n(x)}{n_c} E_0^2(x,\omega)dx$, where S_i is an incident wave energy flux density and $E_0(x,\omega)$ is a solution of (1) calculated in the absence of density fluctuations that gives the distribution of the probing wave electric field in plasma for $S_i = 1$. As it was shown in [5], the relation expressing the turbulence spectrum in terms of CCF takes a form

$$n_\kappa^2 \sim \frac{2}{\sqrt{\pi}} e^{i\frac{\pi}{2}(1+sign(\kappa))} \frac{|\kappa|}{erf^*\sqrt{i\kappa L_0}} \int_{-\infty}^{+\infty} \overline{CCF}(\Delta L) e^{i\kappa\Delta L} d\Delta L \tag{2}$$

where the normalized $\overline{CCF}$ of reflectometry signals $A_s(\omega_0)$ and $A_s(\omega_1)$ is determined as

$$\overline{CCF}(\omega_1) = \frac{\left\langle \left[A_s(\omega_0) - \langle A_s(\omega_0)\rangle\right]\left[A_s(\omega_1) - \langle A_s(\omega_1)\rangle\right]^* \right\rangle}{\sqrt{\left\langle \left|A_s(\omega_0) - \langle A_s(\omega_0)\rangle\right|^2\right\rangle \left\langle \left|A_s(\omega_1) - \langle A_s(\omega_1)\rangle\right|^2\right\rangle}} \tag{3}$$

and averaging is held over random phase sets. The turbulence CCF is easily obtained from the reconstructed spectrum (2) using the Fourier transform.

Numerical reconstruction of the turbulence spectrum and CCF

We present in this section the computation results obtained for plasma density profiles typical for Tore Supra and FT-2 tokamaks where application of the spectrum reconstruction procedure under development in this paper is planned. For Tore Supra tokamak we add the example of numerical computations made with plasma density profile [7] shown in figure 1. The computation parameters are as follows: experimental probing frequency range in O-mode $43.3GHz < f < 50.9GHz$, which corresponds to the probing interval *0.26m<L<0.46m*; reference frequency: $f_0 = 47.7GHz$; reference frequency cut off point $L_0 = 0.36m$; central plasma density $n(0.72m) = 3.9\cdot 10^{13} cm^{-3}$. The averaging is performed over ensemble of typically 500 random phase sets. The turbulence spectrum used in the analysis is presented in figure 2. As it could be clearly seen the turbulence CCF (shown in figure 4) is reconstructed more precisely than the corresponding turbulence spectrum (in figure 3). The oscillations in the reconstructed functions are caused by imperfect averaging and discontinuities in the CCF extrapolation.

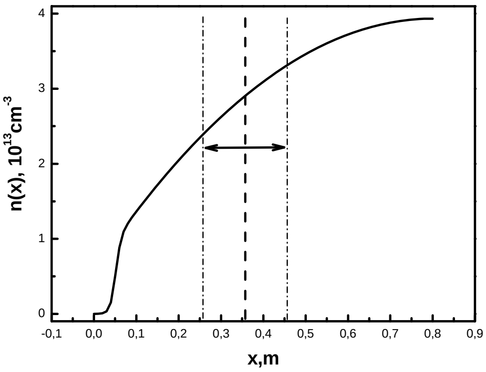

Figure 1. The density profile for Tore Supra [7] as a function of the radial position, dashed lines showing the border of the computation probing interval.

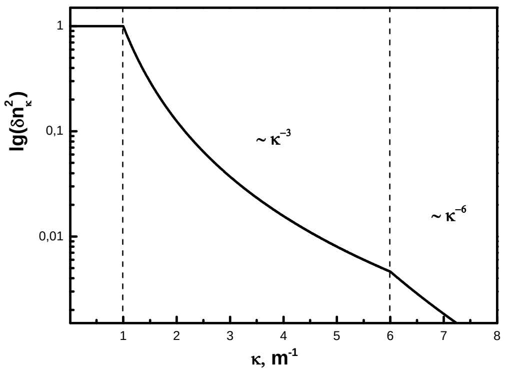

Figure 2. Radial wave number spectrum close to experimental one [8] except for the very low values $\kappa < 1m^{-1}$.

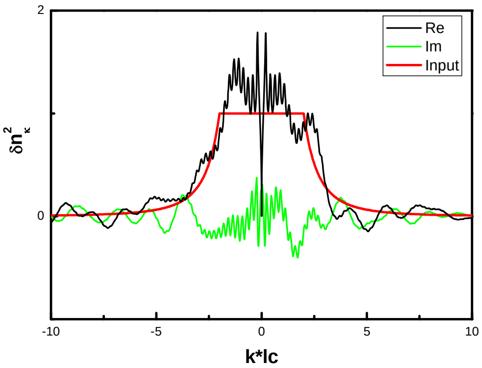

Figure 3. The calculated spectrum versus normalized wave number shown in relative units. Reconstructed real part is shown by black line, imaginary part by green line and input spectrum by red line.

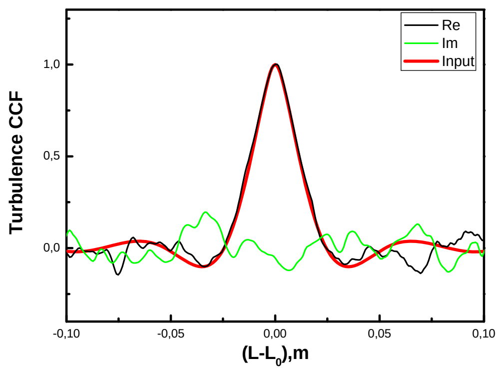

Figure 4. The turbulence CCF (real part by black line and imaginary part by green line) compared to the input turbulence CCF (red line) shown in relative units, $l_c = 0.027m$.

Coming to reconstruction in the case of FT-2 tokamak we take the following computation parameters: the plasma density profile in FT-2 tokamak according to experimental results [9, 10] could be assumed linear; probing frequency range (O-mode) $24.5GHz < f < 37.5GHz$ which corresponds to probing interval *0.03m<L<0.07m*; reference frequency $f_0 = 31.7GHz$, reference cut off $L_0 = 0.05m$. We use the Gaussian turbulence radial wave number spectrum $\tilde{n}_\kappa^2 = \sqrt{\pi} l_c e^{-l_c^2 \kappa^2/4}$, where $l_c = 0.02m$. The averaging is performed over ensemble of 500 random phase sets. The reconstructed turbulence spectrum shown in figure 5 in comparison to input Gaussian spectrum represents the possibility of turbulence spectrum reconstruction with a high accuracy. The turbulence CCF is also reconstructed, utilizing Fourier transform, rather precisely as it is shown in figure 6 despite the fact the plasma size in FT-2 is only 10 wave lengths and the turbulence correlation length is only twice larger then the wave length. Numerical simulations have shown that the probing range must be greater than $2 \div 4$ correlation lengths to obtain a good reconstruction on FT-2 tokamak.

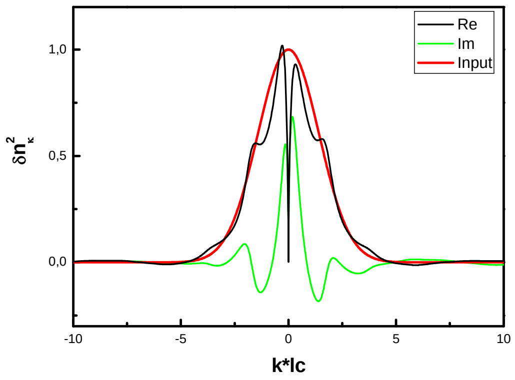

Figure 5. Reconstructed spectrum (black line real part and green line imaginary part) compared to the input Gaussian (red line) shown in relative units.

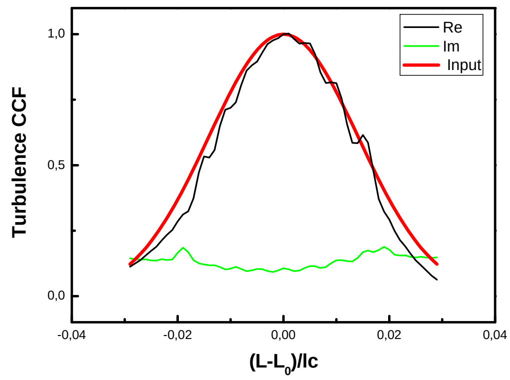

Figure 6. Reconstructed turbulence CCF (black line real part and green line imaginary part) compared to the input Gaussian one (red line) shown in relative units, $l_c = 0.02m$.

Conclusion

Firstly we would like to stress that the performed numerical modeling has confirmed the predictions of the RCR analytical theory developed in [5]. It is worth to underline that application of the proposed procedure to the turbulence spectrum and CCF reconstruction from the RCR data in numerical modeling for realistic plasma density profiles has led to very promising results. The demonstrated possibility of fine reconstruction, at least in 1D geometry, is proving the procedure feasibility and appealing for further optimization and tests in 2D numerical modeling. Nevertheless, it is remarkable that the proposed procedure has its restrictions due to it is based on the linear Born approximation and does not take in account nonlinear effects.

References

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