Neoclassical viscous stress tensor for NTM simulations with XTOR

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1. Introduction

MHD instabilities like the Neoclassical Tearing Modes (NTM) constitute an important issue for ITER. In this device, NTMs are predicted to limit drastically the operational $\beta_N = \beta_{\%} aB/I_p^{MA}$. Thus it is essential to understand better the physics of those modes. Simulations have been already carried out with the XTOR code [1] with a simplified model. We present here the implementation of a more complete formulation based on the viscous stress tensor that is directly linked to the pressure anisotropy. It allows one to recover the neoclassical equilibrium [2] and thus to treat the evolution of NTMs in a consistent manner.

2. Physical model

In general, the velocity distribution function is not isotropic. The pressure anisotropy contribution is modelled in the MHD set of equations by the stress tensor. The latter is composed of three components: parallel, gyroviscous and perpendicular. Considering an ordering where the Larmor Radius is small compared to the plasma size enables to neglect the third component with respect to the parallel term. The gyroviscous cancellation, already implemented in the code, provides the second contribution. Finally, the parallel component that will retain our attention is calculated following [3]. It is written in a Chew-Goldberger-Low (CGL) form:

$$\Pi_{\parallel} = \frac{3}{2}\pi_{\parallel}\left[\mathbf{bb} - \frac{1}{3}\mathbf{I}\right], \qquad \pi_{\parallel} = \frac{2}{3}(p_{\parallel} - p_{\perp}) \tag{1}$$

where $\mathbf{b}$ is the unit vector parallel to the magnetic field. The pressure anisotropy $\pi_{\parallel}$ is given by:

$$\frac{3}{2}\pi_{\parallel} = -n_s m_s \mu_s \frac{\mathbf{B}^2}{\langle(\mathbf{b}\cdot\nabla B)^2\rangle}\left[\left(\mathbf{V_s} + k_s\frac{2\mathbf{q_s}}{5p_s}\right)\cdot\nabla\ln B + \mathbf{b}\cdot\nabla\times(\mathbf{V_s}\times\mathbf{B})/B + \frac{2}{3}\nabla\cdot\mathbf{V_s}\right] \tag{2}$$

where $s = i, e$ corresponds to the ions and electrons, respectively. The first element is the forcing term to the equilibrium flows. The second and third terms have a key role for the damping of fast growing instabilities. The neoclassical coefficients μ_s and $k_s = \mu_{s,2}/\mu_{s,1}$ are calculated according to [4]. The heat flow can be separated in a perpendicular and a parallel component. The first

*See the Appendix of F. Romanelli et al., Proceedings of the 23rd IAEA FEC 2010, Daejeon, Korea one is directly computed from the temperature diamagnetic velocity. The parallel component is determined by using the neoclassical equilibrium heat equation. It has not been retained for the results presented in this paper.

The viscous stress tensor is implemented in the normalized set of MHD equations in XTOR. The momentum equation reads as follows:

$$\rho \partial_t \mathbf{V} = -\rho \left(\mathbf{V} \cdot \nabla \mathbf{V} + \mathbf{V_i^*} \cdot \nabla \mathbf{V}_\perp \right) + \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \Pi_{\parallel i} + \nu \nabla^2 \left(\mathbf{V} + \mathbf{V}_i^* \right) \tag{3}$$

In this relation, the ion neoclassical tensor governs the generation of the poloidal ion equilibrium flow. When the electron contribution is plugged into the Ohm's law, the latter can be written such that the tensor is part of the bootstrap current:

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \left(\mathbf{J}_\parallel - \mathbf{J_{bs}} - \mathbf{J_{CD}} \right) - \alpha \frac{\nabla_\parallel p_e}{\rho} \tag{4}$$

$$\text{with} \qquad \mathbf{J_{bs}} = \frac{\mu_e}{\mu_e + \nu_{ei}} \left[\mathbf{J}_\parallel + \frac{m_i}{m_e} \frac{1}{\alpha} \frac{\left(\nabla \cdot \Pi_{\parallel e} \right)_\parallel}{\mu_e} \right] \tag{5}$$

where $\alpha = 1/(\omega_{ci}\tau_A)$ is a measure of the diamagnetic contribution, τ_A the Alfvén time and ω_{ci} the ion cyclotron frequency. We have assumed that the resistivity with neoclassical effects is $\eta \approx \eta_{SP} \left(\nu_{ei} + \mu_e \right) / \nu_{ei}$ with η_{SP} the Spitzer resistivity.

This model allows one to capture in a consistent way diamagnetic and neoclassical physics.

3. Neoclassical equilibrium

The first simulations including the viscous stress tensor have been undertaken to verify that it satisfies the neoclassical equilibrium. The flux averaged divergence of the viscous stress tensor [2] can be recovered from Eq. 1 and 2:

$$\langle \mathbf{B} \cdot \nabla \cdot \Pi_s \rangle = n_s m_s \mu_s \left\langle B^2 \right\rangle \left(\frac{\langle \mathbf{V_s} \cdot \nabla \theta \rangle}{\langle \mathbf{B} \cdot \nabla \theta \rangle} + k_s \frac{2}{5 p_s} \frac{\langle \mathbf{q_s} \cdot \nabla \theta \rangle}{\langle \mathbf{B} \cdot \nabla \theta \rangle} \right) \tag{6}$$

Assuming that $\langle \mathbf{B} \cdot \nabla \cdot \Pi_i \rangle \approx 0$, we obtain an expression for the ion poloidal velocity:

$$V_{i\theta} = -k_i V_{Ti\theta}^* - k_i \frac{2}{5 p_i} \frac{\left\langle \mathbf{q}_{i\parallel} \cdot \nabla \theta \right\rangle}{\langle \mathbf{B} \cdot \nabla \theta \rangle} \tag{7}$$

The same recipe can be applied to calculate the bootstrap current:

$$\langle \mathbf{B} \cdot \mathbf{J}_{bs} \rangle = \frac{\mu_e}{\mu_e + \nu_{ei}} \frac{\rho}{\alpha} \left\langle B^2 \right\rangle \left(V_{e\theta}^* - V_{i\theta}^* + k_e V_{Te\theta}^* - k_i V_{Ti\theta}^* + k_e \frac{2}{5 p_e} \frac{\left\langle \mathbf{q}_{e\parallel} \cdot \nabla \theta \right\rangle}{\langle \mathbf{B} \cdot \nabla \theta \rangle} - k_i \frac{2}{5 p_i} \frac{\left\langle \mathbf{q}_{i\parallel} \cdot \nabla \theta \right\rangle}{\langle \mathbf{B} \cdot \nabla \theta \rangle} \right) \tag{8}$$

Evaluations on non-inductive discharge from Tore Supra (#32299, $\beta_N = 0.11$, $\beta_p = 0.66$, $q_{min} = 1.34$) and on a hybrid discharge form JET (#77922, $\beta_N = 2.35$, $q_{min} = 1.09$) are shown in Figures 1 and 2, respectively. We have taken experimental values for α ($\alpha = 0.03$ for Tore Supra, $\alpha = 0.02$ for JET). Good agreement is found between the analytical values and the simulation performed with XTOR-2F. Close to the outer boundary, discrepancy is observed for the averaged ion velocity. This is due to the curvature component in $\nabla \cdot \Pi_i$ that stands in the perpendicular direction. Figure 1(right) shows how the flows evolves towards equilibrium after $\sim 10^4 \tau_A$. In fact, balance is reached much faster in the region $\sqrt{\psi} < 0.8$.

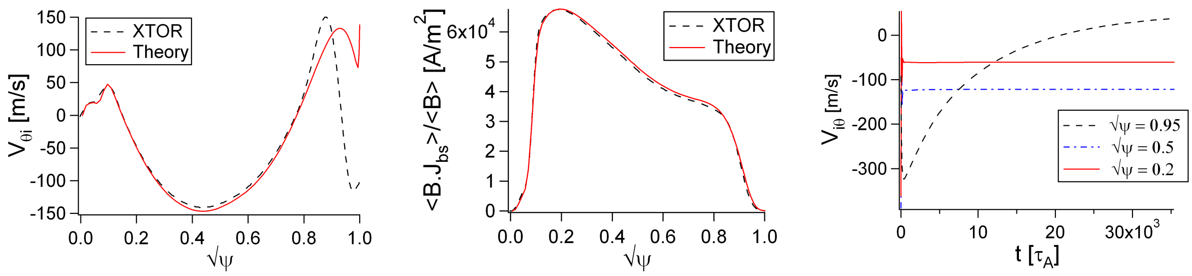

Figure 1: Left: *Equilibrium ion poloidal flow.* Center: *bootstrap current.* Right: *evolution of the ion poloidal flow until equilbrium for Tore Supra shot #32299 at* $t = 233.8s$.

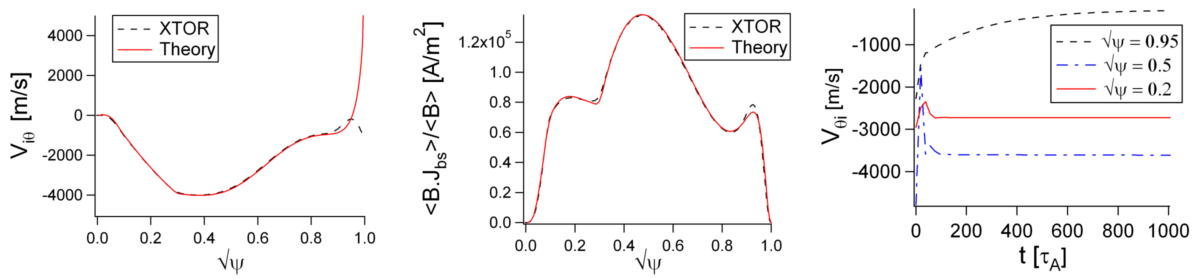

Figure 2: Left: *Equilibrium ion poloidal flow.* Center: *bootstrap current.* Right: *evolution of the ion poloidal flow until equilbrium for JET shot #77922 at* $t = 10s$.

The JET equilibrium is characterized by a strong shaping of the equilibrium while the cross section in Tore Supra is almost circular. The good match between analytical results and simulation confirms that they are verified in a complex geometry. The poloidal ion flow and bootstrap current are much larger and the evolution is much faster than in Tore Supra as the equilibrium is reached after $\sim 10^3 \tau_A$. The faster evolution is mainly due to higher values of the neoclassical coefficients in JET.

4. Growth rate of the (2,1) tearing mode in Tore Supra

Ion flow damping by neoclassical friction is expected to mitigate the growth of tearing modes. Simulations have been performed for situations with ($\alpha = 0.03$) and without ($\alpha = 0$) diamagnetic effects. Figure 3 shows that the neoclassical tensor has a stabilizing effect in both cases as expected from theory. Analytically we have the following relation for $\alpha = 0$: $\left(\frac{\lambda}{\lambda_0}\right)^4 \left(\frac{\lambda}{\lambda_0} + \frac{c_n}{\lambda_0}\right) = 1$ where λ_0 is the growth rate without neoclassical effects and $c_n = \mu_i \tau_A \frac{q^2}{\varepsilon^2 x^2}$.

It is in general much smaller than expected from the simple analytical model. We also notice a discontinuity in the graph for the case without neoclassical effects that could be explained by the quick change of the poloidal velocity flow imposed by the new terms. For $\mu_i \tau_A > 10^{-5}$, the effect of neoclassical terms on the growth rate is larger for $\alpha = 0$.

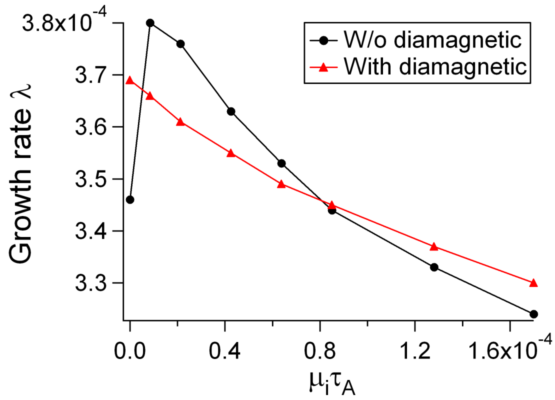

Figure 3: *Mitigation of (2,1) tearing growth by neoclassical friction.*

5. Conclusions

The viscous stress tensor has been implemented in a CGL form in the non linear MHD code XTOR-2F. It enables to recover the neoclassical equilibrium. Simulations show that the growth rate of the (2,1) tearing mode is mitigated by the neoclassical terms as can be expected by theory. When we add a (2,1) magnetic seed in the stable JET equilibrium presented above, the new model enables perturbation of the bootstrap current (Fig. 4). Other modes are presents including the (1,1) kink mode close to the magnetic axis. The holes in the bootstrap current are driving terms for NTMs and provide the basis for a future study with the XTOR-2F code.

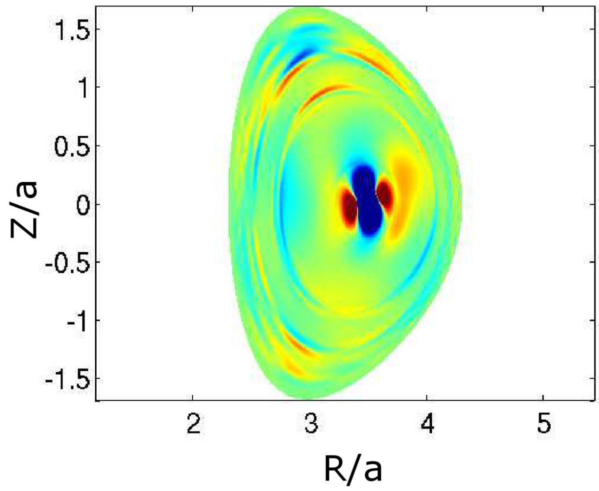

Figure 4: *Perturbed part of the bootstrap current after the injection of a magnetic seed in JET.*

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