Simulation of alpha particle dynamics with experimentally reconstructed displacement eigenfunctions in sawtooth oscillations

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Sawtooth oscillations can redistribute the α particles, thus modifying the power deposition profile and increasing α particle losses and wall loading. In addition, α transport from the core to the outer region can trigger other instabilities. Most previous studies of the effect of sawtooth crashes on α particle confinement have employed a phenomenological description of the evolution of the flux surfaces and included only the dominant (1,1) internal kink mode [1].

Our method consists in calculating the exact α particle trajectories in the total electric and magnetic fields (equilibrium plus perturbation). The approach is similar to that employed by Zhao and White [2] but includes some important differences: 1) The information regarding the space and time dependence of the perturbation is taken from the experimental results published in Ref. [3]. This is the most important feature of our model because it allows us to study the evolution of the α particles in the presence of the experimentally determined perturbations. 2) The exact trajectories are calculated, no guiding center approximation is employed. Although this requires significantly larger computational resources we believe it is justified because the width of the orbits can be larger than the typical scale length of the mode structure.

Equilibrium and perturbed fields

The equilibrium magnetic field is obtained by expanding the Grad-Shafranov (GS) equation in powers of the inverse aspect ratio (ε) and including only the first two terms in the solution. In toroidal coordinates: $\psi(x,\phi) = \psi_0(x) + \varepsilon\psi_1(x,\phi)$, where ψ is the poloidal flux, x the normalized minor radius ($x = \rho/a$) and ϕ the poloidal angle. We normalize ψ with $B_0\pi a^2$, where B_0 is the external toroidal field at the geometric axis (R_0), and assume that the pressure and poloidal current depend on ψ as: $p = p_1\psi^2; I^2 = I_0^2 + I_1^2\psi^2$, where p is normalized with $B_0^2/8\pi$ and I with $B_0 cR_0/2$ (Gaussian units are employed). Substituting this in the GS equation we obtain

$$\psi(x,\phi) = CJ_0(kx) + \varepsilon\frac{\cos\phi}{2}\left\{C\left[xJ_0(kx) - \frac{\sigma x^2}{k}J_1(kx)\right] - DJ_1(kx)\right\} \qquad (1)$$

where $J_0(kx)$ and $J_1(kx)$ are Bessel functions, $\sigma = 4p_1/\varepsilon^2$ and $k^2 = 4(p_1 + I_1^2)/\varepsilon^2$. The boundary of the plasma is at the $x = 1$ surface, where we request the flux to be zero. Then, k has

to be a zero of J_0 and $D = \sigma C/k$. C is determined by fixing the poloidal field at the plasma boundary and I_0 is related to B_0. With the normalization employed $I_0 = 1$. Finally, σ fixes the plasma β. Knowing the poloidal flux we can calculate the equilibrium magnetic field. The q profile obtained for $\varepsilon = 1/3$, $p_1 = 0.05$ $B_{pol}(x = 1, \phi = 0) = 0.155$ is shown in Fig. 1. For ITER-like parameters ($B_0 = 5.3 \times 10^4 G$, $a = 2$ m, $R_0 = 6$ m) the toroidal current equals 8.7 MA, which is consistent with the ITER value (15 MA), if we consider that ITER has $\kappa = 1.7$. The major approximation employed in our model is the use of a "cylindrical", instead of toroidal, displacement eigenfunction. This is justified by the following reasons: 1) The experimental information provided in [3] corresponds to a "cylindrical' mode. The amplitude of the displacement eigenfunction depends only on the minor radius (in toroidal coordinates) and the poloidal and toroidal dependences are included only in the phase (see eq. (2) of [3]). 2) As already noted by Zhao and White [2], the effective aspect ratio of the flux surfaces affected by the sawtooth (inside the $q = 1$ surface) is larger than the aspect ratio of the device (R_0/a). 3) The perturbed fields can be calculated analytically.

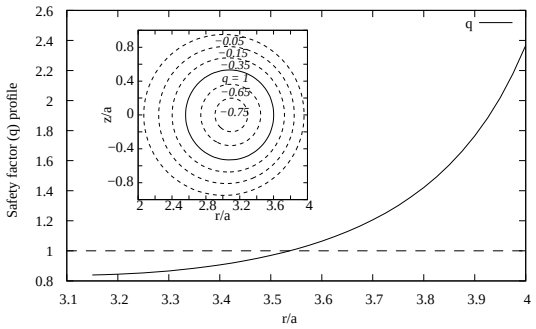

Figure 1: Safety factor and flux surfaces.

Ideal MHD is used to calculate the perturbed magnetic field produced by a known displacement field $\mathbf{B}_1 = \nabla \times (\xi \times \mathbf{B})$, where $\mathbf{B}_1$ is the perturbed magnetic field, ξ is the displacement and $\mathbf{B}$ the equilibrium field calculated above. We note that this equation means that the perturbed vector potential is perpendicular to the equilibrium magnetic field ($\mathbf{A}_1 = \xi \times \mathbf{B}$). This is different from the assumption used in [2], where $\mathbf{A}_1$ is assumed to be parallel to $\mathbf{B}$. To be consistent with the spatial dependence assumed below for the displacement, only the "cylindrical" part of $\mathbf{B}$ is used to calculate $\mathbf{B}_1$ (but the full $\mathbf{B}$ is used to calculate particle orbits).

Following the analysis presented in Ref. [3] we assume that two modes are present (the $(3,3)$ mode is not included in this work) and the x component of the displacement can be written as

$$\xi_x(x,\phi,\theta,t) = \xi_x^{11}(x,t)\cos(\phi - \theta - \omega t) + \xi_x^{22}(x,t)\cos[2(\phi - \theta - \omega t)] \qquad (2)$$

where θ is the toroidal (azimuthal) angle and ω is the frequency. Considering incompressible displacements ($\nabla \cdot \xi = \mathbf{0}$) and following [4] we can write the other components of ξ in terms of ξ_x. The electric field is obtained from the ideal Ohm's law:

$$\mathbf{E}_1 = -\frac{\mathbf{v}_1 \times \mathbf{B}}{c}, \qquad \mathbf{v}_1 = \frac{\partial \xi}{\partial t}. \qquad (3)$$

To proceed we need to specify $\xi_x^{11}(x,t)$ and $\xi_x^{22}(x,t)$, which are taken from the experimental information provided in Fig. 4 (for the x dependence) and Fig. 3 (for the time dependence) of Ref. [3]. Separating the space and time dependence as: $\xi_x^{mn}(x,t) = \xi_0^{mn}(t) f^{mn}(x)$
we introduce the following $f^{mn}(x)$:

$$f^{11}(x) = \frac{1}{2}\{1 - \tanh[\delta(x - x_s)]\}, \qquad f^{22}(x) = \begin{cases} \cos^2\left[\frac{\pi}{2}\left(\frac{x-x_2}{x_2}\right)\right] + \frac{e^{-x^2/x_2^2}}{4} \equiv f_c^{22}(x) \text{ for } 0 \le x \le x_a \\ 0 \text{ for } x_a \le x \le 1 \text{ s.t. } f_c^{22}(x_a) = 0 \end{cases} \tag{4}$$

where x_s is the (minor) radius of the $q = 1$ surface, $x_2 \simeq 0.35$ and δ is a numerical constant adjusted to get the desired slope at $x = x_s$ (typically $\delta \sim 20$). For the time dependence we use the same model as in [2] but with different growth and decay rates for each mode

Results

A fourth order Runge-Kutta method is employed to calculate the exact particle trajectories in the time dependent fields. Collisions are not included because the simulation time is much shorter than the collision time. The time step is taken small enough to guarantee that, when $\mathbf{E}_1$ is not included, the energy and azimuthal (toroidal) component of the canonical momentum (P_θ) are conserved. As a first step in the study of α particle dynamics during sawteeth we consider groups of particles having the same energy distributed on a given flux surface. The initial conditions for each particle are determined using the following procedure: 1) A flux surface is chosen. 2) A random value is generated for the poloidal angle. 3) The energy of the particle is fixed ($E_0 \le 3.5\ MeV$) and a random (isotropic) initial direction is chosen. The particles are followed from the time the $(2,2)$ mode appears until it disappears.

As an example of the effect of the perturbation on individual particle orbits we show, in Fig. 2 (left), the instantaneous position of the guiding center calculated from the exact orbit of a well trapped 3.5 MeV particle. The bounce frequency for this particle is $\omega_{bounce} = 1.3975 \times 10^{-3}\Omega_\alpha$, where Ω_α is the alpha cyclotron frequency calculated with B_0. For a perturbation with $\omega = 2.0 \times 10^{-4}\Omega_\alpha$ including the $\mathbf{E}_1$ and $\mathbf{B}_1$ produced by the $(1,1)$ and $(2,2)$ modes is added to the equilibrium field, the orbit (for the same initial conditions) changes completely and the particle switches from trapped to passing, back to trapped, etc. Its energy, however, changes very little. This is shown in Fig. 2 central, which presents a plot of the particle's energy and parallel velocity as a function of time. When the frequency is changed to $\omega = \omega_{bounce} = 1.3975 \times 10^{-3}\Omega_\alpha$ the energy of the particle changes significantly while it switches between being trapped and passing. This is shown in Fig. 2 right.

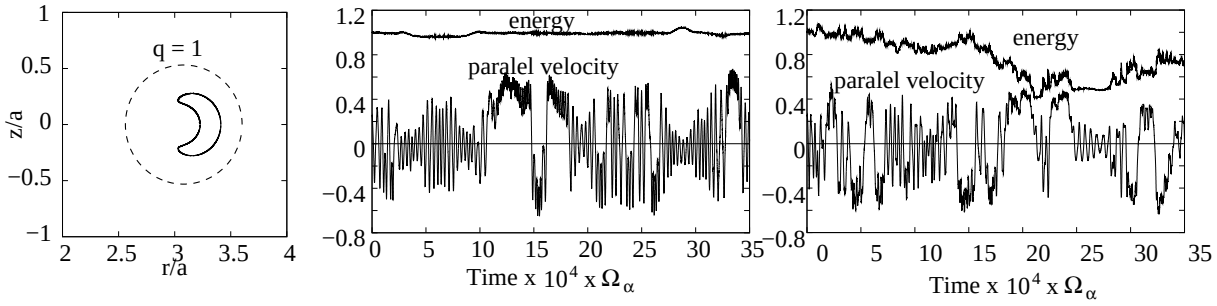

Figure 2: Left: Banana orbit. Middle: Particle energy and parallel velocity for $\omega > \omega_{bounce}$. Right: Particle energy and parallel velocity for $\omega = \omega_{bounce}$.

To quantify the effect of the perturbation we introduce a "diffusion" coefficient defined as:

$$D = \left\langle \left(x_p^f - x_u^f \right)^2 \right\rangle / \Delta\tau \tag{5}$$

where x_p^f and x_u^f are the final values of x (for each particle) with and without perturbation, respectively, $\Delta\tau$ is the time interval and $\langle\rangle$ means average over a large number of particles. Fig. 3 shows D as a function of the energy for several flux surfaces. No sharp maximum is observed.

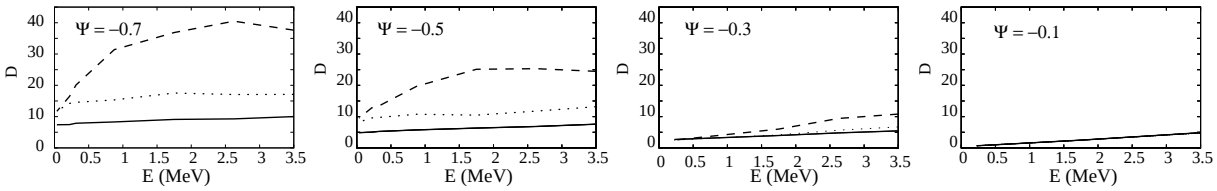

Figure 3: D vs particle energy without perturbation (full line), with b1 (dot line) and with b1 and e1 (dashed line) for several initial flux surfaces.

Future studies will be devoted to calculate energy and particle fluxes following a large number of α particles distributed inside the $q = 1$ surface according to the expected density and velocity distributions.

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References

[1] Ya. I. Kolesnichenko et al., Nucl. Fusion **7**, 1325 (2000)

[2] Y. Zhao and B. White, Phys. Plasmas **4** (4) (1997)

[3] V. Igochine et al., Nuc. Fusion **47**, 23-32 (2007)

[4] J. P. Freidberg, *"Ideal Magneto Hydrodynamics"*, Plenun Press, Chapter 9, (1987)