Thermal forces on dust in magnetised plasmas

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The dynamics of dust particle plays important in diverse settings ranging from star formation to structure formation under controlled laboratory experiments. Some of the structure formed by the low-pressure weakly ionized plasma in the laboratory includes clouds, voids etc. The presence of dust in the plasma provides an important diagnostic tool. For example, in bounded plasma, sheath characteristics can be studied in considerable detail by using fine dust probes.

The dust in the plasma is acted upon by various forces. For example, the momentum exchange between plasma and neutral gas species with the dust is the most common drag force that acts on the dust. The dust could affect the performance of fusion devices as well. In hot and rather dense fusion plasma, neutral gas density is small, while plasma temperature exhibits large gradients. Therefore, thermal plasma gradient could be the dominant force in fusion devices Expression for the thermal force is a function of dust charge. Since both particle and heat flux to the dust are large, the thermionic emission could be important. Therefore, dust in fusion plasma will have either a large negative charge (when thermionic emission is weak) or, a small positive charge (when thermionic emission is strong). Here we shall deal with both negative and positive dust.

The dust surface potential and thus the dust charge is approximated as a nonlinear function of Havnes parameter $P = Zn_d/n_e$ where Z is the number of grain charge and n_d and n_e are grain and electron number densities or, numerically derived from the ambipolar condition, i.e. by balancing the particle fluxes near the grain surface. Recently an analytical formula using the ambipolar condition was derived by Pandey et al. (2011) which is accurate, particularly for dust carrying large negative or, positive charges. Making use of this formula for the dust charge, we show that the only dust parameter that enters the thermal forces is its radius.

The expressions for thermal forces acting on ions as well as dust are well known. The expression for thermal forces have been derived by assuming that relative electron-dust ($\mathbf{w_e}$) ion-dust ($\mathbf{w_i}$) drift are small and dust is stationary [Stepanenko et al., 2011]. The equilibrium grain charge on the other hand is derived by assuming the ambipolar condition near the grain surface, i.e. sum of electron, I_e and ion I_i current is set zero. For negatively charged dust, neglecting the ion drift, the dust charge can be approximated by [Pandey eta al, 2011]

$$Q \approx \frac{4}{\pi} \frac{a\,T_i}{Z_i\,e}, \tag{1}$$

where a is the dust radius, $m_i\,, T_i$ is the ion mass and temperature respectively. It is clear from Eq. (1) that the plasma thermal motion causes the dust charging. Above expression is valid only when the ion kinetic energy is negligible in comparison with the ion thermal energy. However, in general, both ion flow as well as thermal motion will determine the dust charge.

Thermal forces due to ions

Consider the part of ion - dust thermal force due to absorption by a negatively charged dust grain,

$$\mathbf{F^a_{di}} = \mathbf{F^a_{i\|}} + \mathbf{F^a_{i\perp}} + \mathbf{F^a_{iH}}\,. \tag{2}$$

The components are due to thermal gradient parallel and perpendicular to the magnetic field, with the last term along $\left(\mathbf{F^a_{di}} \times \mathbf{b}\right)$. Here $\mathbf{b}$ is the unit vector in the direction of ambient field. Various components [Stepanenko et al., 2011] after using Eq. (1) becomes

$$\mathbf{F^a_{i\|}} = -0.37\,Y_i^a\,\nabla_\| T_i^3\,, \quad \mathbf{F^a_{i\perp}} = Y_i^a\,A^a_{i\perp}\,\nabla_\perp T_i^3\,, \quad \mathbf{F^a_{iH}} = Y_i^a\,A^a_{iH}\,\left(\nabla T_i^3 \times \mathbf{b}\right)\,, \tag{3}$$

where

$$\begin{aligned} Y_i^a &= \frac{1}{ln\Lambda_{ii}} \left(\frac{a}{Z_i^2\,e^2}\right)^2\,, \\ A^a_{i\perp} &= -0.11\,\frac{\beta_i^2 + 0.28}{\beta_i^4 + 0.68\,\beta_i^2 + 0.043}\,\left[1 - 0.81\,\Delta_1\left(\beta_i\right)\right]\,, \\ A^a_{iH} &= -0.24\,\frac{\beta_i^3 + 0.42\,\beta_i}{\beta_i^4 + 0.68\,\beta_i^2 + 0.043}\,\left[1 - 0.63\,\Delta_2\left(\beta_i\right)\right]\,, \end{aligned} \tag{4}$$

and

$$\Delta_1\left(\beta_i\right) = 1 - \frac{0.08}{\beta_i^2 + 0.28}\,, \quad \Delta_2\left(\beta_i\right) = 1 - \frac{0.09}{\beta_i^2 + 0.42}\,. \tag{5}$$

Therefore, for a give dust size, $\mathbf{F^a_{i\|}}$ is a function of ambient plasma parameters, i.e. ion charge and parallel thermal gradient. Similarly, $\mathbf{F^a_{i\perp}}$ and $\mathbf{F^a_{iH}}$ becomes a function of purely plasma Hall parameter, $\beta_i = \omega_{ci}/\nu_{ii}$ which is a measure of ion diffusion across the field lime.

The force on the dust due to scattering of ions can be written in component form similar to Eq. (2)

$$\mathbf{F^s_{di}} = \mathbf{F^s_{i\|}} + \mathbf{F^s_{i\perp}} + \mathbf{F^s_{iH}}\,. \tag{6}$$

after using Eq. (1) becomes

$$\mathbf{F^s_{i\|}} = 2.21\,Y_i^s\,\nabla_\| T_i^3\,, \quad \mathbf{F^s_{i\perp}} = Y_i^s\,A^s_{i\perp}\,\nabla_\perp T_i^3\,, \quad \mathbf{F^s_{iH}} = Y_i^s\,A^s_{iH}\,\left(\nabla T_i^3 \times \mathbf{b}\right)\,, \tag{7}$$

where

$$Y_i^s = (ln\Lambda_{di})\, Y_i^a\,,$$
$$A_{i\perp}^s = 0.69\,\frac{\beta_i^2+0.15}{\beta_i^4+0.68\,\beta_i^2+0.043}\,,$$
$$A_{iH}^s = 1.06\,\frac{\beta_i^3+0.24\,\beta_i}{\beta_i^4+0.68\,\beta_i^2+0.043}\,. \tag{8}$$

Thermal forces due to electrons

The thermal forces due to electron absorption on the negatively charged dust becomes can be written in the component form similar to Eq. (2) with subscript i replaced by e and

$$\mathbf{F_{e\|}^a} = A_{e\|}^a\, Y_e^a\, \nabla_\| T_e\,, \quad \mathbf{F_{e\perp}^a} = A_{e\perp}^a\, Y_e^a\, \nabla_\perp T_e\,, \quad \mathbf{F_{eH}^a} = A_{eH}^a\, Y_e^a\, (\nabla T_e \times \mathbf{b})\,, \tag{9}$$

where

$$\chi_e = \frac{T_i}{Z_i\, T_e}\,, Y_e^a = \frac{4\,\chi_e}{\pi\, ln\Lambda_{ei}}\left(\frac{a\,T_i}{Z_i^2\, e^2}\right)^2,$$
$$A_{e\|}^a = -0.40\,\exp(-\chi_e)\left[\nu\,\gamma_\| - \frac{\mu}{14}\,\mu_\|\right], \tag{10}$$

and

$$\nu = 1+4.5\,\chi_e+\chi_e^2\,, \quad \mu = 1+2.5\,\chi_e+2\,\chi_e^2\,(1-\chi_e)\,. \tag{11}$$

The coefficients $\gamma_\|$ and $\mu_\|$ are independent on the dust charge and thus are same as given by Stenaneko et al. [2011] and we do not give them here. The expressions for $A_{e\perp}^a$ and A_{eH}^a are identical to Eq. (10) except now $\gamma_\|$ and $\mu_\|$ needs to be replaced by perpendicular components.

In order to give the thermal forces due to electron absorption on the positively charged dust, we note the dust charge in this case can be approximated by [Pandey eta al, 2011]

$$Q \approx -\frac{a\,T_e}{e}\,. \tag{12}$$

Writing the thermal force corresponding to electron absorption as

$$\mathbf{F_{de}^a} = \mathbf{F_{e\|}^a} + \mathbf{F_{e\perp}^a} + \mathbf{F_{eH}^a}\,. \tag{13}$$

We can express each component like Eq. (9). Similarly, the thermal force due to Coulomb scattering can be expressed like Eq. (13). The dust dependent Y_e^a is

$$Y_e^a = \frac{1}{ln\Lambda_{ei}}\left(\frac{a\,T_e}{Z_i\, e^2}\right)^2, \tag{14}$$

and corresponding Y_e^s using expression from Stepanenko et al. [2011] and Eq. (12) gives

$$Y_e^s = (ln\Lambda_{de})\, Y_e^a\,. \tag{15}$$

Clearly, except for the constant Coulomb logarithm, the contribution of the dust is identical to both the thermal absorption and scattering forces. The expressions for $A^a_{e\|}$, $A^a_{e\perp}$ and A^a_{eH} are functions of ambient plasma parameters only [Stepanenko et al., 2011].

To summarise, we have shown here that the expressions for thermal force acting on either negatively or weakly positively charged dust due to plasma collision depends only on the dust size. The dependence on the dust charge can be easily eliminated in favour of dust size and thus either of the force due to scattering or, absorption will dominate depending on the dust size.

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References

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