General Features and Solutions of Master Equation for equilibrium structures in complex plasmas

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1.**Drag and diffusion coefficients for non-linear dust screening.** The balance of forces in self-organized in Master Equations are using dust-ion interaction cross-section with exact formulation of non-linear screening in most developed model [1,2]. For typical experiments in laboratories and on board of International Space Station (ISS) the individual dust screening is non-linear, i.e the parameter β, the ratio of electrostatic energy of ion-grain interaction at the distance of Debye radius to the ion average kinetic energy T_i , is large $\beta \gg 1$ (typically $\beta \approx 20-30$). Dusty plasmas are considered to be partially ionized and the ion neutral collisions, determined by ion-neutral mean free path λ, are taken into account together with ion-dust collisions. Normalization of forces F, distances r, dust sizes a, densities and Havnes parameter P is the following: $F \to F\lambda/T_i; r \to r/\lambda, a \to a/\lambda, n \to n_i 4\pi e^2\lambda^2/T_i; n_e \to n_e 4\pi e^2\lambda/T_i; P \to Z_d n_d n 4\pi e^2\lambda/T_i; z \to Z_d e^2/a\lambda T_e; \tau \to T_i/T_e$. Plasma flux Φ and ion drift velocity u are normalized with respect to ion thermal velocity v_{Ti}: $\Phi \to \Phi T_i/\sqrt{2} v_{Ti} e^2\lambda^2; u \to u_i/\sqrt{2} v_{Ti}$. For this normalization non-linear parameter β the drag coefficient f_{dr} and diffusion coefficient D are determined by expressions: $\beta = za\sqrt{n}/\tau; F_{dr} = Z_d f_{dr} u\sqrt{n}; \Phi = nu - D(dn/dr)$. The transport cross-sections for scattering of ions on grains is calculated taking into account both large angle scattering and reflection from potential barriers. It enters both in f_{dr} and D. Linear drag coefficient (for $\beta \ll 1$)is $f_{dr} \propto \beta \ln(1/\beta)$ and cannot be extrapolated to large β and is much larger than the non-linear screening coefficient, but the absolute value of calculated non-linear coefficient is larger than the maximum possible linear one estimated for $\beta \approx 1$. The numerical calculation of f_{dr} as function of u and β have shown that after some increase with β the drag coefficient is decreasing with an increase of β. The maximum drag is 2-3 times larger than that calculated in [3] and is close to that obtained in some numerical simulation of [4]. Contrary to [4] the calculations are able to give f_{dr} in broad range of $u(-4 < u < 4)$ and β $(3 < \beta < 90$. The results are numerical continuous functions of f_{dr} an D appropriate for solutions of Master Equations for structures. An example of these results for $f_{dr}$are presented on Fig. 1. Both ion-neutral and ion dust collisions are taken into account in diffusion coefficient which was found as numerical continuous function of 3 parameter $u, \beta, p = P/2\sqrt{n}$ and an examples of these results is presented on Fig. 2.

Also the ion flux friction force F_{fr} in neutral gas is used for the model of constant cross-sections ion neutral collisions $F_{fr} = \sqrt{u^2 + (8\pi/3sqrt\pi)^2}$ together with the friction force for ion flux due to dust drag $-Euf_{dr}(P/\sqrt{n})$.

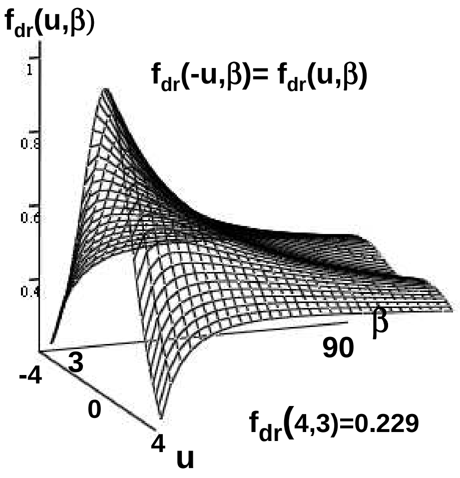

Figure 1: Surface diagram for dependence of $f_{dr}(u,\beta)$ on u in the range $-4 < u < 4$ and on β in the range $3 < \beta < 90$. The minimum value of f_{dr} on this figure is 0.229 and is only about 5 times less than its maximum value

2.General Master equations for dust structures.

The master equations describe: 1. Balance of fluxes (sum of convection and diffusion flux) including volume ionization and volume absorption on dust grains, 2. Balance of forces of grains including electric field force $-Z_dE$ and the drag force ($E = f_{dr}u\sqrt{n}u$) 3. Balance of forces on ions including the electric field force, friction on neutrals [5] and grain and ion pressure force. 4. Balance of micro-fluxes on individual grains including the electron thermal flux and sun of the thermal and convection flux on ions. 5. The Poison equation resulting in algebraic equation for Havnes parameter P that can be solved numerically as function of local values of n, n_e, u, z, Φ, r. The solution of self-consistent strongly nonlinear Master Equations is found from the equilibrium requirement relations of main structures parameters at the center. It is demonstrated that the general features of equilibrium structures are determined only by two parameters related with plasma flux and power of external ionization. The equilibrium is shown to be possible only in restricted range of these parameters. The finite range of possible equilibrium is used to investigate and scan all possible types of self-organized dust structures. An important role is played by suppression of diffusion by ion dust collisions. Previously the role of ion dust collision on diffusion in low temperature plasma was not investigated even for weak non-linearities in dust screening. In present report this effect is investigated for the first time for the case of strong non-linear dust screening. The suppression of diffusion by these collision **is a new effect which influences strongly the dust self-organized structures** for big phase space area of global parameters where the equilibrium states can exist.

3.**General Features of Master Equation solutions for compact equilibrium dust structures**. Instead of natural global parameters (external plasma flux and external volume ionization power)in numerical calculation is used determined by them ion density at the center of the structure $n(0)$ and the coefficient of ionization α_i (defined by the relation that the space derivative of the total flux is equal to $\alpha_i n_e$. The equilibrium asymptotic valued at the center of the structures $(n(0), n_e(0), z(0), du/dr(0), d\Phi/dr(0))$ and therefore the whole distributions of n, n_e, u, z, Φ, E in equilibrium self-organized described by self-consistent equations linear in derivatives with respect to distance are completely determined by only two parameters $n(0), \alpha_i$. The whole phase space $\{n(0), \alpha_i\}$ available for existence of equilibria of compact dust structures was scanned. This phase space is shown to narrow with an increase of α_i. For $\alpha_i = 0$ it is determined by $n_{min} < n(0) < n_{max}$ where n_{min} being about 1-3 is completely determined by the value of drag coefficient at the center and is larger the larger is the non-linearity in screening, while n_{max} is determined by the ratio of ion-neutral mean free path to the grain size and can reach the values 800 or larger. An increase of the ionization coefficient mainly increases n_{min} and slightly increases n_{nax} narrowing the available phase space for compact dust structures without voids in the center. At some large critical α_i (about $10-15$) this phase space vanishes and only structures with voids at the center are available. Stable structures with voids inside them are allowed for lower α_i outside the mention range of $n(0)$ but they occupy also a restricted phase space, determined by condition of stability. All structures have finite size, are charged with degree of quasi-neutrality regulated by the drag coefficient.

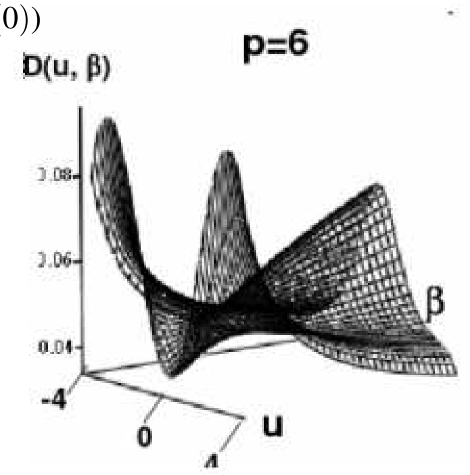

Figure 2: 2D diagram for dependencies of diffusion coefficient D on u and β. The numbers on the axes u and β are only for indication of curves, the ranges of change of the parameters u and β on these axes is $-4 < u < 4, 3 < \beta < 90$ and the particular value of p for this case is given on the top of the figure.

4. **Compact structure with small diffusion**. Investigation has been performed of type of dust structures obtained as solutions of master equations neglecting diffusion using the founded novel effect of suppression of diffusion by ion-dust collisions on non-linearly screened grains. The diffusion fluxes have been neglected in first approximation and there role was calculated by perturbations to obtain the range of applicability of assumption of small role of diffusion. The solution of Poisson equation gives still cumbersome expression for Havnes parameter P as a ratio $P = N/M$ of two functions N and M depending on $n, n_e, u, \Phi, r, f_{dr}$ and derivatives df_{dr}/du and $df_{dr}/d\beta$. The latter are computed numerically as continuous functions of u and β and used in Master Equations. The final result for $\alpha_i = 0$ shows that the structures can be of 3 types: 1)for $n_{min} < n(0) < n_{cr}^{(1)}$ they have continuous and smooth decrease of P from the center $r = 0$ up to $r = R_{str}$ where P vanishes and determines the structure size, 2)for $n_{cr}^{(1)} < n(0) < n_{cr}^{(2)}$ with an increase of $n(0)$ the distribution of P inside the structures starting from $r = 0$ became

more flat and it is decreasing to $P = 0$ at the edge more rapidly, at $n(0) = n_{cr}^{(2)}$ the derivative of P at the center became zero and at the edge the P changes to zero abrupt in a thin layer, 3) for $n_{cr}^{(2)} < n(0) < n_{max}$ the P is starting to increase from the structure center up to certain distance where it increases several times and have a peak corresponding in rough approximation to change the sign of derivative of P abruptly with a smooth decrease of P after peak up to the structure edge where it is changing abruptly to zero. The functions N and M can have zeros with an increase of distance from the center. For the structures of the first type the zero of N is reached first, for structures of the third type the zeros of M have a tendency to reach zero first but since the master equations contain P which cannot be infinite the self-consistency of solutions require that the zeros of numerator N move to zeros of denominator M to give uncertainty which corresponds to finite value of P. It can be resolved only using very high precision calculations close to the peak. Ended the calculations with high precision in small distance range around the peak where the derivative dP/dr is changing sign abruptly shows that all parameters in the peak are continuous including dP/dr and the perturbation theory shows that the diffusion can be neglected everywhere including the peak. The diffusion can play role only in a thin layer at the structure edge where $P \to 0$ and the effect of suppression of diffusion by ion-dust collisions is not operating. Thus the structure of the peak has been completely resolved showing unimportance of diffusion fluxes. The type of master equations with such properties is quite new in the theory of self-organization. Numerical computation of critical densities have been made for typical parameters in existing experiment $a = 0.01, \tau 0.01$, gas Argon with result $n_{min} = 2.5, n_{cr}^{(0)} \approx 30, n_{cr}^{(2)} = 400, n_{max} = 800$. The dust structures with dust density peaks have been observed recently in dusty plasma experiment [6]. The direct experimental evidence of establishing for dust the balance of collective electric field and drug force for non-linear dust screening was obtained in experiments on board of ISS where the small grains have been injected in the dust-void structure created by larger grain [6], since according to the present results the drag coefficient for non-linear screening decreases with grain size and the electric field created by larger grains is smaller and the larger grains should be ejected from the region of smaller grains which is exactly the phenomenon observed in [7,8].

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