EFFECTS OF DUST SIZE DISTRIBUTION AND TWO TEMPERATURE ELECTRONS ON DUST ACOUSTIC WAVES

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Abstract. Taking into account dust size distribution**,** we study the effect of two-temperature electrons on the dust acoustic soliton in multi-component dusty plasma. We use reductive perturbation method to derive the spherical Kadomtsev-Petviashvili (SKP) equation that describes the propagation of dust acoustic solitary waves. The modification in the amplitude and the width of the solitary wave structure due to the inclusion of two types of isothermal electrons are investigated.

Introduction

Nowadays, there has been a great deal of interest in understanding different types of collective processes in dusty plasmas, which are very common in laboratory and astrophysical environments **[1-7].** It has been found that the presence of charged dust grains modifies the existing plasma wave spectra, whereas the dust dynamics may even introduce new eigenmodes in the plasma**[8-17]**. Rao *et al.* **[18]** were the first to predict theoretically the existence of extremely low-phase velocity dust acoustic waves in unmagnetized dusty plasmas whose constituents are inertial charged dust grains and Boltzmann distributed ions and electrons. These waves have been reported experimentally and their nonlinear features investigated by Barkan *et al.* **[19].** Recently, it has been proved that the solitary wave characteristic may be modified by taking into account some parameters and the most important of them is indubitably «temperature». Indeed, the effect of dust temperature in hot dusty plasma was studied by several authors **[20,21,22]**, effect of ion temperature was also investigated **[23,24].** Although, just a little work is concerned with the effect of electrons temperature **[25].** In addition, due to their importance, the solitary waves in unmagnetized plasma without geometry distorsion and the dissipation effects have been extensively investigated and found to be described by the Kotweg-de Vries (KdV) equation or Kadomtesv-Petviashvili (KP) equation**[26-28].** However, new theoretical studies indicate that the properties of solitary waves in bounded nonplanar spherical geometry differ from that in unbounded planar geometry **[29]**. It is well known that the transverse perturbation (which always exist in the higher dimensional system) may not only introduce anisotropy into the system but also modify the structure and stability. The combined effects of both nonplanar geometry and the transverse perturbation on the DAWs have been considered by some authors, but most of them have paid attention to wave propagation in mono-sized dust grains because it is easier to study**[30-33].** Although, in real case, the dust grains have many different sizes both in space and laboratory plasmas **[34].**

Motivated by all those previous works and trying to study a more realistic dusty plasma, in this paper, dust acoustic waves DAWs are investigated by including four effects, namely; ***i/*** two-temperature electron, ***ii/*** dust size distribution (by considering the different dust size distribution, viz., power law distribution), ***iii/*** nonplanar spherical geometry and ***iv/*** the transverse perturbation. A spherical Kadomtsev –Petviashvili equation is obtained analytically by using the well-known reductive perturbation method **[35].**

Formulation

We consider dusty plasma with extremely massive, high negatively charge dust grain, Boltzmann distributed ions and two types of electrons with different temperature T_{el} (lower) and T_{eh} (higher). We assume that there are N different dust grains whose masses are m_j, $(j=1.2....,N)$, whose sizes are different. Charge neutrality at equilibrium requires that $n_{he0}+n_{le0}=n_{i0}-\sum_{j=1} Z_{d0j}n_{d0}$ where $n_{i\mathbf{0}}$, $n_{d\mathbf{0}}$, $n_{he\mathbf{0}}$ and $\mathrm{n}_{\mathrm{le0}}$ are the unperturbed ions density, *j*th dust grains number densities, and electrons number densities at two different temperature T_{el} and T_{eh}, respectively. Z_{doj} is the unperturbed number of charges residing on the *j*th dust grain measured in the unit of electron charge. So, the usual dusty fluid equations govern the dust acoustic waves in spherical geometry is

$$\begin{cases} \dfrac{\partial n_{dj}}{\partial t}+\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r^2 n_{dj}u_{dj}\right)+\dfrac{1}{r}\dfrac{\partial}{\partial\theta}\left(n_{dj}v_{dj}\right)+\dfrac{n_{dj}v_{dj}}{r}\cot g\theta=0, \\ \dfrac{\partial u_{dj}}{\partial t}+u_{dj}\dfrac{\partial u_{dj}}{\partial r}+\dfrac{v_d}{r}\dfrac{\partial u_{dj}}{\partial\theta}-\dfrac{v_{dj}^2}{r}=\dfrac{Z_{dj}}{m_{dj}}\dfrac{\partial\phi}{\partial r}, \\ \dfrac{\partial u_{dj}}{\partial t}+u_{dj}\dfrac{\partial v_{dj}}{\partial r}+\dfrac{v_d}{r}\dfrac{\partial v_{dj}}{\partial\theta}-\dfrac{u_{dj}v_{dj}}{r}=\dfrac{1}{r}\dfrac{Z_{dj}}{m_{dj}}\dfrac{\partial\phi}{\partial r}, \\ \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial\phi}{\partial r}\right)+\dfrac{1}{r}\dfrac{\partial^2\phi}{\partial\theta^2}+\dfrac{\cot g\theta}{r^2}\dfrac{\partial\phi}{\partial\theta}=\sum_{j=1}^{N}Z_{dj}n_{dj}+n_{le}+n_{he}-n_i. \end{cases} \quad \mathbf{(1)}$$

Where *r*, *θ* are the radial and angle coordinates and u_{dj}, v_{dj} are the dust fluid velocity in *r* and *θ* directions, respectively. n_d, n_d, ϕ represent the dust density and the electrostatic potential. The variables t, r, n_d, u_d, v_d, *and* ϕ are normalilized to the dust plasma frequency $\omega_p^{-1}=\sqrt{4\pi n_{d0}Z_d^2e^2/m_d}$, Debye radius $\lambda_D=\sqrt{k_BT_i/4\pi n_{d0}Z_d^2e^2}$, unperturbed equilibrium dust density n_d , effective dust acoustic velocity $C_D=\sqrt{Z_dk_BT_i/m_d}$, and k_BT/e , respectively.

Here we have denoted $\dfrac{\mathbf{1}}{T_{eff}}=\mathbf{1}\Big/ Z_{d\mathbf{0}}n_{d\mathbf{0}}\left(\dfrac{n_{i\mathbf{0}}}{T_i}+\dfrac{n_{eh\mathbf{0}}}{T_h}+\dfrac{n_{el\mathbf{0}}}{T_l}\right)$ The dimensionless number densities are

$n_{el}=\mu_1\exp(s\Phi),\ n_{eh}=\mu_2\exp(\beta_1 s\Phi),\ n_i=\mu_3\exp(-\Phi/\beta_2).$

$$\mu_1=\frac{n_{el0}}{Z_d n_{d0}},\ \mu_2=\frac{n_{eh0}}{Z_d n_{d0}},\ \mu_3=\frac{n_{i0}}{Z_d n_{d0}},\quad \beta_1=\frac{T_{el}}{T_{eh}},\ \beta_2=\frac{T_i}{T_{eff}},\ \beta=\frac{\beta_1}{\beta_2},\ s=\left(\frac{\mu_3-1}{\mu_1+\beta_1\mu_2}\right),\ \delta_1=\frac{n_{il0}}{n_{e0}},\ \delta_1=\frac{n_{ih0}}{n_{e0}}.$$

To study the dynamics of small amplitude dust-acoustic solitary waves, we use the so-called reductive perturbation method **[36].** We can then expend the variables n_d, u_d and ϕ about the unperturbed states in power series of ε (ε is a small parameter) that means, we let,

$$n_d=1+\varepsilon n_{dj1}+\varepsilon^2 n_{dj2}+\ldots,/u_d=\varepsilon u_{dj1}+\varepsilon^2 u_{dj2}+\ldots,/\phi=\varepsilon\phi_1+\varepsilon^2\phi_2+\ldots/Z_d=1+\varepsilon Z_{dj1}+\varepsilon^2 Z_{dj2}+\ldots \quad \mathbf{(2)}$$

We can rewrite **Eqs.(1-4)** taking into account **Eqs.(5-7)** and the stretched coordinates $\xi=\varepsilon^{1/2}(r-v_0 t),\tau=\varepsilon^{3/2}t,\ \eta=\varepsilon^{-1/2}\theta$, where v_0 is the wave velocity, to get the following system equations, to the lowest order in ε we have

$$n_{d1j}=-\frac{Z_{dj}}{v_0^2 m_{dj}}\phi_1,\ u_{d1j}=-\frac{Z_{dj}}{v_0 m_{dj}}\phi_1,\ v_0^2=\left[\frac{1}{(\mu_3/\beta_2)+\mu_1 s+\mu_2 s\beta_1}\right]\left(\sum_{j=1}^{N} Z_{dj}^2\Big/m_{dj}\right)\frac{\partial v_{dj}}{\partial\xi}=-\frac{1}{v_0^2\tau}\frac{Z_{dj}}{m_{dj}}\frac{\partial\phi_1}{\partial\eta} \quad \mathbf{(3)}$$

To the next order in ε we get the following set of equations;

$$\begin{cases}\dfrac{\partial n_1}{\partial\tau}-v_0\dfrac{\partial n_2}{\partial\xi}+\dfrac{\partial u_2}{\partial\xi}+\dfrac{\partial(n_{dj}u_{dj})}{\partial\xi}+\dfrac{1}{v_0\tau}\dfrac{\partial v_1}{\partial\eta}+\dfrac{1}{v_0\tau}\left(2v_1+\dfrac{1}{\eta}v_1\right)=0,\\ \dfrac{\partial u_1}{\partial\tau}-v_0\dfrac{\partial u_2}{\partial\xi}+u_1\dfrac{\partial u_1}{\partial\xi}-\sum_{j=1}^{N}\dfrac{Z_{dj}}{m_{dj}}\dfrac{\partial\phi_2}{\partial\xi}=0,\\ \dfrac{\partial^2\phi_1}{\partial\xi^2}=\sum_{j=1}^{N}Z_{dj}n_{d2}+\left(\dfrac{\mu_3}{\beta_2}+\mu_1 s+\mu_2\beta_1 s\right)\phi_2+\dfrac{1}{2}\left(-\dfrac{\mu_3}{\beta_2^2}+\mu_1 s^2+\mu_2 s^2\beta_1^2\right)\phi_1^2.\end{cases} \quad \mathbf{(4)}$$

Using **Eqs**. **(2), (3)** and eliminate $n_2, u_2, and\ \phi_2$ from **(4)**, we obtain the SKP equation,

$$\frac{\partial}{\partial\xi}\left[\frac{\partial\phi_1}{\partial\tau}+A\phi_1\frac{\partial\phi_1}{\partial\xi}+B\frac{\partial^3\phi_1}{\partial\xi^3}+\frac{1}{\tau}\phi_1\right]+C\left[\frac{\partial^2\phi_1}{\partial\eta^2}+\frac{1}{\eta}\frac{\partial\phi_1}{\partial\eta}\right]=0, \quad \mathbf{(5)}$$

$$A=-\frac{v_0^3}{2}\left[\frac{3}{v_0^4}-\frac{\mu_3}{\beta_2^2}+\mu_1 s^2+\mu_2 s^2\beta_1^2+\frac{Z_{dj}^2}{v_0^4 m_{dj}}\right]B=\frac{v_0^3}{2},\ C=\frac{1}{2v_0\tau^2},$$

It is important to point out, that if the wave propagate without the transverse perturbation, the last term in the left side of **Eq. (5)** disappear and the SKP equation **(5)** reduce to the ordinary spherical KdV equation. We can find an exact solitary wave solution for the SKP equation **(5)** by using a suitable variable transformation. In **Eq. (5),** the two terms with variable coefficient, can be canceled if we assume $\zeta=\xi-\frac{v_0}{2}\eta^2\tau,\ \phi_1=\Phi(\zeta,\tau)$. Then the SKP **Eq.(5)** is reduced to the standard KdV equation

$$\frac{\partial\Phi}{\partial\tau}+A\Phi\frac{\partial\Phi}{\partial\xi}+B\frac{\partial^3\Phi}{\partial\xi^3}=0 \quad (6) \quad \Rightarrow \quad \Phi(\zeta)=\frac{3U_0}{A}\mathbf{sech}^2\left[\sqrt{\frac{U_0}{4B}}(\zeta-U_0\tau)\right]$$

U_0 is a constant represent wave velocity. Thus we get an exact solitary wave solution of the

SKP **Eq.(6)** $\Phi(\zeta)=\frac{3U_0}{A}\mathbf{sech}^2\left[\sqrt{\frac{U_0}{4B}}\left(\xi-\left(U_0+\frac{v_0}{2}\right)\tau\right)\right]$ **(**7)

On one hand we can notice that the amplitude and wave velocity of our solitary wave described by SKP **Eq.(5**) are exclusively determined by the parameters of the system and only depending on the initial conditions. **Eq.(7)** indicates that the phase velocity of the solitary wave is angle dependent in the phase. This means that the spherical wave described by the SKP **Eq.(5**) will slightly deform as time goes on. The solution of **Eq.(7)** gives us an additional information; a spherical soliton with constant amplitude can exist if the transverse perturbation is considered.

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