Propagation and conversion of non-axisymmetric guided modes in strongly non-uniform helicon plasma

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Abstract. Systematic analytical and numerical studies of the 2nd-order wave equations describing decoupled electrostatic and helicon modes were carried out. The conversion of non-axisymmetric ($m \neq 0$) helicon modes into strongly-damped electrostatic plasma modes is investigated on the basis of the general 4th-order wave equation. For steep radial density gradient, the plasma density, at which mode conversion occurs, can be considerably lower as in weakly inhomogeneous plasma. This case is studied for a plasma column with a peaked density distribution in the center.

I. Helicon mode propagation. To analyse helicon mode propagation along a non-uniform magnetized plasma cylinder we solve the wave equation for the azimuthal component of the electric field intensity, $E_\theta(r,z,\theta,t) = E_\theta(r)\exp(-i\omega t + im\theta + ik_z z)$. This equation is obtained from the full system of Maxwell's equations for high plasma conductivity along the external magnetic field ($\mathbf{B}_0 = B_0\hat{\mathbf{z}}$) so that $E_z = 0$. For the frequency range $|\omega_{ce}| \gg \omega \gg \omega_{LH} \equiv \sqrt{|\omega_{ce}|\omega_{ci}}$ the wave equation becomes in thin-cylinder approximation ($1 \gg k_z^2 r^2$) and $m^2 = 1$ [1,2]

$$\frac{\partial^2}{\partial r^2}E_\theta + \frac{3}{r}\frac{\partial}{\partial r}E_\theta + \frac{g}{N_z^2}\left(-\frac{m}{r}\frac{\partial \ln n}{\partial r} + \frac{\omega^2}{c^2}g\right)E_\theta = 0\,, \tag{1}$$

where $g \equiv \omega_{pe}^2(r)/(\omega\omega_{ce})$, $\omega_{ce} = eB_0/m_e c$, $\omega_{pe}^2(r) = 4\pi e^2 n(r)/m_e$, $N_z = k_z c/\omega$. Eq.(1) has been studied analytically and numerically in Ref.2. The effect of density gradient becomes determinant if $\left|\frac{m}{r}\frac{\partial \ln n}{\partial r}\right| \gg \frac{\omega^2}{c^2}|g|$. In particular, the quadratic scaling, $\omega \propto k_z^2$, was confirmed by computations satisfying the above condition. The validity of the helicon approximation, $E_z = 0$, is examined by computations based on fully electromagnetic treatment solving the exact 4th order wave equation. Fig.1 shows typical field profiles for a high-density ($|g| \gg N_z^2$) plasma with a Gaussian profile (width l = 2 cm). Deviations from the helicon calculations based on Eq.(1) occur only in the outer region where Trivelpiece-Gould modes are excited due to coupling with the helicon mode. However, their contribution to the total power absorption can be only significant (in the present case not fulfilled) when the edge density is not too low. As computations further show, the helicon solution of eq.(1) deviates from the exact solution increasingly with decreasing central density indicating the helicon approximation to be no longer valid in the central region.

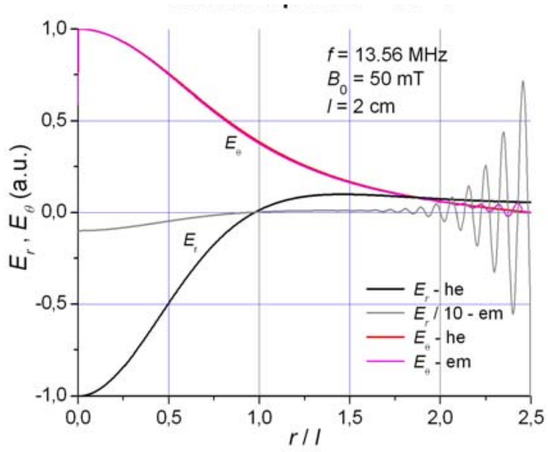

Fig.1. Electric field profiles of helicon mode (m = +1); comparison of HE (eq.(1)) and EM treatment

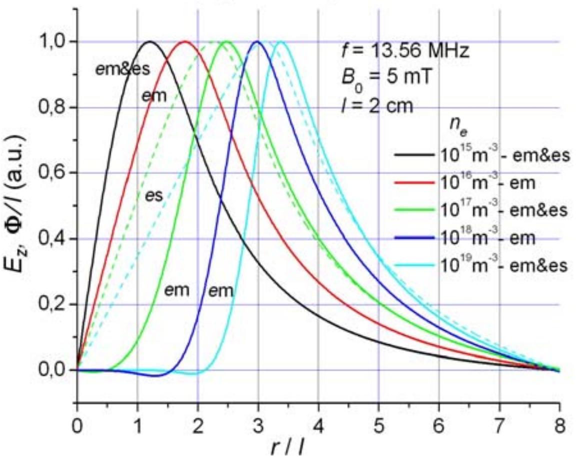

Fig.2. Electric field profiles of ES modes (m = +1) comparison of ES (eq.(2)) and EM treatment

II. Electrostatic mode propagation. The gradient density effect on the propagation of electrostatic modes is governed by the wave equation reading in terms of the potential $\Phi(r)$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right)+L_N^{-1}\frac{\partial\Phi}{\partial r}+\left[\frac{|\omega_{ce}|}{\omega}\frac{m}{r}L_N^{-1}-\frac{m^2}{r^2}-k_z^2\frac{\varepsilon_{\parallel}}{\varepsilon_{\perp}}\right]\Phi=0\,, \qquad (2)$$

where $\varepsilon_{\perp}(r)=1+\omega_{pe}^2(r)/\omega_{ce}^2$, $\varepsilon_{\parallel}(r)=1-\omega_{pe}^2(r)/\omega^2$, $L_N^{-1}(r)\equiv d\ln\varepsilon_{\perp}(r)/dr$. The results based on the treatment of Eq.(2) for the lowest radial mode for a Gaussian density profile (width l = 2 cm) are presented in Fig.2. The exact computations (4th order wave equation) agree well with the ES computations at low densities ($|g|<N_z^2$); however, considerable deviations from the electrostatic case arise in the central plasma region when the density and, thus, g increases so that $|g|>N_z^2$. As a result, the maximum power absorption (not shown here) shifts outwards; yet, the dispersion (quadratic scaling, $\omega\propto k_z^2$, at low frequencies) changes only weakly in case of the exact calculations.

III. Mode conversion. Motivated by high-density helicon experiments, in which the enhanced *rf* power absorption might be associated with the pronounced density peak observed close to the axis [6,7], the role of mode conversion in the central plasma region was studied. This mechanism is favoured by the fact that it may occur at considerably lower density ($\varepsilon_{\perp}\ll N_z^2$) [4] than in weakly inhomogeneous plasma, where $\varepsilon_{\perp}=\mathrm{O}(N_z^2)$ [5].

Let us first discuss the role of the plasma density gradient for the two modes involved in mode conversion. From the wave equation (2) follows that the wavelength of electrostatic modes can strongly increase with the density gradient if

$$\frac{|\omega_{ce}|}{\omega}\frac{m}{r}L_N^{-1}(r)-\frac{m^2}{r^2}-k_z^2\frac{\varepsilon_{\|}(r)}{\varepsilon_{\perp}(r)}\approx 0\,. \qquad (3)$$

According to (3), the helicon wavelength becames shorter in the region of strong density gradient, if $\left|\frac{m}{r}\frac{\partial \ln n}{\partial r}\right| \gg \frac{\omega^2}{c^2}|g|$, as it can be seen from eq.(1). To simulate the experimental situation, we chose a double-Gaussian density profile in the computations. The radial structure of the ES mode and helicon mode fields for such a profile is illustrated in Fig 3. We see that the ES wavelength increases and the helicon wavelength decreases in the region of strong density gradient.

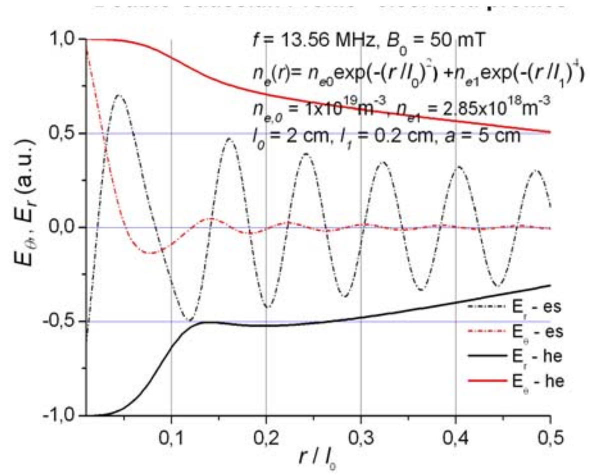

Fig.3. ES and Helicon field profiles near the axis (*m* = *+1*)

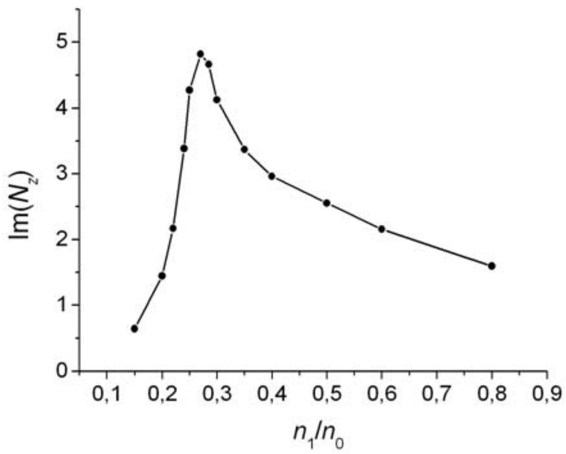

Fig.4. Damping decrement as a function of central density peak height (*EM*; *m* = *+1*)

To describe the propagation of the coupled modes in this region the 4-th order differential (wave) equation as derived from the full set of Maxwell's equation (EM) has to be taken. Coupling of ES and Helicon modes resulting in maximum power absorption is expected in the region of strong gradient if the condition (3) is satisfied. This can clearly be seen from Fig. 4, in which the dependence of the axial damping rate of the coupled mode on the ratio n_1 / n_0 is shown. It is worth emphasizing that the the *rf* power absorption and the axial damping decrement of the mode is a factor of 10 higher than for s single Gaussian profile (without central peak).

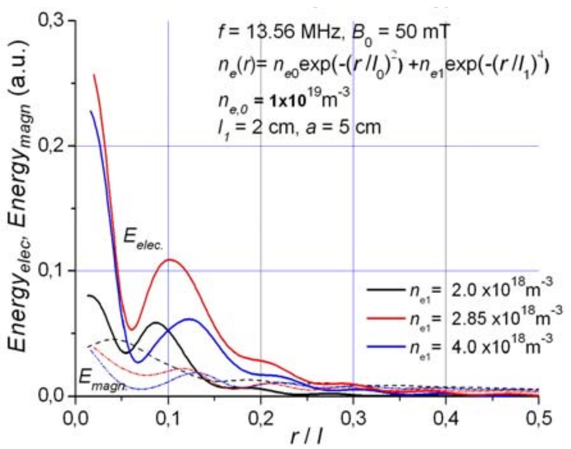

Fig.5. *EM energy profiles* (*m* = *+1*)

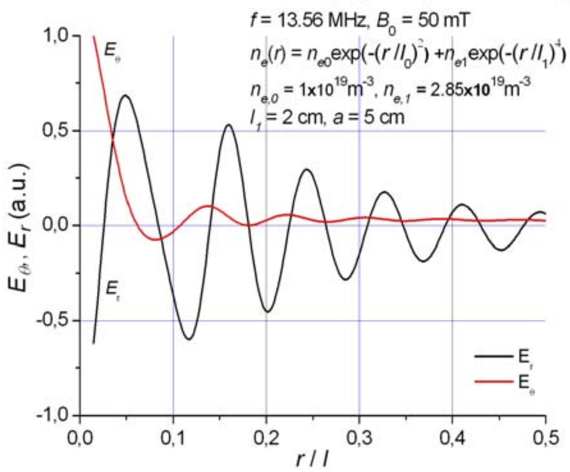

Fig.6. Electric field profiles (*EM*; *m* = *+1*)

In Fig.5 the distributions of electrostatic and magnetic energy absorption with radial distance are presented. It turns out that the electric wave energy strongly grows if the condition is fulfilled (3). The profiles of the electric field are plotted in Fig.6 for maximum power absorption. Comparing the field profiles with those obtained from the ES theory shown in Fig.3 it is obvious that the ES mode plays a dominant role for the mode coupling. The enhanced density of ES wave energy near the axis may explain the *rf* power deposition in high-density helicon plasma experiments [6,7].

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