Landau damping of Langmuir waves in non Maxwellian plasmas: effect of density

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Abstract. The non Maxwellian character of the electron distribution function for free electrons moving in the nearest neighbor ion's potential well is proved to bring about a correction to Langmuir wave dispersion relation, viz., the oscillating frequency as well as the Landau damping rate (c.f. Ref.[1]). In this note, the effect of density on the Landau damping rate for Langmuir waves is investigated and found salient for dense plasmas.

Introduction

The Langmuir wave (LW) is an electrostatic wave that propagates in the plasma with a phase velocity greater than the particles thermal velocity. The waves have a tremendous potential as they are encountered in many environments, e.g, astrophysical plasmas, fusion plasmas, plasma based acceleration schemes [2]. Bohm and Gross derived for the first time the dispersion relation for the Langmuir wave propagating in a plasma, the particles of which obey a Maxwell velocity distribution function [3]. The authors (c.f.Ref.[1]) rederived the Langmuir wave dispersion relation in the framework of the velocity distribution function first proposed by Wright and Theimer [4]. It has been found that the modified dispersion relation and consequently the characteristics of the wave depart appreciably from the Maxwellian one, for dense and cool plasmas. Moreover, as pointed out by Landau [5], a wave propagating in a plasma, interacts inevitably with plasma particles and loses energy in favour of the latters. In this note, the Landau damping rate is investigated and show that the departure from the non Maxwellian distribution function, the latter is noticeable for cold and dense plasmas.

Model Equations

In the work by Ouazene and Annou (c.f. Ref.[1]), it has been shown that the appropriate Langmuir dispersion relation for a non Maxwellian distribution function is given by the expression,

$$1=\frac{\omega_p^2}{k^2}\left[1+\frac{3}{2}\frac{k^2 v_{th}^2}{\omega^2}\left(1+\varepsilon\right)\right]+i\pi\frac{\omega_p^2}{k^2}\tilde{f}'\left(v_x=\frac{\omega}{k}\right) \qquad (1)$$

where, ε is the correction that arose due to the non-Maxwellian distribution function and $v_{th}^2 = \frac{2T}{m}$, is the squared thermal velocity. The complex frequency may be written as, $\omega = \omega_r + i\delta$, where the imaginary part is small $\delta \langle\langle \omega_r$, consequently the squared frequency is cast as, $\omega^2 \approx \omega_r^2 + 2i\delta\omega_r$. To evaluate the Landau damping contribution, we consider the non-Maxwellian velocity distribution function [4] to get the Landau damping rate,

$$\frac{\delta}{\omega_p} = \frac{-\pi^{1/2} t_0^3}{4\sqrt{2}} \frac{1}{\left(k\lambda_D^*\right)} \left(\frac{n}{n^*}\right)^{3/2} \exp\left(-\frac{1}{2k^2\lambda_D^{*2}} \frac{n}{n^*}\right)[A] - \frac{-\pi^{1/2} t_0^3}{8\sqrt{2}} \frac{1}{\left(k\lambda_D^*\right)} \left(\frac{n}{n^*}\right)^{3/2} \exp(t_0)[B] \qquad (2)$$

where,

$$A = \ln(t_0) + C + Ei(t_0) + \left(2t_0^{-3} + t_0^{-2} + t_0^{-1}\right)\exp(t_0) + \frac{1}{t_0}\frac{1}{2k^2\lambda_D^{*2}}\frac{n}{n^*} - \frac{1}{2t_0^2}\frac{1}{2k^4\lambda_D^{*4}}\left(\frac{n}{n^*}\right)^2$$
$$+ \frac{1}{2k^2\lambda_D^{*2}}\frac{n}{n^*} + t_0 + \frac{1}{4}\left(\frac{1}{2k^2\lambda_D^{*2}}\frac{n}{n^*} + t_0\right)^2 + \frac{1}{18}\left(\frac{1}{2k^2\lambda_D^{*2}}\frac{n}{n^*} + t_0\right)^3 + \frac{1}{96}\left(\frac{1}{2k^2\lambda_D^{*2}}\frac{n}{n^*} + t_0\right)^4$$

and,

$$B = \frac{4}{\left(\frac{1}{2(k\lambda_D^*)^2}\frac{n}{n^*} + t_0\right)^3} + \frac{2}{\left(\frac{1}{2(k\lambda_D^*)^2}\frac{n}{n^*} + t_0\right)^2} + \frac{1}{\sqrt{\frac{1}{2} + t_0 + \frac{1}{2\left(k\lambda_D^*\right)^2}\frac{n}{n^*}}\left(\frac{1}{2} + \sqrt{\frac{1}{2} + t_0 + \frac{1}{2\left(k\lambda_D^*\right)^2}\frac{n}{n^*}}\right)}$$
$$- \frac{1}{\sqrt{\frac{1}{2} + t_0 + \frac{1}{2\left(k\lambda_D^*\right)^2}\frac{n}{n^*}}\left(-\frac{1}{2} + \sqrt{\frac{1}{2} + t_0 + \frac{1}{2\left(k\lambda_D^*\right)^2}\frac{n}{n^*}}\right)}$$

where $t_0 = \cap^2 /$, $\cap = T^{21}$, C is the Euler constant and $r_0 = (4M\ \ )^{21\beta}$ is the average inter-particle distance, n being the number density. Θ_D^* is $\sqrt{2}$ times the Debye length at a reference density $n^* = 10^6\ cm^{-3}$ and T=10 K. Is plotted in figures 1 and 2 the normalized Landau damping rate versus the normalized wave number for T=10 K and two densities, i.e., n=10^7 cm^{-3}, n=10^9 cm^{-3}. It is found that the Landau damping rate increases (in magnitude) for an increasing density. Besides, the absolute value of the Landau damping rate calculated at the minimum of the curves for the above mentioned densities is, 1.21 (21%) and 1.58 (58%) times the absolute value of Landau damping rate calculated with the Maxwellian distribution function, respectively. It is shown also that the pick of each curve is shifted to short wavelengths; this shift being more pronounced for low temperatures.

Conclusion

In conclusion, we recall that for denser plasmas the departure from Landau damping rate in Maxwellian plasmas of the proposed expression of Landau damping rate is clear. At low temperatures the latter being severe and the trough of the curves is shifted to short wavelengths.

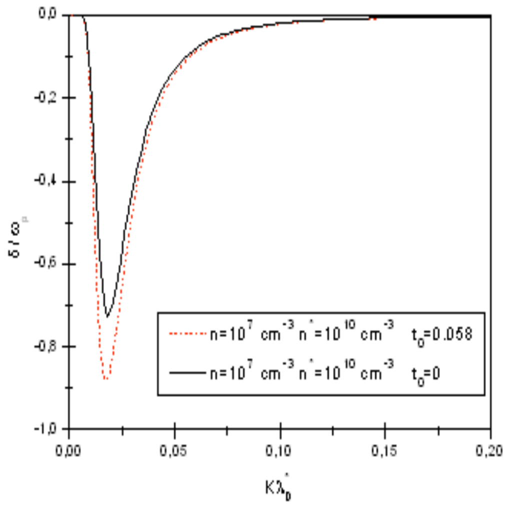

Figure 1

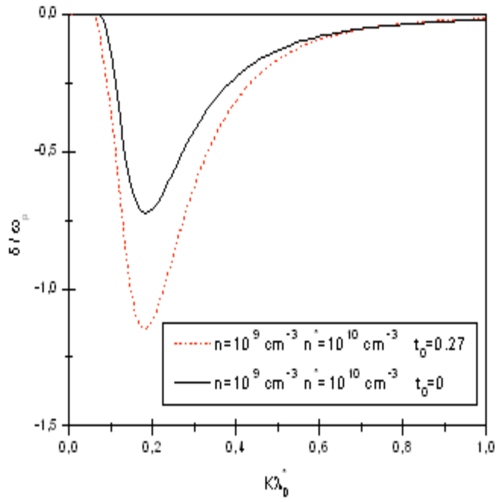

Figure 2

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