Theory of radial correlation Doppler reflectometry

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Introduction. Radial correlation Doppler reflectometry (RCDR) is a new diagnostic technique which has been developed at ASDEX Upgrade [1]. In this method a probing microwave beam at two probing frequencies is launched into the plasma with a finite tilting angle with respect to the density gradient. A back-scattered signal in the two frequency channels is collected by a nearby antenna and then analyzed using the correlation method. Due to the backscattering experimental geometry at oblique probing this technique is believed to be less sensitive to the poor localized forward scattering component leading to gradual decay of correlation theoretically predicted for the radial correlation reflectometry (RCR) in the frame of the realistic 2D model [2]. Therefore in principle this diagnostic should combine high enough localization of Doppler reflectometry measurements of plasma rotation velocity with sensitivity to radial wave number spectrum of turbulence enhanced in comparison to the radial correlation reflectometry. It is expected to provide localized measurements of several plasma characteristics such as the radial electric field, the radial correlation length of the density fluctuations and their poloidal wave number spectrum. Though, a number of observations provided by this technique or utilizing similar justification were reported recently [1,3], a lack of the theoretical understanding of transition from standard RCR to RCDR remains appealing for the systematic analytical study. In this work we investigate analytically in the frame of theory linear in density perturbation amplitude the scheme of RCDR, focusing on the method feasibility for the turbulence radial wave number spectrum or at least correlation length determination. Performing the analysis, we restrict ourselves to the case of the O- mode RCDR and for the sake of simplicity neglect the magnetic surface curvature. Based on the expression obtained for the diagnostic's cross-correlation function the possibility of the turbulence radial wave number spectrum reconstruction is discussed. The possible role of rotation velocity shear in suppression of correlation in different frequency channels is mentioned.

The model. In order to simplify the analysis of the O- mode RCDR in 2D model we introduce a slab geometry with coordinates (x, y), where x and y are «radial» and «poloidal» coordinates, and assume the linear dependence of the unperturbed density on x: $n_e(x) = n_c\, x/L_0$ where $n_c = m_e\omega_0^2/4\pi e^2$, $L_0^{-1} = \partial \ln n(x)/\partial x|_{x_c}$ and ω_0 is the probing wave frequency. According to reciprocity theorem [4] the amplitude of the reflectometry signal at frequency $\omega = \omega_0 + \Omega$ in the linear regime of the scattering is given by $A_S(\omega) = i\dfrac{e^2}{4m_e\omega}\int E_a^2(\omega_0,\vec{r})\tilde{n}_\Omega(\vec{r})d\vec{r}$ where $E_a(\omega_0,\vec{r})$ is the O-wave amplitude in an unperturbed plasma at frequency ω_0, $\tilde{n}_\Omega(\vec{r})$ represents the density fluctuation at frequency Ω. The CCF of two scattered signals at frequencies ω and $\omega' = \omega_1 + \Omega'$ is given by

$$CCF \triangleq < A_S(\omega)A_S^*(\omega') >= \frac{e^4}{(4m_e\omega)^2}\iint d\vec{r}d\vec{r}'E_a^2(\omega_0,\vec{r})E_a^2(\omega_1,\vec{r}') < \tilde{n}_\Omega\tilde{n}_{\Omega'}^* > \left(\vec{r}-\vec{r}', \frac{\vec{r}+\vec{r}'}{2}\right) \quad (1)$$

where $<...>$ means statistical averaging over an ensemble of the fluctuations. In weakly inhomogeneous plasma the average cross-correlation spectrum $<\tilde{n}_\Omega\tilde{n}_{\Omega'}^*>$, exhibiting much stronger dependence on the first argument $\vec{r}-\vec{r}'$ than on the second - $(\vec{r}+\vec{r}')/2$ associated with the plasma poloidal velocity inhomogeneity, can be represented in terms of wave number spectrum as

$$< \tilde{n}_\Omega\tilde{n}_{\Omega'}^* >= 2\pi\delta(\Omega-\Omega')\int\frac{d\kappa dq}{(2\pi)^2}\exp\left[i\kappa(x-x') + iq(y-y')\right]\delta n_\Omega^2\left(\kappa, q, \frac{x+x'}{2}\right) \quad (2)$$

where κ, q are the «radial» and «poloidal» components of the fluctuation wave vector, and $\delta n_\Omega^2(\kappa,q,x) = \pi l_{cx}l_{cy}\delta n_0^2(x)\exp\left[-\kappa^2 l_{cx}^2/4 - q^2 l_{cy}^2/4 - (\Omega - qV)^2 t_c^2/4\right]$ with l_{cx}, l_{cy}, t_c, V, being the radial and the poloidal correlation lengths, the correlation time of the fluctuations and the turbulence poloidal velocity. In the case of the linear density profile E_a is given by $E_a(\omega,x,y) = \int\frac{dk_y}{2\pi}f(k_y)\exp(ik_y y)Ai\left[\{x - L(k_y)\}/\ell\right]$ where $f(k_y)$ is the Gaussian antenna diagram $f(k_y) = (2\sqrt{\pi}\rho)^{1/2}\exp\left[-(k_y - K)^2\rho^2/2\right]$ over the poloidal wave number k_y with 2ρ being a probing microwave waist, $K = \omega/c\sin\vartheta$ and ϑ denoting the tilting angle of the probing wave with respect to the density gradient, $Ai(...)$ - Airy function, $\ell = \left(c^2L_0/\omega_0^2\right)^{1/3}$ and $L(k_y) = L(1 - k_y^2c^2/\omega^2)$, $L\omega_0^2 = L_0\omega^2$. For the sake of simplicity the homogeneous radial profiles of both the fluctuations and the poloidal velocity $\delta n_0^2, V = const$ are assumed. Substituting (2) and $E_a(\omega,x,y)$ into (1) and using the integral representation of the Airy function affords evaluating of the multiple integral by saddle point method that, after neglecting the non-Bragg contributions to the scattering signal [2], yields:

$$CCF \simeq \delta n_0^2 P_i \ell^3 \rho^2 \left(\frac{e^2}{m_e c^2}\right)^2 \exp\left\{-i\frac{4}{3}\frac{L\Delta\omega}{c}\right\} J(\Delta\omega),$$
$$J = l_{cy} l_{cx} \int d\kappa dq \frac{\exp\left\{-i\Delta\omega/\omega_0\left[q^2 L_0 c/\omega_0 - 2\kappa L_0\right] - \kappa^2 l_{cx}^2/4 - (q-2K)^2\rho^2/2 - q^2 l_{cy}^2/4\right\}}{\sqrt{\kappa^2\rho^4 + \left[2L_0 c/\omega_0\kappa + L_0\left(qc/\omega_0\right)^2\right]^2}} \quad (3)$$

where P_i is the probing wave power, $\Delta\omega = \omega_0 - \omega_1$. There are two contributions to the integral over κ in (3). The first one comes from a branching point $\kappa_{FS}^* = -q^2 c/\left[2\omega(1 \pm i\rho^2\omega_0/(2L_0 c))\right]$ and corresponds to a non-local forward scattering (FS) all over the path of the probing wave. When the cut off is situated in the wave zone of the antenna the fluctuations able to produce the FS signal coming back to the antenna should satisfy the condition $\kappa_{FS}^* \simeq -q^2 c/(2\omega)$. This condition is broadened due to the finite radii of the antenna. With antenna radius growing and transition of the cut-off to the near zone the condition turns less restrictive. The second contribution being originated due to Bragg Back scattering (BS) near the cut-off comes from vicinity of the saddle point $\kappa_{BS}^* = i4\Delta\omega/\omega_0 \cdot L/l_{cx}^2$. The relative role of these two contribution is controlled by a dimensionless parameter depending on the tilting angle of the probing wave $\sigma(\vartheta) = \kappa_{FS}^* l_{cx} = 2\omega_0/c \cdot l_{cx}\left(1+\rho^4\omega_0^2/(2L_0 c)^2\right)^{-1/2}\sin^2\vartheta$. For the tilting angles $\vartheta < \vartheta^*$ ensuring an inequality $\sigma < 1$ the contribution of the fluctuations responsible for FS dominates, the contribution of the saddle point is negligible and, thus, the CCF possesses the large-weighted logarithmic tail (or slowly decay with $\Delta\omega$ increasing). In the opposite case of the oblique enough propagation of the probing wave ($\vartheta > \vartheta^*$, $\sigma > 1$) the spectral density of the fluctuations capable of producing the FS contribution is exponentially small and the measurements of the radial correlation length are possible. In Figs. 1 and 2 the dependence of the normalized CCF, defined by (3), and evaluated numerically, versus the dimensionless cut off separation $\Delta = 4(\Delta\omega/\omega_0)(L_o/l_{cx})$ is given for two scenarios of the RCDR experiments on ASDEX-U, referred below as “near antenna zone” ($L_0 = 30\ cm$, $\rho = 4cm$, $l_{cx} = 0.3\ cm$, $\omega/(2\pi) = 65$ GHz, $\rho^2\omega_0/(2L_0 c) \simeq 3.7 \gg 1$) and “wave zone” ($L_0 = 120\ cm$, $\rho = 4cm$, $l_{cx} = 0.3\ cm$, $\omega/(2\pi) = 75$ GHz, $\rho^2\omega_0/(2L_0 c) \simeq 0.8 < 1$). As we can see in Figs. 1, 2, the tilting angles, at which CCF slow decay with Δ increasing is suppressed, is larger when the cut off is in the antenna near zone than when it is in the wave zone. This computational result well agrees with prediction of the criterion $\sigma(\vartheta) > 1$. It is expressed by a stronger suppression

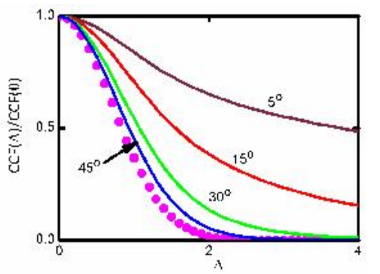

Fig.1. Normalized *CCF* versus the dimensionless cut off separation for the different tilting angles. Circles – the correlation function of the fluctuations; $\vartheta^* = 30^o$, $\rho^2\omega_0/(2L_o c) \simeq 3.7$ (near zone).

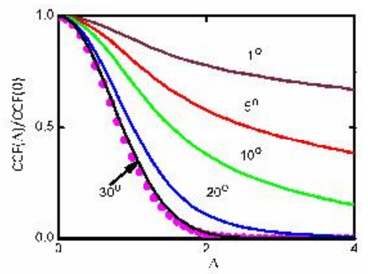

Fig.2. Normalized *CCF* versus the dimensionless cut off separation for the different tilting angles. Circles – the correlation function of the fluctuations; $\vartheta^* = 20^o$, $\rho^2\omega_0/(2L_o c) \simeq 0.8$ (wave zone).

of the poor localized FS component of the DR signal in the case of wave zone cut off position. What can be anticipated as well is that the strong enough plasma rotation velocity shear, leading to decorrelation of two nearby radial points contributions into the correlation function of two signals, can suppress the slowly decaying contribution of the forward scattering. This phenomenon will be analyzed in detail elsewhere. At the close of the section we note that in the case of isotropic long scale $(l_c \gg c/\omega)$ turbulence the RCDR, operating with the probing wave propagating obliquely, allows measuring of the radial correlation length and the radial wave number spectrum corresponding only to the tail of the poloidal wave number spectrum.

Conclusions. In this paper we have investigated analytically in the frame of theory linear in density perturbation amplitude the experimental scheme of radial correlation Doppler reflectometry, focusing on the method feasibility for the turbulence radial wave-number spectrum or at least correlation length determination. As shown in the case of large incidence angles satisfying the derived criterion suppression of poor localized forward scattering contribution to the Doppler reflectometry signal leads to fast decay of correlation in the two frequency channels similar to that expected from the turbulence correlation function decay. Based on the expression obtained for the diagnostic's cross-correlation function the possibility of the turbulence radial wave-number spectrum measurements for the fluctuations possessing large poloidal wave number is confirmed.

Acknowledgments. *Financial support by RFBR grant 09-02-00453, NWO-RFBR Centre of Excellence grant (047.018.002), and scientific school grant 6214.2010.2 is acknowledged.*

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